Permutations

Exercise 8A

Q. 1. Compute:

(i) $\frac{9!}{(5!)\times(3!)}$!

(ii) $\frac{32!}{29!}$

(iii) $\frac{(12!)-(10!)}{9!}$

Answer : (i) To Find : Value of $\frac{9!}{(5!)\times(3!)}$

Formulae :

- $n! = n \times (n-1)!$
- $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let,

$$x = \frac{9!}{(5!) \times (3!)}$$

By using above formula, we can write,

$$\therefore x = \frac{9 \times 8 \times 7 \times 6 \times (5!)}{(5!) \times (3 \times 2 \times 1)}$$

Cancelling (5!) from numerator and denominator we get,

$$\therefore x = \frac{9 \times 8 \times 7 \times 6}{3 \times 2 \times 1}$$

∴x = 504

Conclusion : Hence, value of the expression $\frac{9!}{(5!) \times (3!)}$ is 504.

(ii) To Find : Value of $\frac{32!}{29!}$

Formula : $n! = n \times (n-1)!$

Let,

$$x = \frac{32!}{29!}$$

By using the above formula we can write,

$$\therefore x = \frac{32 \times 31 \times 30 \times (29!)}{29!}$$

Cancelling (29!) from numerator and denominator,

∴x = 32 × 31 × 30

∴x = 29760

Conclusion : Hence, the value of the expression $\frac{32!}{29!}$ is 29760.

(iii) To Find : Value of $\frac{(12!) - (10!)}{9!}$

Formula : $n! = n \times (n-1)!$

Let,

$$x = \frac{(12!) - (10!)}{9!}$$

By using the above formula we can write,

$$\therefore x = \frac{[12 \times 11 \times 10 \times (9!)] - [10 \times (9!)]}{9!}$$

Taking (9!) common from numerator,

$$\therefore x = \frac{(9!)[(12 \times 11 \times 10) - 10]}{9!}$$

Cancelling (9!) from numerator and denominator,

∴x = (12 × 11 × 10) - 10

∴x = 1310

Conclusion : Hence, the value of the expression $\frac{(12!) - (10!)}{9!}$ is 1310.

Q. 2. Prove that LCM {6!, 7!, 8!} = 8!

Answer : To Prove : LCM {6!, 7!, 8!} = 8!

Formula : $n! = n \times (n-1)!$

LCM is the smallest possible number that is a multiple of two or more numbers.

Here, we observe that (8!) is the first number which is a multiple of all three given numbers i.e. 6!, 7! and 8!.

$$1 \times (8!) = 8!$$

$$8 \times (7!) = 8!$$

$$8 \times 7 \times (6!) = 8!$$

Therefore, 8! is the LCM of {6!, 7!, 8!}

Conclusion : Hence proved

Q. 3. Prove that $\frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145!}{12!}$ **.**

Answer : To Prove :

$$\frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145}{12!}$$

Formula : n! = n × (n - 1)!

$$L.H.S. = \frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!}$$

$$= \frac{12 \times 11}{12 \times 11 \times (10!)} + \frac{12}{12 \times (11!)} + \frac{1}{12!}$$

$$= \frac{132}{12!} + \frac{12}{12!} + \frac{1}{12!}$$

$$= \frac{145}{12!}$$

= R.H.S.

∴L.H.S. = R.H.S.

Conclusion : $\therefore \frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145}{12!}$

Q. 4. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$', find the value of x.

Answer : Given Equation :

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

To Find : Value of x.

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\therefore \frac{8 \times 7}{8 \times 7 \times 6!} + \frac{8}{8 \times 7!} = \frac{x}{8!}$$

By using the above formula we can write,

$$\therefore \frac{56}{8!} + \frac{8}{8!} = \frac{x}{8!}$$

$\therefore \frac{64}{8!} = \frac{x}{8!}$

Cancelling (8!) from both the sides,

$\therefore x = 64$

Conclusion : Value of x is 64.

Q. 5. Write the following products in factorial notation:

(i) 6 × 7 × 8 × 9 × 10 × 11 × 12
(ii) 3 × 6 × 9 × 12 × 15

Answer : (i) Formula : $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let,

$x = 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$

Multiplying and dividing by $(5 \times 4 \times 3 \times 2 \times 1)$

$\therefore x = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$

From the above formula,

$x = \frac{12!}{5!}$

Conclusion :

$\therefore (12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6) = \frac{12!}{5!}$

(ii) Formula : $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let,

$x = 3 \times 6 \times 9 \times 12 \times 15$

Above equation can be written as

$x = 3(1) \times 3(2) \times 3(3) \times 3(4) \times 3(5)$

$\therefore x = 3^5 \times (5 \times 4 \times 3 \times 2 \times 1)$

By using above formula,

$\therefore x = 3^5 \times (5!)$

Conclusion :

$\therefore (3 \times 6 \times 9 \times 12 \times 15) = 3^5 \times (5!)$

Q. 6. Which of the following are true of false?

(i) (2 + 3) ! = 2 ! + 3!
(ii) (2 × 3)! = (2!) × (3!)

Answer : Option (i) and (ii) both are <u>false</u>

Proofs <u>:</u>

For option (i),

L.H.S. = (2 + 3)! = (5!) = 120

R.H.S. = (2!) + (3!) = 2 + 6 = 8

∴ L.H.S. ≠R.H.S.

For option (ii),

L.H.S. = (2 × 3)! = (6!) = 720

R.H.S = (2!) × (3!) = 4 × 6 = 24

∴L.H.S. ≠R.H.S.

Important Notes : for any two whole numbers a and b,

- $(a + b)! \neq (a!) + (b!)$
- $(a \times b)! \neq (a!) \times (b!)$

Q. 7. If (n + 1) ! = 12 × (n – 1) !, find the value of n.

Answer : Given Equation :

(n + 1)! = 12 × (n - 1)!

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

(n + 1)! = 12 × (n - 1)!

By using above formula we can write,

∴(n + 1) × (n) × (n - 1)! = 12 × (n - 1)!

Cancelling the term (n - 1)! from both the sides,

∴(n + 1) × (n) = 12 …….. eq(1)

∴(n + 1) × (n) = (4) × (3)

Comparing both the sides, we get,

∴n = 3

Conclusion : Value of n is 3.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 12 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 8. If (n + 2) ! = 2550 × n!, find the value of n.

Answer : Given Equation :

$(n + 2)! = 2550 \times n!$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

(n + 2)! = 2550 × n!

By using above formula we can write,

∴ $(n + 2) \times (n + 1) \times (n!) = 2550 \times n!$

Cancelling the term $(n)!$ from both the sides,

∴ $(n + 2) \times (n + 1) = 2550$

∴ $(n + 2) \times (n + 1) = (51) \times (50)$

Comparing both the sides, we get,

∴ $n = 49$

Conclusion : Value of n is 49.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 2550 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 9. If (n + 3) ! = 56 × (n + 1) !, find the value of n.

Answer : Given Equation :

$$(n + 3)! = 56 \times (n + 1)!$$

To Find : Value of n

Formula : $n! = n \times (n - 1)!$

By given equation,

$(n + 3)! = 56 \times (n + 1)!$

By using above formula we can write,

∴ $(n + 3) \times (n + 2) \times (n + 1)! = 56 \times (n + 1)!$

Cancelling the term $(n + 1)!$ from both the sides,

∴ $(n + 3) \times (n + 2) = 56$

∴ $(n + 3) \times (n + 2) = (8) \times (7)$

Comparing both the sides, we get,

∴ n = 5

Conclusion : Value of n is 5.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 56 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 10. If $\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!}$ **= 2 : 1, find the value of n.**

Answer : Given Equation :

$$\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!} = 2:1$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{n!}{(2!)\times(n-2)!}:\frac{n!}{(4!)\times(n-4)!} = 2:1$$

$$\therefore \frac{\frac{n!}{(2!)\times(n-2)!}}{\frac{n!}{(4!)\times(n-4)!}} = \frac{2}{1}$$

$$\therefore \frac{n!}{(2!)\times(n-2)!}\times\frac{(4!)\times(n-4)!}{n!} = 2$$

By using above formula,

$$\therefore \frac{(4\times 3\times 2!)\times(n-4)!}{(2!)\times[(n-2)\times(n-3)\times(n-4)!]} = 2$$

Cancelling terms (n - 4)! And (2!),

$$\therefore \frac{(4 \times 3)}{[(n-2) \times (n-3)]} = 2$$

∴ (n - 2) × (n - 3) = 6

∴ (n - 2) × (n - 3) = (3) × (2)

By comparing both the sides,

$$\therefore n = 5$$

Conclusion : Value of n is 5.

Note : Instead of taking product of two brackets in eq(1), it is easy to convert the constant term that is 6 into product of two consecutive numbers and then by observing two sides of equation we can get value of n.

Q. 11. If $\frac{(2n)!}{(3!)\times(2n-3)!} : \frac{n!}{(2!)\times(n-2)!}$ = 44 : 3, find the value of n.

Answer : Given Equation :

$$\frac{(2n)!}{(3!) \times (2n-3)!} : \frac{n!}{(2!) \times (n-2)!} = 44:3$$

To Find : Value of n

Formula : $n! = n \times (n-1)!$

By given equation,

$$\frac{(2n)!}{(3!) \times (2n-3)!} : \frac{n!}{(2!) \times (n-2)!} = 44:3$$

$$\therefore \frac{\frac{(2n)!}{(3!) \times (2n-3)!}}{\frac{n!}{(2!) \times (n-2)!}} = \frac{44}{3}$$

$$\therefore \frac{(2n)!}{(3!) \times (2n-3)!} \times \frac{(2!) \times (n-2)!}{n!} = \frac{44}{3}$$

By using above formula,

$$\therefore \frac{(2n) \times (2n-1) \times (2n-2) \times (2n-3)!}{(3 \times 2!) \times (2n-3)!} \times \frac{(2!) \times (n-2)!}{n \times (n-1) \times (n-2)!} = \frac{44}{3}$$

Cancelling terms (n - 2)! , (2!), (2n - 3)! & n, we get,

$$\therefore \frac{2 \times (2n-1) \times 2(n-1)}{3} \times \frac{1}{(n-1)} = \frac{44}{3}$$

….. taking 2 common from the term (2n - 2)

$$\therefore (2n-1) = \frac{44 \times 3}{3 \times 2 \times 2}$$

∴ (2n - 1) = 11

∴n = 6

Conclusion : Value of n is 6.

Q. 12. Evaluate $\frac{n!}{(r!)\times(n-r)!}$, when n = 15 and r = 12.

Answer : Given : n = 15 and r = 12

To Find : Value of $\frac{n!}{(r!) \times (n-r)!}$ at given n and r

Formula :

- $n! = n \times (n-1)!$
- $n! = n \times (n-1) \times (n-2) \ldots \ldots \ldots \ldots 3 \times 2 \times 1$

Let ,

$$x = \frac{n!}{(r!) \times (n-r)!}$$

Substituting n = 15 and r = 12 in above equation,

$$\therefore x = \frac{(15!)}{(12!) \times (15-12)!}$$

$$\therefore x = \frac{(15!)}{(12!) \times (3)!}$$

By using above formula,

$$\therefore x = \frac{15 \times 14 \times 13 \times 12!}{(12!) \times (3 \times 2 \times 1)}$$

Cancelling (12!) from numerator & denominator,

$$\therefore x = \frac{15 \times 14 \times 13}{3 \times 2 \times 1}$$

∴x = 455

Conclusion : Value of $\frac{n!}{(r!) \times (n-r)!}$ at n = 15 and r = 12 is 6.

Q. 13. Prove that (n + 2) × (n!) + (n + 1) ! = (n!). (2n + 3)

Answer : To Prove : $(n + 2) \times (n!) + (n + 1)! = (n!) \times (2n + 3)$

Formula : $n! = n \times (n-1)!$

L.H.S. = (n + 2) × (n!) + (n + 1)!

= (n + 2) × (n!) + (n + 1) × (n!)

= (n!) × [(n + 2) + (n + 1)]

= (n!) × (2n + 3)

= R.H.S.

∴L.H.S. = R.H.S.

Conclusion : (n + 2) × (n!) + (n + 1)! = (n!) × (2n + 3)

Q. 14. Prove that

(i) $\frac{n!}{r!}$ **= n(n – 1) (n – 2) …. (r + 1)**

(ii) (n – r + 1). $\frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

(iii) $\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$

Answer :

(i) To Prove : $\frac{n!}{r!} = n(n-1)(n-2)\ldots.(r + 1)$

Formula : $n! = n \times (n-1)!$

$$L.H.S. = \frac{n!}{r!}$$

Writing (n!) in terms of (r!) by using above formula,

$= \frac{n(n-1)(n-2)\ldots\ldots(r+1)(r!)}{r!}$

Cancelling (r!),

= n(n - 1)(n - 2)…. (r + 1)

= R.H.S.

∴ LHS = RHS

Note : In permutation and combination r is always less than n, so we can write n! in terms of r! by using given formula.

(ii) To Prove : $(n - r + 1).\frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

Formula : $n! = n \times (n-1)!$

$$L.H.S. = (n - r + 1)\frac{n!}{(n - r + 1)!}$$

By using above formula,

$$= (n - r + 1)\frac{n!}{(n - r + 1)(n - r)!}$$

Cancelling (n - r + 1),

$$= \frac{n!}{(n - r)!}$$

= R.H.S.

∴LHS = RHS

(iii) To Prove : $\frac{n!}{(r!) \times (n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)!} = \frac{(n+1)!}{(r!) \times (n-r+1)!}$

Formula : $n! = n \times (n - 1)!$

$$L.H.S. = \frac{n!}{(r!) \times (n - r)!} + \frac{n!}{(r - 1)! \times (n - r + 1)!}$$

By using above formula,

$$= \frac{(n - r + 1)n!}{(r!) \times (n - r + 1)(n - r)!} + \frac{(r) \times n!}{(r)(r - 1)! \times (n - r + 1)!}$$

$$= \frac{(n - r + 1)n!}{(r!) \times (n - r + 1)!} + \frac{(r) \times n!}{(r)! \times (n - r + 1)!}$$

Taking $\left(\frac{(n!)}{(r!) \times (n-r+1)!}\right)$ common,

$$= \frac{n!}{(r!) \times (n - r + 1)!}(n - r + 1 + r)$$

$$= \frac{(n + 1) \times n!}{(r!) \times (n - r + 1)!}$$

$$= \frac{(n + 1)!}{(r!) \times (n - r + 1)!}$$

= R.H.S.

∴ LHS = RHS

Exercise 8B

Q. 1. There are 10 buses running between Delhi and Agra. In how many ways can a man go from Delhi to Agra and return by a different bus?

Answer : Given: 10 buses running between Delhi and Agra.

To Find: Number of ways a man can go from Delhi to Agra and return by a different bus.

There are 10 buses running between Delhi and Agra so there are 10 different ways to go from Delhi to Agra. The man cannot return from the same bus he went so number of ways are reduced to 9.

These second event occur in completion of first event so there are: $10 \times 9 = 90$ ways in which a man can go from Delhi to Agra and return by a different bus.

Q. 2. A, B and C are three cities. There are 5 routes from A to B and 3 routes from B to C. How many different routes are there from A to C via B?

Answer : Given: 5 routes from A to B and 3 routes from B to C.

To find: number of different routes from A to C via B.

Let E_1 be the event : 5 routes from A to B

Let E_2 be the event : 3 routes from B to C

Since going from A to C via B is only possible if both the events E_1 and E_2 occur simultaneously.

So there are $5 \times 3 = 15$ different routes from A to C via B.

Q. 3. There are 12 steamers plying between A and B. In how many ways could the round trip from A be made if the return was made on (i) the same steamer? (ii) a different steamer?

Answer : Given: 12 steamers plying between A and B.

To find: number of ways the round trip from A can be made.

(i) The steamer which will go from A to B will be returning back, since the given condition is that same steamer should return.

There are 12 steamers available so there are 12 different ways to make around trip between A & B if done on same steamer.

(ii) If the return trip is done on different steamer than the once used in trip on going from A to B then the possible number of ways are:$12 \times 11=132$.

(11 because the once used in going from A to B cannot be used in returning hence, reduced by 1.)

Q. 4. In How many ways can 4 people be seated in a row containing 5 seats?

Answer : To find : Number of ways in which 4 people can be seated in a row containing 5 seats.

The possible number of ways in which 4 people be seated in a row containing 5 seats $={}^7P_4$ (There are 5 places to be filled with 4 persons where arrangement doesn't matter.)

${}^7P_4 = \frac{7!}{(7-4)!}$ …(${}^nP_r = \frac{n!}{(n-r)!}$)

$= \frac{7!}{3!}$

$=7 \times 6 \times 5 \times 4$

$= 840$

Q. 5. In How many ways can 5 ladies draw water from 5 taps, assuming the no tap remains unused?

Answer : To find: number of ways in which 5 ladies draw water from 5 taps.

Condition: no tap remains unused

The condition given is that no well should remain unused.

So possible number of ways are: $5 \times 4 \times 3 \times 2 \times 1=120$.

Q. 6. In a textbook on mathematics there are three exercises A, B and C consisting of 12, 18 and 10 questions respectively. In how many ways can three questions be selected choosing one from each exercise?

Answer : Given: three exercises A, B and C consisting of 12, 18 and 10 questions respectively.

To find: number of ways in which three questions be selected choosing one from each exercise.

Ways of selecting one question from exercise A:$^{12}C_1$ (way of selecting one element from n number of elements.)

Ways of selecting one question from exercise B:$^{18}C_1$

Ways of selecting one question from exercise C:$^{10}C_1$

So number of ways of choosing one question from each exercise A ,B,C
$=^{12}C_1 \times {}^{18}C_1 \times {}^{10}C_1$

$=12 \times 18 \times 10$

$=2160$

Q. 7. In a school, there are four sections of 40 students each in XI standard. In how many ways can a set of 4 student representatives be chosen, one from each section?

Answer : Given: there are four sections of 40 students each in XI standard.

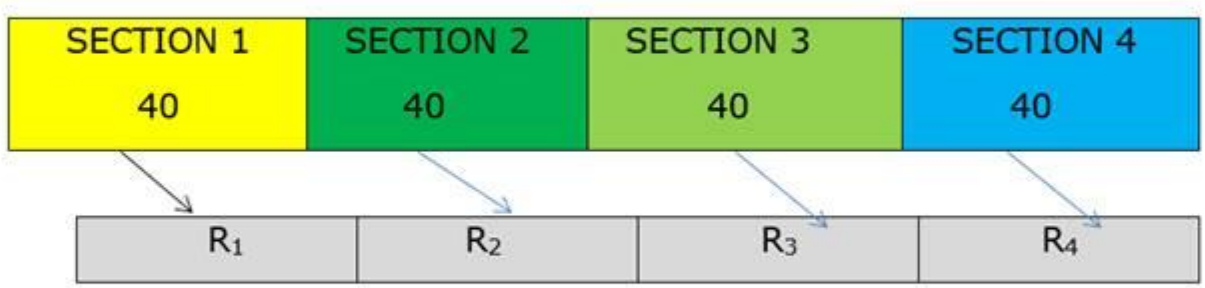
To find : number of ways in which a set of 4 student representatives be chosen, one from each section.

Ways of selecting one student from section 1:$^{40}C_1$

Ways of selecting one student from section 2:$^{40}C_1$

Ways of selecting one student from section 3:$^{40}C_1$

Ways of selecting one student from section 4:$^{40}C_1$



So number of ways of choosing a set of 4 student representatives one from each section=${}^{40}C_1 \times {}^{40}C_1 \times {}^{40}C_1 \times {}^{40}C_1$

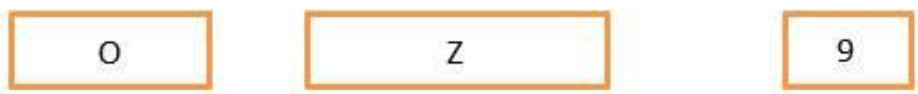
$= 40 \times 40 \times 40 \times 40$

$= 2560000$

Q. 8. In how many ways can a vowel, a consonant and a digit be chosen out of the 26 letters of the English alphabet and the 10 digits?

Answer : To find: number of ways in which a vowel, a consonant and a digit be chosen out of the 26 letters of the English alphabet and the 10 digits.

e.g.



Way of selecting a vowel from 5 vowels=${}^{5}C_1$

Way of selecting a consonant from 26 consonants=${}^{21}C_1$

Way of selecting a digit from 10 digits=${}^{10}C_1$

So ways of choosing a vowel, a consonant, a digit=${}^{5}C_1 \times {}^{21}C_1 \times {}^{10}C_1$

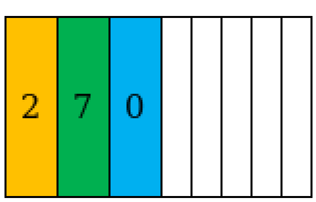
$= 5 \times 21 \times 10$

$= 1050$

Q. 9. How many 8-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 270 and n digit appears more than once?

Answer : Given:8 digit telephone number starts with 270 .

To find: How many 8-digit telephone numbers can be constructed?



There are 10 digits between 0 to 9,and three of them are utilized in filling up the first three digits i.e.270 of the 8 digit phone number, so remaining number of digits=10-

3=7,and this need to be used in filling up the remaining 8-3=5 places of the telephone number.

i.e. the remaining 5 places need to filled up with any one of:1,3,4,5,6,8,9

So, number of ways=7 × 6 × 5 × 4 × 3=2520.

Q. 10. (ac and the outcomes are recorded. How many possible outcomes are there?

(b) How many possible outcomes if the coin is tossed.
(i) four times? (ii) five times? (iii) n times?

Answer : (a) A coin is tossed three times

So possible number of outcomes=2^3=8

(HHH,HHT,HTH,HTT,THH,THT,TTH,TTT)

(b) i) A coin is tossed four times

So possible number of outcomes=2^4=16

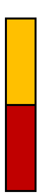
(HHHH,HHHT,HHTH,HHTT,HTHH,HTHT,HTTH,HTTT,THHH,THHT,THTH,THTT,TTHH,TTHT,TTTH,TTTT)

(ii) A coin is tossed n times

So possible number of outcomes=2^n

Q. 11. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Answer : Given:5 Flags



Way of generating signal using 2 different flags =${}^5P_2$ (way of selecting 2 things out of 5 things with considering arrangement.)

Way of generating signal using 3 different flags =${}^5P_3$

Way of generating signal using 4 different flags $= {}^5P_4$

Way of generating signal using 5 different flags $= {}^5P_5$

So total number of ways $= {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$=20 + 60 + 120 + 120$

$=320$

Q. 12. How many 4-letter codes can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Answer : Given: first 10 letters of the English alphabet.

In 4 letter code for first position there are 10 possibilities for second position there are 9 possibilities, for third position there are 8 possibilities and for fourth position there are 7 possibilities since repetition is not allowed.

So total numbers of combination $=10 \times 9 \times 8 \times 7 = 5040$

Q. 13. Given, A = {2, 3, 5} and B = {0, 1}. Find the number of different ordered pairs in which the first entry is an element of A and the second is an element of B.

Answer : This is the example of Cartesian product of two sets.

The pairs in which the first entry is an element of A and the second is an element of B are :

(2,0),(2,1),(3,0),(3,1),(5,0),(5,1)

$\Rightarrow 3 \times 2 = 6$

Q. 14. How many arithmetic progressions with 10 terms are there whose first term in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4}?

Answer : Given: Two sets: {1, 2, 3} & {2, 3, 4}

To find: number of A.P. with 10n terms whose first term is in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4}

Number of arithmetic progressions with 10 terms whose first term are in the set {1, 2, 3} and whose common difference is in the set {2, 3, 4} are: $3 \times 3=9$

(3 because there are three elements in the set {1, 2, 3} and another 3 because there are three elements in the set {2, 3, 4})

Q. 15. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible (correct or incorrect) answers are there to this question?

Answer :

COLUMN A	COLUMNB
ITEM1	MATCH1
ITEM2	MATCH2
ITEM3	MATCH3
ITEM4	MATCH4
ITEM5	MATCH5
ITEM6	MATCH6

As we can see that For Item2 there can be any of the match

So, For each item in column A there are 6 different options in column B since we don't have to think about correct or incorrect matching.

So possible number of combinations possible to answer:

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Q. 16. A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of February calendars should it prepare to serve for all the possibilities in the future years?

Answer : To find: types of February calendars that can be prepared.

There are two factors to develop FEBRUARY metallic calendars

(1) The day on the start of the year of which possibility=7

(2) Whether the year is leap year or not of which possibility is =2

So, number of FEBRUARY calendars possibilities to serve in future years=$7 \times 2 = 14$

Q. 17. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

Answer : Given: 36 teachers are there in a school.

To find: Number of ways in which one principal and one vice-principal can be appointed.

There are 36 options of appointing principal and 35 option of appointing vice-principal since same teacher cannot be appointed as principal and vice-principal.

Total number of ways=$36 \times 35 = 1260$

Q. 18. A sample of 3 bulbs is tested. A bulb is labeled as G if it is good and D if it is defective. Find the number of all possible outcomes.

Answer : A bulb can be good or defective, so there are 2 different possibilities of a bulb.

So number of all possible outcomes (of all bulbs)=$2 \times 2 \times 2 = 8$

Q. 19. For a set of five true or false questions, no student has written the all correct answer and no two students have given the same sequence of answers. What is the maximum number of students in the class for this to be possible?

Answer : Given: a set of five true – false questions.

To find: the maximum number of students in the class.

Condition: no student has written the all correct answer and no two students have given the same sequence of answers.

The total number of answering a set of 5 true or false question=2^5=32

Since, no two students have given the same sequence of answers and no student has written the all correct answer.

Therefore total possibilities reduces by 1(of no student has written the all correct answer)

$\Rightarrow 2^5-1=32-1=31$

Q. 20. In how many ways can the following prizes be given away to a class of 20 students : first and second in mathematics; first and second in chemistry; first in physics and first in English?

Answer : Given: 20 students.

The number of ways of giving first and second prizes in mathematics to a class of 20 students=20 × 19.

(First prize can be given to any one of the 20 students but the second prize cannot be given to the student that received the first prize so the number of candidates for the second prize is 19.)

The number of ways of giving first and second prizes in chemistry

To a class of 20 students=20 × 19.

The number of ways of giving first prize in physics to a class of 20 students=20

The number of ways of giving first prize in English to a class of 20 students=20

So total number of ways=20 × 19 × 20 × 19 × 20 × 20 = 57760000

Q. 21. Find the total number of ways of answering 5 objective-type question, each question having 4 choices.

Answer : Given: 5 objective-type question, each question having 4 choices.

To find: the number of ways of answering them.

Each objective-type question has 4 choices.

So the total number of ways of answering 5 objective-type question, each question having 4 choices=$4 \times 4 \times 4 \times 4 \times 4=4^5$

Q. 22. A gentleman has 6 friends to invite. In how many ways can be send invitation cards to them, if he has 3 servants to carry the cards?

Answer : Given: A gentleman has 6 friends to invite. He has 3 servants to carry the cards.

Each friend can be invited by 3 possible number of servants.

So the number of ways of inviting 6 friends using 3 servants=$3 \times 3 \times 3 \times 3 \times 3 \times 3=3^6$

Q. 23. In how many ways 6 rings of different types can be worn in 4 fingers?

Answer : Given:6 rings and 4 fingers.

Each ring has 4 different fingers that they can be worn.

So total number of ways in which 6 rings of different types can be worn in 4 fingers =$4 \times 4 \times 4 \times 4 \times 4 \times 4=4^6$

Q. 24. In how many ways can 5 letters be posted in 4 letter boxes?

Answer : Each letter has 4 possible letter boxes option.

So the number of ways in which 5 letters can be posted in 4 letter boxes =$4 \times 4 \times 4 \times 4 \times 4=4^5$ (Each 4 for each letter.)

Q. 25. How many 3-letters words can be formed using a, b, c, d, e if

(i) Repetition of letters is not allowed?
(ii) Repetition of letters is allowed

Answer :

any one of 6 letters	any one of the remaining 5 letters	any one of th

(i) if repetition of letters is not allowed then number of many 3-letters words that can be formed using a, b, c, d, e are

$5 \times 4 \times 3=60$

any one of 5 letters	any one of 5 letters	any one of 5 letters

(ii) if repetition of letters is allowed then number of many 3-letters words that can be formed using a, b, c, d, e are

$5 \times 5 \times 5 = 125$

Q. 26. How many 4-digit numbers are there, when a digit may be repeated any number of times?

Answer : To find: Number of 4 digit numbers when a digit may be repeated any number of times

The first place has possibilities of any of 9 digits.

(0 not included because 0 in starting would make the number a 3 digit number.)

The second place has possibilities of any of 10 digits.

The third place has possibilities of any of 10 digits.

The fourth place has possibilities of any of 10 digits.

Since repetition is allowed.

So there are $9 \times 10 \times 10 \times 10 = 9000$ 4-digit numbers when a digit may be repeated any number of times.

Q. 27. How many numbers can be formed from the digits 1, 3, 5, 9 if repetition of digits is not allowed?

Answer : To find: number of numbers that can be formed from the digits 1, 3, 5, 9 if repetition of digits is not allowed

Forming a 4 digit number:$4!$

Forming a 3 digit number:${}^4C_3 \times 3!$

Forming a 2 digit number:${}^4C_2 \times 2!$

Forming a 1 digit number:4

So total number of ways=$4! + ({}^4C_3 \times 3!) + ({}^4C_2 \times 2!) + 4$

$=24 + 24 + 12 + 4$

$=64$

Q. 28. How many 3-digit numbers are there with no digit repeated?

Answer :

Any 1 of 9 digits Not ZERO	Any 1 of 9 digits	Any 1 of 8 digits

In forming a 3 digit number the 100's place can be occupied by any 9 out of 10 digits (0 not included because it will lead to formation of 2 digit number.)

The 10's place can be occupied by any of the remaining 9 digits (here 0 can or cannot be used.)

In one's place any of the remain 8 digits can be used.

So total 3-digit numbers with no digit repeated are: $9 \times 9 \times 8=648$.

Q. 29. How many 3-digit numbers can be formed by using the digits 0, 1, 3, 5, 7 while each digit may be repeated any number of times?

Answer : 100's place 10's place Unit's place

Any one of 1,3,5,7	Any one of 0,1,3,5,7	Any one of 0,1,3,5,7

There are total 5 digits available, for forming a 3 digit number, in 100's place only 1,3,5,7 can be used(0 not included because it will lead to formation of 2 digit number.)

In 10's place any of the 5 can be used and same is the case with one's place.

So total number of 3 digit numbers formed=$4 \times 5 \times 5=100$

Q. 30. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7, 9 when no digit is repeated? How many of them are divisible by 10?

Answer :

Any one of 1,3,5,7,9					For divisibility by 10 this place should contain 0

There are total 6 digits available ,for forming a 6 digit number, in 100000’s place only 1,3,5,7,9 can be used(0 not included because it will lead to formation of 2 digit number.)

In 10000’s place any of the remaining 5 digits can be used(even 0 can be used.)

In 1000’s place any of the remaining 4 digits can be used.

In 100’s place any of the remaining 3 digits can be used.

In 10’s place any of the remaining 2 digits can be used.

In one’s place the remaining digit can be used.

So total number of 6 digit numbers possible=$5 \times 5 \times 4 \times 3 \times 2 \times 1=600$

For finding the number of 6 digit numbers divisible by 10 the one’s place should contain 0 so possibilities=$5 \times 4 \times 3 \times 2 \times 1=120$

Q. 31. How many natural numbers less than 1000 can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times?

Answer : To find: number of natural numbers less than 1000 that can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times

For forming a 3 digit number less than 1000 possible ways are:

$5 \times 6 \times 6$…(in 100’s place 5 digits are only possible 0 not included.)

=180

For forming a 2 digit number less than 1000 possible ways are:

5 × 6……(in 10’s place 5 digits are only possible 0 not included.)

= 30

For forming a 1 digit number less than 1000 possible ways are:

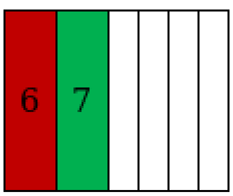
5…(0 not included because it is a whole number and natural number is asked in question.)

So total number of numbers less than 1000 that can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times=180 + 30 + 5=215

Q. 32. How many 6-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Answer : To find: 6-digit telephone numbers that can be constructed using the digits 0 to 9.

Condition: each number starts with 67 and no digit appears more than once



There are 10 digits between 0 to 9,and two of them are utilized in filling up the first two digits i.e.67 of the 6 digit phone number, so remaining number of digits=10-2=8,and this need to be used in filling up the remaining 6-2=4 places of the telephone number.

So, number of ways=8 × 7 × 6 × 5=1680.

Q. 33. In how many ways can three jobs, I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job?

Answer : Given: three jobs, I, II and III to be assigned to three persons A, B and C.

To find: In how many ways this can be done.

Condition: one person is assigned only one job and all are capable of doing each job.

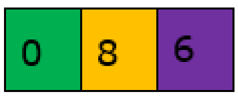
It is given that one person is assigned only one job and all are capable of doing each job.

So if for person one 3 options are available, for person two 2 options and for person three only one option is available.

So total number of ways in which three jobs, I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job=3 × 2 × 1 = 6

Q. 34. A number lock on a suitcase has three wheels each labeled with ten digits 0 to 9. if opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible? Also, find the number of unsuccessful attempts to open the lock.

Answer :



e.g

The number of sequences possible = 10 × 9 × 8 = 720(since no repeated digits is the given condition.)

There will be only one successful attempt so the number of unsuccessful attempts to open the lock=720 – 1 = 719.

Q. 35. A customer forgets a four-digit code for an automated teller machine (ATM) in a bank. However, he remembers that this code consists of digits 3, 5, 6, 9. Find the largest possible number of trials necessary to obtain the correct code.

Answer : Given: code consists of digits 3, 5, 6, 9.

To find: the largest possible number of trials necessary to obtain the correct code.

The customer remembers that this 4 digit code consists of digits 3, 5, 6, 9.

So the largest possible number of trials necessary to obtain the correct code = 4! = 4 × 3 × 2 × 1 = 24

Q. 36. In how many ways can 3 prizes be distributed among 4 girls, when

(i) no girl gets more than one prize?
(ii) a girl may get any number of prizes?
(iii) no girl gets all the prizes?

Answer : (i) To distribute 3 prizes among 4 girls where no girl gets more than one prize the possible number of permutation possible are:${}^4P_3$=24

(ii) To distribute 3 prizes among 4 girls where a girl may get any number of prizes the number of possibilities are:$4 \times 4 \times 4=64$.

(Since a prize can be given to any of the 4 girls.)

(iii) To distribute 3 prizes among 4 girls where no girl gets all the prizes the number of possibilities are: $(4 \times 4 \times 4)-(4)=64-4=60$

(The situation where a single girl gets all the prizes has to reduced from the situation where a girl may get any number of prizes.)

Exercise 8C

Q. 1. A. Evaluate:

$^{10}P_4$

Answer : To find: the value of $^{10}P_4$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

Therefore,

$^{10}P_4 = \frac{10!}{(10-4)!}$

$^{10}P_4 = 10 \times 9 \times 8 \times 7$

$^{10}P_4 = 5040$

Thus, the value of $^{10}P_4$ is 5040.

Q. 1. B. Evaluate:

$^{62}P_3$

Answer : To find: the value of $^{62}P_3$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

Therefore,

$^{62}P_3 = \frac{62!}{(62-3)!}$

$^{62}P_3 = 62 \times 61 \times 60 \times 59 = 226920$

Thus, the value of $^{62}P_3$ is 226920.

Q. 1. C. Evaluate:

$^6P_6$

Answer : To find: the value of $^6P_6$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

Therefore,

$^6P_6 = \frac{6!}{(6-6)!}$

$^{62}P_3 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Thus, the value of $^6P_6$ is 720.

Q. 1. D. Evaluate:

$^9P_0$

Answer : To find: the value of $^9P_0$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

Therefore,

$^9P_0 = \frac{9!}{(9-0)!}$

$^9P_0 = 1$

Thus, the value of $^9P_0$ is 1.

Q. 2. Prove that $^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

Answer : To Prove: $^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

The equation given below needs to be proved i.e

$^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

$$\frac{9!}{(9-3)!} + \left(3 \times \frac{9!}{(9-2)!}\right) = \frac{10!}{(10-3)!}$$

$(9 \times 8 \times 7) + (3 \times 9 \times 8) = 10 \times 9 \times 8$

$10 \times 9 \times 8 = 10 \times 9 \times 8$

Hence, proved.

$^9P_3 + 3 \times {}^9P_2 = {}^{10}P_3$.

Q. 3. (i) If $^nP_5 = 20 \times {}^nP_3$, find n.

(ii) If 16 × $^{n}P_3$ = 13 × $^{n+1}P_3$, find n.
(iii) If $^{2n}P_3$ = 100 × $^{n}P_2$, find n.

Answer : (i) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$$^{n}P_r = \frac{n!}{(n-r)!}$$

$^{n}P_5 = 20 \times {}^{n}P_3$.

$$\frac{n!}{(n-5)!} = \left(20 \times \frac{n!}{(n-3)!}\right)$$

$$\frac{1}{(n-5)!} = \left(20 \times \frac{1}{(n-3)(n-4)(n-5)!}\right)$$

$$1 = \left(20 \times \frac{1}{(n-3)(n-4)}\right)$$

20 = (n - 3)(n - 4)

$$n^2 - 7n + 12 = 20$$

$$n^2 - 7n - 8 = 0$$

(n - 8)(n + 1) = 0

n = 8, -1

We know, that n cannot be a negative number.

Hence, value of n is 8

(ii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

${}^nP_r = \frac{n!}{(n-r)!}$

$16 \times {}^nP_3 = 13 \times {}^{n+1}P_3$.

$$16 \times \frac{n!}{(n-3)!} = \left(13 \times \frac{(n+1)!}{(n-2)!}\right)$$

$$16 \times \frac{n!}{(n-3)!} = \left(13 \times \frac{(n+1)n!}{(n-2)(n-3)!}\right)$$

$$16 = 13 \times \frac{(n+1)}{(n-2)}$$

16n – 32 = 13n + 13

3n = 45

n = 15

Hence, value of n is 15.

(iii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

${}^nP_r = \frac{n!}{(n-r)!}$

${}^{2n}P_3 = 100 \times {}^nP_2$

$$\frac{2n!}{(2n-3)!} = \left(100 \times \frac{n!}{(n-2)!}\right)$$

$$\frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = \left(100 \times \frac{n(n-1)(n-2)!}{(n-2)!}\right)$$

$$\frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = \left(100 \times \frac{n(n-1)(n-2)!}{(n-2)!}\right)$$

$2n(2n - 1)(2n - 2) = 100 \times n(n - 1)$

$4n(2n - 1)(n - 1) = 100 \times n(n-1)$

$$8n^2 - 4n - 100n = 0$$

$$8n^2 - 104n = 0$$

$8n(n - 13) = 0$

$n = 0, 13$

We know that n should be greater than zero.

Hence, value of n is 13

Q. 4. (i) If If $^5P_r = 2 \times {}^6P_{r-1}$, find r.

(ii) If $^{20}P_r = 13 \times {}^{20}P_{r-1}$, find r.
(iii) If $^{11}P_r = {}^{12}P_{r-1}$, find r.

Answer : (i) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^5P_r = 2 \times {}^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \left(2 \times \frac{6!}{(7-r)!}\right)$$

$$\frac{5!}{(5-r)!} = \left(2 \times \frac{6 \times 5!}{(7-r)(6-r)(5-r)!}\right)$$

$$1 = \left(\frac{12}{(7-r)(6-r)}\right)$$

$$r^2 - 13r + 30 = 0$$

r = 10, 3

Hence, value of r is 3, 10

(ii) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

${}^{20}P_{r} = 13 \times {}^{20}P_{r-1}$

$$\frac{20!}{(20-r)!} = \left(13 \times \frac{20!}{(21-r)!}\right)$$

$$\frac{1}{(20-r)!} = \left(13 \times \frac{1}{(21-r)(20-r)!}\right)$$

21 – r = 13

r = 8

Hence, value of r is 8.

(iii) To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^{11}P_r = {}^{12}P_{r-1}$

$$\frac{11!}{(11-r)!} = \left(\frac{12!}{(13-r)!}\right)$$

$$\frac{11!}{(11-r)!} = \left(\frac{12 \times 11!}{(13-r)(12-r)(11-r)!}\right)$$

$$1 = \frac{12}{(13-r)(12-r)}$$

$$r^2 - 25r + 144 = 0$$

(r - 16)(r - 9) = 0

r = 16, 9

Since r cannot be 16 as it creates a negative factorial in denominator. Therefore, r = 16 is not possible.

Hence, value of r is 9.

Q. 5. (i) If $^nP_4 : {}^nP_5$, = 1 : 2, find n.

(ii) If $^{n-1}P_3 : {}^{n+1}P_3$, = 5 : 12, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^nP_4 : {}^nP_5 = 1:2$

$$\frac{n!}{(n-4)!} : \frac{n!}{(n-5)!} = \frac{1}{2}$$

$$\frac{n!}{(n-4)(n-5)!} : \frac{n!}{(n-5)!} = \frac{1}{2}$$

$$\frac{n!}{(n-4)(n-5)!} \times \frac{(n-5)!}{n!} = \frac{1}{2}$$

$$\frac{1}{(n-4)} = \frac{1}{2}$$

n – 4 = 2

n = 6

Hence, value of n is 6.

(ii) To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{n-1}P_3 : {}^{n+1}P_3, = 5 : 12$

$$\frac{(n-1)!}{(n-4)!} : \frac{(n+1)!}{(n-2)!} = \frac{5}{12}$$

$$\frac{(n-1)!}{(n-4)!} : \frac{(n+1)n(n-1)!}{(n-2)(n-3)(n-4)!} = \frac{5}{12}$$

$$\frac{(n-1)!}{(n-4)!} \times \frac{(n-2)(n-3)(n-4)!}{(n+1)n(n-1)!} = \frac{5}{12}$$

$$\frac{(n-2)(n-3)}{(n+1)n} = \frac{5}{12}$$

$$\frac{n^2-5n+6}{n^2+n} = \frac{5}{12}$$

$12 n^2 - 60n + 72 = 5n^2 + 5n$

$7n^2 - 65n + 72 = 0$

n = 8, 2.25

Since n cannot be 2.25 as it creates a negative factorial in denominator. Therefore, n = 2.25 is not possible.

Hence, value of n is 8.

Q. 6. If $^{15}P_{r-1} : {}^{16}P_{r-2}$, = 3 : 4, find r.

Answer : To find: the value of r

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

$^{15}P_{r-1} : {}^{16}P_{r-2}$, = 3 : 4

$$\frac{15!}{(16-r)!} : \frac{16!}{(18-r)!} = \frac{3}{4}$$

$$\frac{15!}{(16-r)!} : \frac{16 \times 15!}{(18-r)(17-r)(16-r)!} = \frac{3}{4}$$

$$\frac{15!}{(16-r)!} \times \frac{(18-r)(17-r)(16-r)!}{16 \times 15!} = \frac{3}{4}$$

$$\frac{(18-r)(17-r)}{4} = 3$$

$$\frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$r^2 - 35r + 306 = 12$$

$r^2 - 35r + 294 = 0$

$(r - 21)(r - 14) = 0$

$r = 21, 14$

Since r cannot be 21 as it creates a negative factorial in denominator. Therefore, $r = 14$ is not possible.

Hence, value of r is 14

Q. 7. If $^{2n-1}P_n : {}^{2n+1}P_{n-1}$, = 22 : 7, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^{2n-1}P_n : {}^{2n+1}P_{n-1}$, = 22 : 7

$$\frac{(2n-1)!}{(n-1)!} : \frac{(2n+1)!}{(n+2)!} = \frac{22}{7}$$

$$\frac{(2n-1)!}{(n-1)!} : \frac{(2n+1)(2n)(2n-1)!}{(n+2)(n+1)n(n-1)!} = \frac{22}{7}$$

$$\frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+1)n(n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{22}{7}$$

$$\frac{(n+2)(n+1)}{(2n+1)2} = \frac{22}{7}$$

$$\frac{n^2 + 3n + 2}{2n + 1} = \frac{44}{7}$$

$7n^2 + 21n + 14 = 88n + 44$

$7n^2 - 67n - 30 = 0$

n = 10, -0.42

Since n cannot be -0.42

Hence, value of n is 10.

Q. 8. Find n, If $^{n+5}P_{n+1} = \frac{11}{2}$ (n – 1). $^{n+3}P_n$, find n.

Answer : To find: the value of n

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$^nP_r = \frac{n!}{(n-r)!}$

$^{n+5}P_{n+1} = \frac{11}{2}$ (n – 1). $^{n+3}P_n$

$$\frac{(n+5)!}{4!} = \frac{11}{2}(n-1)\frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)(n+3)!}{4 \times 3!} = \frac{11}{2}(n-1)\frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)}{2} = 11(n-1)$$

$$n^2 + 9n + 20 = 22n - 22$$

$$n^2 - 13n + 42 = 0$$

(n - 7)(n - 6) = 0

n = 7, 6

Hence, values of n are 7 & 6

Q. 9. Prove that $1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

Answer : To Prove: $1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$1 + 1.\ {}^{1}P_{1} + 2.\ {}^{2}P_{2} + 3.\ {}^{3}P_{3} + \ldots.\ n.\ {}^{n}P_{n} = {}^{n+1}P_{n+1}$.

$1 + (2! - 1!) + (3! - 2!) + (4! - 3!) + \ldots\ldots.((n + 1)! - n!) = (n + 1)!$

$1 + ((n + 1)! - 1!) = (n + 1)!$

$(n + 1)! = (n + 1)!$

Hence proved.

Q. 10. Find the number of permutations of 10 objects, taken 4 at a time.

Answer : To find: the number of permutations of 10 objects, taken 4 at a time.

Formula Used:

Total number of ways in which n objects can be arranged in r places (Such that no object is replaced) is given by,

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}^{10}P_{4} = \frac{10!}{6!}$$

$${}^{10}P_{4} = 10 \times 9 \times 8 \times 7$$

$${}^{10}P_{4} = 5040$$

Hence, the number of permutations of 10 objects, taken 4 at a time is 5040.

Exercise 8D

Q. 1. In how many ways can 5 persons occupy 3 vacant seats?

Answer : To find: number of arrangements of 5 people in 3 seats.

Consider three seats A B C

Now, place A can be occupied by any 1 person out of 5.

Then place B can be occupied by any 1 person from remaining 4 and for C there are 3 people to occupy the seat.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 5 different objects in 3 places is

P(5,3) = $\frac{5!}{(5-3)!}$

= $\frac{5!}{2!}$ = $\frac{120}{2}$ = 60.

Therefore, the number of possible solutions is 60.

Q. 2. In how many ways can 7 people line up at a ticket window of a cinema hall?

Answer : To find: number of arrangements of 7 people in a queue.

Here there are 7 spaces to be occupied by 7 people.

Therefore 7 people can occupy first place.

Similarly, 6 people can occupy second place and so on.

Lastly, there will be a single person to occupy the 7 positions.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 7 different objects in 7 places is

P(7,7) = $\frac{7!}{(7-7)!}$

= $\frac{7!}{0!}$ = $\frac{5040}{1}$ = 5040.

Therefore, the number of possible ways is 5040

Q. 3. In how many ways can 5 children stand in a queue?

Answer : To find: number of arrangements of 5 children in a queue.

Here, 5 places are needed to be occupied by 5 children.

Therefore any one of the 5 children can occupy first place.

Similarly, any 4 children can occupy second place and so on.

Lastly, there will be a single person to occupy the 5 position

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 5 different objects in 5 places is

P(5,5) = $\frac{5!}{(5-5)!}$

= $\frac{5!}{0!}$ = $\frac{120}{1}$ = 120.

Hence, this can be done in 120 ways.

Q. 4. In how many ways can 6 women draw water from 6 wells if no well remains unused?

Answer : To find: number of arrangements of 6 women drawing water from 6 wells

Here, 6 wells are needed to be used by 6 women.

Therefore any one of the 6 women can draw water from the 1 well.

Similarly, any 5 women can draw water from the 2nd well and so on.

Lastly, there will be single women left to draw water from the 6th well.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 6 different objects in 6 places is

P(6,6) = $\frac{6!}{(6-6)!}$

= $\frac{6!}{0!}$ = $\frac{720}{1}$ = 720.

Hence, this can be done in 720 ways.

Q. 5. In how many ways can 4 different books, one each in chemistry, physics, biology and mathematics, be arranged on a shelf?

Answer : To find: number of arrangements of 4 different books in a shelf.

There are 4 different books.

Any one of the four different books can be placed on the shelf first.

Similarly, in the next position, 1 book out of 3 can be placed.

Finally, the last book will have a single place to fit.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 4 different objects in 4 places is

$P(4,4) = \frac{4!}{(4-4)!}$

$= \frac{4!}{0!} = \frac{24}{1} = 24.$

Hence they can be arranged in 24 ways.

Q. 6. Six students are contesting the election for the president ship of the students, union. In how many ways can their names be listed on the ballot papers?

Answer : To find: number of arrangements of names on a ballot paper.

There are six contestants contesting in the elections.

Name of any 1 student out of six can appear first on the ballot paper.

2 position on the ballot paper can be filled by rest of the five names and so on.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!}$

$= \frac{6!}{0!} = \frac{720}{1} = 720.$

Hence, their name can be arranged in 720 ways.

Q. 7. It is required to seat 5 men and 3 women in a row so that the women occupy the even places. How many such arrangements are possible?

Answer : To find: number of arrangements in which women sit in even places

Condition: women occupy even places

Here the total number of people is 8.

_ _ _ _ _ _ _ _

1 2 3 4 5 6 7 8

In this question first, the arrangement of women is required.

The positions where women can be made to sit is 2^{nd}, 4^{th}, 6^{th}, 8^{th}. There are 4 even places in which 3 women are to be arranged.

Women can be placed in P (4,3) ways. The rest 5 men can be arranged in 5! ways.

Therefore, the total number of arrangements is P (4,3) ×5!

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 3 places and the arrangement of 5 men are

P (4,3) ×5! = $\frac{4!}{(4-3)!} \times 5!$

= $\frac{24}{1} \times 120$

= 2880.

Hence number of ways in which they can be seated is 2880.

Q. 8. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible, correct or incorrect answers are there to this question?

Answer : To find: number of possibilities of a selection of answers.

Each item in column A can select another item in column B.

Therefore the question involves selecting each item from column A to each item in column B. this can be done in P(6,6)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

P(6,6) = $\frac{6!}{(6-6)!}$

= $\frac{6!}{0!}$ = $\frac{720}{1}$ = 720.

Therefore, the possible number of selecting an incorrect or correct answer is 720.

Q. 9. Five letters F, K, R, R and V one in each were purchased from a plastic warehouse. How many ordered pairs of letters, to be used as initials, can be formed from them?

Answer : (i) The number of initials is 1

In this case, all letters have one chance (i.e. letters F, K, R, V).

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 1 place is

P(4,1) = $\frac{4!}{(4-1)!}$

= $\frac{4!}{3!}$ = $\frac{24}{6}$ = 4.

So no of ways is 4.

(ii) The number of initials is 2

There are two cases here

(a) When two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 2 places is

P(4,2) = $\frac{4!}{(4-2)!}$

= $\frac{4!}{2!}$ = $\frac{24}{2}$ = 12.

A number of arrangements here are 12.

(b) When two R occurs in initials

When two R are chosen then 1 pair is included twice.

Selection of 0 letters remaining from 3 letters can be done in P(3,0) ways.

Formula:

A number of permutations of n objects in which p objects are alike of one kind are =n!/p!

Selections = P(3,0) × $\frac{2!}{2!}$

= $\frac{3!}{3!}$ × $\frac{2!}{2!}$ = 1

Therefore, the total number of pairs 13.

(iii) The number of initial is 3

(a) two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 3 places is

P(4,3) = $\frac{4!}{(4-3)!}$

= $\frac{4!}{1!}$ = $\frac{24}{1}$ = 24.

A number of arrangements here are 24.

(b) two R occurs in initials

When two R are chosen then 1 pair is included twice.

Selection of 1 letter from the remaining 3 letters is P(3,1)

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = P(3,1) × $\frac{3!}{2!}$

= $\frac{3!}{2!}$ × $\frac{3!}{2!}$ = 9

total number of arrangements for 3 initials are 33

(iv) The number of initials is 4

(a) Two R do not occur in initials

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 4 places is

P(4,4) = $\frac{4!}{(4-4)!}$

= $\frac{4!}{0!}$ = $\frac{24}{1}$ = 24.

A number of arrangements here are 24.

(b) Two R occurs in the initials

When two R are chosen then 1 pair is included twice.

Selection of 2 letters from the remaining 3 letters is P(3,2)

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = P(3,2) × $\frac{4!}{2!}$

= $\frac{3!}{1!}$ × $\frac{4!}{2!}$ = 36

Total number of arrangements for 4 initials are 60

(v) The number of initials is 5

Formula:

A number of permutations of n objects in which p objects are alike of one kind = n!/p!

Selections = $\frac{5!}{2!}$ = 60.

Total number of arrangements are 4 + 13 + 33 + 60 + 60 = 170

Q. 10. Ten students are participating in a race. In how many ways can the first three prizes be won?

Answer : To find: number of ways of winning the first three prizes.

The first price can go to any of the 10 students.

The second price can go to any of the remaining 9 students.

The third price can go to any of the remaining 8 students.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 10 different objects in 3 places is

P(10,3) = $\frac{10!}{(10-3)!}$

= $\frac{10!}{7!}$ = $\frac{3628800}{5040}$ = 720.

Therefore, there are 10 × 9 × 8 = 720 ways to win first three prizes.

Q. 11. If there are 6 periods on each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least one period?

Answer : To find: number of ways of arranging 5 subjects in 6 periods.

Condition: at least 1 period for each subject.

5 subjects in 6 periods can be arranged in P (6,5).

Remaining 1 period can be arranged in P (5,1)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Total arrangements = P(6,5) × P(5,1) = $\frac{6!}{(6-5)!}$ × $\frac{5!}{(5-1)!}$

= $\frac{6!}{1!}$ × $\frac{5!}{4!}$ = 720 × 5 = 3600.

Total number of ways is 3600 ways.

Q. 12. In how many ways can 6 pictures be hung from 4 picture nails on a wall?

Answer : To find: number of ways of hanging 6 pictures on 4 picture nails.

There are 6 pictures to be placed in 4 places.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 4 places is

P(6,4) = $\frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{720}{2}$ = 360

This can be done by 360 ways.

Q. 13. Find the number of words formed (may be meaningless) by using all the letters of the word 'EQUATION', using each letter exactly once.

Answer : There are 8 alphabets in the word EQUATION.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 8 different objects in 8 places is

P(8,8) = $\frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{40320}{1}$ = 40320

Hence there are 40320 words formed.

Q. 14. Find the number of different 4-letter words (may be meaningless) that can be formed from the letters of the word 'NUMBERS',

Answer : To find: 4 lettered word from letters of word NUMBERS

There are 7 alphabets in word NUMBERS.

The word is a 4 different letter word.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 4 places is

P(7,4) = $\frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{5040}{6}$ = 840.

Hence, they can be arranged in 840 words.

Q. 15. How many words can be formed from the letters of the word 'SUNDAY'? How many of these begin with D?

Answer : There are 6 letters in the word SUNDAY.

Different words formed using 6 letters of the word SUNDAY is P(6,6)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

P(6,6) = $\frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{720}{1}$ = 720.

720 words can be formed using letters of the word SUNDAY.

When a word begins with D.

Its position is fixed, i.e. the first position.

Now rest 5 letters are to be arranged in 5 places.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 5 different objects in 5 places is

P(5,5) = $\frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1}$ = 120.

Therefore, the total number of words starting with D are 120.

Q. 16. How many words beginning with C and ending with Y can be formed by using the letters of the word 'COURTESY'?

Answer : To find: number of words starting with C and end with Y

There are 8 letters in word COURTESY.

Here the position of the letters C and Y are fixed which is 1^{st} and 8^{th}.

$$\underline{C}\ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}\ \underline{Y}$$

Rest 6 letters are to be arranged in 6 places which can be done in P(6,6).

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 6 different objects in 6 places is

$P(6,6) = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{720}{1} = 720.$

Therefore, total number of words starting with C and ending with Y is 720.

Q. 17. Find the number of permutations of the letters of the word 'ENGLISH'. How many of these begin with E and end with I?

Answer : There are 7 letters in the word ENGLISH.

Permutation of 7 letters in 7 places can be done in P(7,7) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 7 places is

$P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{5040}{1} = 5040.$

Hence, the total number of permutations is P 5040.

To find a number of words starting with E and ending with I, let us consider their position which is 1st and 7th.

_ _ _ _ _ _ _

E ? ? ? ? ? I

The rest 5 letters are to be arranged in 5 places which can be done in P (5,5)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 5 different objects in 5 places is

$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = 120.$

Therefore, there are 120 words starting with E and ending with I.

Q. 18. In how many ways can the letters of the word 'HEXAGON' be permuted? In how many words will the vowels be together?

Answer : There are 7 letters in the word HEXAGON.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 7 different objects in 7 places is

$P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{5040}{1} = 5040.$

They can be permuted in P (7,7) = 5040 ways.

The vowels in the word are E, A, O.

Consider this as a single group.

Now considering vowels as a single group, there are total 5 groups (4 letters and 1 vowel group) can be permuted in P (5,5)

Now vowel can be arranged in 3! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the arrangement of 5 groups and vowel group is

$P(5,5)\times 3! = \frac{5!}{(5-5)!} \times 3! = \frac{5!}{0!} \times 3! = \frac{120}{1} \times 6 = 720.$

Hence total number of arrangements possible is 720.

Q. 19. How many words can be formed out of the letters of the word 'ORIENTAL' so that the vowels always occupy the odd places?

Answer : To find: number of words formed

Condition: vowels occupy odd places

There are 8 letters in the word ORIENTAL and vowels are 4 which are O, I, E,A respectively.

$\underline{O}\ \underline{E}\ \underline{O}\ \underline{E}\ \underline{O}\ \underline{E}\ \underline{O}\ \underline{E}$

There is 4 odd places in which 4 vowels are to be arranged.

The rest 4 letters can be arranged in 4! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total arrangement is

P(4,4) × 4! = $\frac{4!}{(4-4)!}$ × 4! = $\frac{4!}{0!}$ ×4! = $\frac{24}{1}$ × 24 = 576.

Therefore, total number of words formed are 576.

Q. 20. In how many ways can the letters of the word 'FAILURE' be arranged so that the consonants may occupy only odd positions?

Answer : To find: number of words

Condition: consonants occupy odd places

There are total of 7 letters in the word FAILURE.

There are 3 consonants, i.e. F, L, R which are to be arranged in 4 places.

The rest 5 letters can be arranged in 4! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total number of words are

P(4,3) ×4! = $\frac{4!}{(4-3)!}$ ×4! = $\frac{4!}{1!}$ ×4! = $\frac{24}{1}$ ×24 = 576.

Hence total number of arrangements is 576.

Q. 21. In how many arrangements of the word 'GOLDEN' will the vowels never occur together?

Answer : To find: number of words

Condition: vowels should never occur together.

There are 6 letters in the word GOLDEN in which there are 2 vowels.

Total number of words in which vowels never come together =

Total number of words – total number of words in which the vowels come together.

A total number of words is 6! = 720 words.

Consider the vowels as a group.

Hence there are 5 groups that can be arranged in P(5,5) ways, and vowels can be arranged in P(2,2,) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Total arrangements = P(5,5) × P(2,2) = $\frac{5!}{(5-5)!} \times \frac{2!}{(2-2)!}$

= $\frac{5!}{0!} \times \frac{2!}{0!}$ = 120 × 2 = 240.

Hence a total number of words having vowels together is 240.

Therefore, the number of words in which vowels don't come together is 720 – 240 = 480 words.

Q. 22. Find the number of ways in which the letters of the word 'MACHINE' can be arranged such that the vowels may occupy only odd positions.

Answer : To find: number of words

Condition: vowels occupy odd positions.

There are 7 letters in the word MACHINE out of which there are 3 vowels namely A C E.

There are 4 odd places in which 3 vowels are to be arranged which can be done P(4,3).

The rest letters can be arranged in 4! ways

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, the total number of words is

$P(4,3) 4! \times = \frac{4!}{(4-3)!} \times 4!$

$= \frac{4!}{1!} \times 4! = \frac{24}{1} \times 24 = 576.$

Hence the total number of word in which vowel occupy odd positions only is 576.

Q. 23. How many permutations can be formed by the letters of the word ‘VOWELS’, when

(i) there is no restriction on letters;
(ii) each word begins with E;
(iii) each word begins with O and ends with L;
(iv) all vowels come together;
(v) all consonants come together?

Answer : (i) There is no restriction on letters

The word VOWELS contain 6 letters.

The permutation of letters of the word will be 6! = 720 words.

(ii) Each word begins with

Here the position of letter E is fixed.

Hence, the rest 5 letters can be arranged in 5! = 120 ways.

(iii) Each word begins with O and ends with L

The position of O and L are fixed.

Hence the rest 4 letters can be arranged in 4! = 24 ways.

(iv) All vowels come together

There are 2 vowels which are O, E.

Consider this group.

Therefore, the permutation of 5 groups is 5! = 120

The group of vowels can also be arranged in 2! = 2 ways.

Hence the total number of words in which vowels come together are 120×2 = 240 words.

(v) All consonants come together

There are 4 consonants V,W,L,S. consider this a group.

Therefore, a permutation of 3 groups is 3! = 6 ways.

The group of consonants also can be arranged in 4! = 24 ways.

Hence, the total number of words in which consonants come together is 6×24 = 144 words.

Q. 24. How many numbers divisible by 5 and lying between 3000 and 4000 can be formed by using the digits 3, 4, 5, 6, 7, 8 when no digit is repeated in any such number?

Answer : For a number to be divisible by 5, the last digit should either be 5 or 0.

In this case, 5 is only possible.

For a four digit number to be between 3000 to 4000, in this case, should start with 3.

Therefore, the other 2 digits can be arranged by 4 numbers in P(4,2)

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, a permutation of 4 different objects in 2 places is

P(4,2) = $\frac{4!}{(4-2)!}$

= $\frac{4!}{2!}$ = $\frac{24}{2}$ = 12.

Therefore, there are 12 numbers present between 3000 to 4000 formed by using numbers 3,1,5,6,7,8.

Q. 25. In an examination, there are 8 candidates out of which 3 candidates have to appear in mathematics and the rest in different subjects. In how many ways can they are seated in a row if candidates appearing in mathematics are not to sit together?

Answer : Candidates in mathematics are not sitting together = total ways – the

Students are appearing for mathematic sit together.

The total number of arrangements of 8 students is 8! = 40320

When students giving mathematics exam sit together, then consider

Them as a group.

Therefore, 6 groups can be arranged in P(6,6) ways.

The group of 3 can also be arranged in 3! Ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, total arrangement are

$P(6,6) \times 3! = \frac{6!}{(6-6)!} \times 3!$

$= \frac{6!}{0!} \times 3! = \frac{720}{1} \times 6 = 4320.$

The total number of possibilities when all the students giving

Mathematics exam sits together is 4320 ways.

Therefore, number of ways in which candidates appearing

Mathematics exam is 40320 – 4320 = 36000.

Q. 26. In how many ways can 5 children be arranged in a line such that

(i) two of them, Rajan and Tanvy, are always together?
(ii) two of them, Rajan and Tanvy, are never together,

Answer : (i) two of them, Rajan and Tanvy, are always together

Consider Rajan and Tanvy as a group which can be arranged in 2! = 2 ways.

The 3 children with this 1 group can be arranged in 4! = 24 ways.

The total number of possibilities in which they both come together is 2×24 = 48 ways.

(ii) Two of them, Rajan and Tanvy, are never together

Two of them are never together = total number of possible ways of sitting – total number of ways in which they sit together.

A total number of possible way of arrangement of 5 students is 5! = 120 ways.

Therefore, the total number of arrangement when they both don't sit together is = 120 – 48 = 72.

Q. 27. when a group photograph is taken, all the seven teachers should be in the first row, and all the twenty students should be in the second row. If the tow corners of the second row are reserved for the two tallest students, interchangeable only between them, and if the middle seat of the front row is reserved for the principal, how many arrangements are possible?

Answer : For the first row:

There are 7 teachers in which the position of principal is fixed.

Therefore, the teachers can be arranged in p(7,7) = 5040.

For the second row:

The tallest students are at the ends and can be arranged in 2! = 2 ways.

Rest 18 students can be arranged in P(18,18) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of 18 different objects in 18 places is

P(18,18) = $\frac{18!}{(18-18)!}$

= $\frac{18!}{0!}$ = $\frac{18!}{1}$ = 18!

Therefore, a total number of arrangements of the second row is 2 × 18!

Total arrangements = 2 × 18! × 5040 = 10080 × 18!

The total number of arrangements is 10080 × 18!

Q. 28. Find the number of ways in which m boys and n girls may be arranged in a row so that no two of the girls are together; it is given that m > n.

Answer : In this question, n girls are to be seated alternatively between m boys.

There are m+1 spaces in which girls can be arranged.

1	2	3	4	5	....	m-1	m	m+1
Girl	boy	girl	boy	girl		girl	boy	girl

The number of ways of arranging n girls is P(m+1,n) = $\frac{(m+1)!}{(m-n+1)!}$ ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = n!/(n-r)!

Therefore, permutation of n different objects in m+1 places is

P(m+1,n) = $\frac{(m+1)!}{(m+1-n)!}$

= $\frac{(m+1)!}{(m-n+1)!}$

The arrangement of m boys can be done in P(m, m) ways.

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n, r) = n!/(n-r)!

Therefore, a permutation of m different objects in m places is

P(m, m) = $\frac{m!}{(m-m)!}$ = $\frac{m!}{0!}$ = m!

Therefore the total number of arrangements is $\frac{(m+1)!}{(m-n+1)!} \times m!$.

Exercise 8E

Q. 1. Find the total number of permutations of the letters of each of the words given below:

(i) APPLE (ii) ARRANGE
(iii) COMMERCE (iv) INSTITUTE
(v) ENGINEERING
(vi) INTERMEDIATE

Answer : To find: number of permutations of the letters of each word

Number of permutations of n distinct letters is n!

Number of permutations of n letters where r letters are of one kind, s letters of another kind, t letters of a third kind and so on = $\frac{n!}{r!s!t!...}$

(i) Here n = 5

P is repeated twice

So the number of permutations = $\frac{5!}{2!} = 5 \times 4 \times 3 = 60$

(ii) Here n = 7

A is repeated twice, and R is repeated twice

So, the number of permutations = $\frac{7!}{2!.2!} = \frac{7\times6\times5\times4\times3}{2} = 1260$

(iii) Here n = 8

M and E are repeated twice

So, the number of permutations = $\frac{8!}{2!2!} = \frac{8\times7\times6\times5\times4\times3\times2}{4} = 10080$

(iv) Here n = 9

I is repeated twice, T is repeated thrice

So, the number of permutations = $\frac{9!}{2!3!} = 30240$

(v) Here n = 11

E, N is repeated thrice, I,G are repeated twice

So the number of permutations = $\frac{11!}{3!3!2!2!} = 277200$

(vi) Here n = 12

I and T are repeated twice, E is repeated thrice

So, the number of permutations = $\frac{12!}{2!2!3!} = 19958400$

Q. 2. In how many ways can the letters of the expression $x^2y^2z^4$ be arranged when written without using exponents?

Answer : To find: number of ways the letters can be arranged

The following table shows the possible arrangements

Power Alphabet	2	2	4
Case 1	x	y	Z
Case 2	x	z	Y
Case 3	y	z	X
Case 4	y	x	Z
Case 5	z	x	Y
Case 6	z	y	X

However, we see that case 1 = $x^2y^2z^4$ is the same as case 4 = $y^2x^2z^4$

Similarly (case2,case 5), (case 3,case 6) are the same

So there are only 3 distinct cases

Hence the letters can be arranged in 3 distinct ways

Q. 3. There are 3 blue balls, 4 red balls and 5 green balls. In how many ways can they are arranged in a row?

Answer : To find: no of ways in which the balls can be arranged in a row where some balls are of the same kind

Total number of balls = 3+4+5 = 12

3 are of 1 kind, 4 are of another kind, 5 are of the third kind

Number of ways = $\frac{12!}{3!4!5!} = 27720$

They can be arranged in 27720 ways

Q. 4. A child has three plastic toys bearing the digits 3, 3, 5 respectively. How many 3 - digit numbers can he make using them?

Answer : To find: number of 3 digit numbers he can make

If all were distinct, he could have made 3! = 6 numbers

But 2 number are the same

So the number of possibilities = $\frac{3!}{2!} = \frac{6}{2} = 3$

He can make 3 three - digit numbers using them

Q. 5. How many different signals can be transmitted by arranging 2 red, 3 yellow and 2 green flags on a pole, if all the seven flags are used to transmit a signal?

Answer : To find: Number of distinct signals possible

Total number of fags = 7

2 are of 1 kind, 3 are of another kind, and 2 are of the 3rd kind

⇒ Number of distinct signals = $\frac{7!}{2!3!2!} = 210$

Hence 210 different signals can be made

Q. 6. How many words can be formed by arranging the letters of the word 'ARRANGEMENT', so that the vowels remain together?

Answer : To find: number of words where vowels are together

Vowels in the above word are: A,A,E,E

Consonants in the above word: R,R,N,G,M,N,T

Let us denote the all the vowels by a single letter say Z

⇒ The word now has the letters, R,R,N,G,M,N,T,Z

R and N are repeated twice

Number of permutations = $\frac{8!}{2!2!}$

Now Z is comprised of 4 letters which can be permuted amongst themselves

A and E are repeated twice

⇒ Number of permutations of Z = $\frac{4!}{2!2!}$

⇒ Total number of permutations = $\frac{8!\times4!}{2!^4} = 60480$

The number of words that can be formed is 60480

Q. 7. How many words can be formed by arranging the letters of the word 'INDIA', so that the vowels are never together?

Answer : To find: Number of words that can be formed so that vowels are never together

Number of words such that vowels are never the together = Total number of words - Number of words where vowels are together

Total number of words = $\frac{5!}{2!} = 60$

To find a number of words where vowels are together

Let the vowels I, I, A be represented by a single letter Z

⇒the new word is NDZ

A number of permutations = 3! = 6

Z is composed of 3 letters which can be permuted amongst themselves.

Number of permutations of Z = $\frac{3!}{2!} = 3$

Number of words where vowels are together = 6×3 = 18

⇒ Number of words where vowels are not together = 60 - 18 = 42

There are 42 words where vowels are not together

Q. 8. Find the number of arrangements of the letters of the word ‘ALGEBRA’ without altering the relative positions of the vowels and the consonants.

Answer : To find: number of arrangements without changing the relative position

The following table shows where the vowels and consonants can be placed

Consonants can be placed in the blank places

vowel				vowel			vowel

There are 3 spaces for vowels

There are 3 vowels out of which 2 are alike

Vowels can be placed in $\frac{3!}{2!} =$ 3 ways

There are 4 consonants, and they can be placed in $4!$ = 24 ways

⇒ Total number of arrangements = 24×3 = 72 ways

72 arrangements can be made

Q. 9. How many words can be formed from the letters of the word ‘SERIES’, which start with S and end with S?

Answer : To find: number of words which start and end with S

S	....	....	....	....	S

There are 4 places to fill up with 4 letters out of which 2 are of the same kind

⇒Number of words = $\frac{4!}{2!} = 12$

12 words are possible

Q. 10. In how many ways can the letters of the word 'PARALLEL' be arranged so that all L's do not come together?

Answer : To find: number of words where L do not come together

Let the three L's be treated as a single letter say Z

Number of words with L not the together = Total number of words - Words with L's together

The new word is PARAEZ

Total number of words = $\frac{8!}{2!3!} = 3360$

Words with L together = $6! = 720$

⇒ Words with L, not together = 3360 – 720 = 2640

There are 2640 words where L do not come together

Q. 11. How many different words can be formed with the letters of the word 'CAPTAIN'? In how many of these C and T are never together?

Answer : To find: number of words such that C and T are never together

Number of words where C and T are never the together = Total numbers of words - Number of words where C and T are together

Total number of words = $\frac{7!}{2!} = 2520$

Let C and T be denoted by a single letter Z

$\Rightarrow$New word is APAINZ

This can be permuted in $\frac{6!}{2!}$ = 360 ways

Z can be permuted among itself in 2 ways

$\Rightarrow$ Number of words where C and T are together = 360 × 2 = 720

$\Rightarrow$ Number of words where C and T are never together = 2520 - 720 = 1800

There are 1800 words where C and T are never together

Q. 12. In how many ways can the letters of the word 'ASSASSINATION' be arranged so that all S's are together?

Answer : To find: number of ways letters can be arranged such that all S's are together

Let all S's be represented by a single letter Z

New word is AAINATIONZ

Number of arrangements = $\frac{10!}{3!2!2!} = 151200$

Letters can be arranged in 151200 ways

Q. 13. (i)How many arrangements can be made by using all the letters of the word 'MATHEMATICS'?

(ii) How many of them begin with C?
(iii) How many of them begin with T?

Answer : (i) There are 11 letters of which 2 are of 1 kind, 2 are of another kind, 2 are of the 3rd kind

Total number of arrangements = $\frac{11!}{2!2!2!} = 4989600$

(ii)

C	….	….	….	….	….	…	….	….	…	…

There are 10 spaces to be filled by 10 letters of which 2 are of 3 different kinds

Number of arrangements = $\frac{10!}{2!2!2!} =$ 453600

(iii)

T	…	….	….	….	….	….	….	…	….	…

There are 10 spaces to be filled by 10 letters of which 2 are of 2 different kinds

Number of arrangements = $\frac{10!}{2!2!} = 907200$

Q. 14. In how many ways can the letters of the word 'INTERMEDIATE' be arranged so that:

(i) The vowels always occupy even places?
(ii) The relative orders of vowels and consonants do not change?

Answer : (i)

	Even		Even		Even		Even		Even		even

There are 6 even places and 6 vowels out of which 2 are of 1 kind, 3 are of the 2nd kind

The vowels can be arranged in $\frac{6!}{2!3!} =$ 60

There are 6 consonants out of which 2 is of one kind

Number of permutations = $\frac{6!}{2!} = 360$

⇒ Total number of words = $360 \times 60 = 21600$

(ii)

Vowel			Vowel			Vowel		Vowel	Vowel		vowel

There are 6 vowels to arrange in $\frac{6!}{2!3!}$

There are 6 consonants which can be arranged in $\frac{6!}{2!}$

⇒ Total number of ways = $\frac{6!}{2!3!} \times \frac{6!}{2!} = 21600$

Q. 15. (i)Find the number of different words by using all the letters of the word, 'INSTITUTION'.

In how many of them
(ii) are the three T' s together
(iii) are the first two letters the two N' s?

Answer : (i) There are 11 letters of which 3 are of 1 kind, 2 are of the 2nd kind ,3 are of the 3rd kind

Number of arrangements = $\frac{11!}{3!2!3!} = 554400$

(ii) Let all the three T's be denoted by a single letter Z

New word is INSIUIONZ

Number of permutations = $\frac{9!}{3!2!} = 30240$

(iii)

N	N	...	...	...	...	...	...	...	...	...

There are 9 places to be filled by 9 letters of which 3 are of 2 different kinds

Number of permutations = $\frac{9!}{3!3!} = 10080$

Q. 16. How many five - digit numbers can be formed with the digits 5, 4, 3, 5, 3?

Answer : To find: Number of 5 - digit numbers that can be formed

2 numbers are of 1 kind, and 2 are of another kind

Total number of permutations = $\frac{5!}{2!2!} = 30$

30 number can be formed

Q. 17. How many numbers can be formed with the digits 2, 3, 4, 5, 4, 3, 2 so that the odd digits occupy the odd places?

Answer : The table shows the places where the odd digits can be placed

Odd		odd		odd		Odd

There are 4 places

And 3 odd digits out of which 2 are of the same kind

Choose any 3 places out of the four places in ${}^4C_3$ ways = 4 ways

In each way, the 3 digits can be placed in $\frac{3!}{2!}$ ways = 3 ways

⇒ Total number of ways in which odd digits occupy odd places = 4×3 = 12

Now there are 4 remaining digits out of which 2 are same of 1 kind, and 2 are same as another kind

⇒ They can be arranged in the remaining places in $\frac{4!}{2!2!}$ = 6 ways

⇒ Total number of numbers where odd digit occupies odd places = 12 × 6 = 72

There are 72 such numbers

Q. 18. How many 7 - digit numbers can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Answer : To find: number of 7 digit

0 cannot be in the first place because that would make a 6 digit number

Total number of 7 - digit numbers = Total number of number possible - Number of numbers with 0 at the first place

Total number of numbers possible = $\frac{7!}{3!2!} = 420$

Number of numbers with 0 at first place = $\frac{6!}{3!2!} = 60$

⇒ Number of 7 - digit numbers = 420 - 60 = 360

360 seven - digit numbers are possible

Q. 19. How many 6 - digit numbers can be formed by using the digits 4, 5, 0, 3, 4, 5?

Answer : To find: number of 6 digit

0 cannot be in the first place because that would make a 5 - digit number

Total number of 6 - digit numbers = Total number of number possible - Number of numbers with 0 at the first place

Total number of numbers possible = $\frac{6!}{2!2!} = 180$

Number of numbers with 0 at first place = $\frac{5!}{2!2!} = 30$

⇒ Number of 6 - digit numbers = 180 - 30 = 150

150 six - digit numbers are possible

Q. 20. The letters of the word 'INDIA' are arranged as in a dictionary. What are the 1^{st}, 13^{th}, 49^{th} and 60^{th} words?

Answer : Alphabetical arrangement of letters: A,D,I,N

⇒ 1^{st} word: ADIIN

To find other words:

Case 1: words starting with A

Number of words = $\frac{4!}{2!} = 12$

⇒ 13^{th} word starts with D and is DAIIN

Case 2: words starting with D

Number of words = $\frac{4!}{2!} = 12$

Case 3: Words starting with I

Number of words = $4! = 24$

⇒ $(12+12+24+1)^{th} = 49^{th}$ word starts with N and is NAIID

Case 4: Words starting with N

Number of words = $\frac{4!}{2!} = 12$

⇒ $(48+12)^{th}$ word is the last word which starts with N

⇒ 60^{th} word = NDIIA

1st word: ADIIN

13^{th} word: DAIIN

9^{th} word: NAIID

60th word: NDIIA

Exercise 8F

Q. 1. A child has 6 pockets. In how many ways, he can put 5 marbles in his pocket?

Answer : The first marble can be put into the pockets in 6 ways,

i.e. Choose 1 Pocket From 6 by $\underline{{}^6C_1=6}$

Similarly second, third, Fourth, fifth & Sixth marble. Thus, the number of ways in which the child can put the marbles is $\underline{6^5}$

Q. 2. In how many ways can 5 bananas be distributed among 3 boys, there being no restriction to the number of bananas each boy may get?

Answer : As there is 5 banana, So suppose it as B_1, B_2, B_3, B_4, B_5 And Let the Boy be A_1, A_2, A_3

So B_1 can Be distributed to 3 Boys (A_1, A_2, A_3) by 3 ways,

Similarly, B_2, B_3, B_4, B_5 Can be distributed to 3 Boys by 3^4

So total number of ways is 3^5

Q. 3. In how many ways can 3 letters can be posted in 2 letterboxes?

Answer : Let Suppose Letterbox be B_1, B_2 and letters are L_1, L_2, L_3

So L_1 can be posted in any 2 letterboxes (B_1, B_2) by 2 ways

Similarly, L_2 can be posted in any 2 letterbox (B_1, B_2) by 2 ways

Similarly, L_3 can be posted in any 2 letterbox (B_1, B_2) by 2 ways

So total number of ways is $2^3 = 8$

Q. 4. How many 3-digit numbers are there when a digit may be repeated any numbers of time?

Answer : Let Suppose 3 digit number as 3 boxes as shown below. First Box is at 100^{th}place, the Second box is at 10^{th} place, and the Third box be at 1^{st} place.

1^{st}	2^{nd}	3^{rd}

To make a 3 digit number,

1^{st} box can be filled with nine numbers(1, 2, 3, 4, 5, 6, 7, 8, 9) if we include 0 in 1^{st} box then it become 2 digit number(i.e 010 is 2 digit number not 3 digit)

2^{nd} box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0) as repetition is allowed.

Similarly 3^{rd} box can be filled with ten numbers(1, 2, 3, 4, 5, 6, 7, 8, 9, 0)

Total number of ways is $9 \times 10 \times 10 = 900$

Q. 5. How many 4-digit numbers can be formed with the digits 0, 2, 3, 4, 5 when a digit may be repeated any numbers of time in any arrangement?

Answer : Let Suppose 4 digit number as 4 boxes as shown below. First Box is at 1000^{th}place, the Second box is at 100^{th} place, the Third box is at 10^{th} place, and Fourth box is at 1^{st} place.

1^{st}	2^{nd}	3^{rd}	4^{th}

The 1^{st} box can be filled with four numbers(2, 3, 4, 5) if we include 0 in the 1^{st}box then it becomes 3 digit number(i.e. 0234 is 3 digit number, not 4 digits)

The 2^{nd} box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Similarly, the 3^{rd} box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Similarly, the 4^{th} box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.

Total number of ways is $4 \times 5 \times 5 \times 5 = 500$

Q. 6. In how many ways can 4 prizes be given to 3 boys when a boy is eligible for all prizes?

Answer : Let suppose 4 prizes be P_1, P_2, P_3, P_4 and 3 boys be B_1, B_2, B_3

Now P_1 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

Similarly, P_2 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

Similarly, P_3 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways,

And P_4 can be distributed to 3 boys(B_1, B_2, B_3) by 3 ways

So total number of ways is $3 \times 3 \times 3 \times 3 = 81$

Q. 7. There are 4 candidates for the post of a chairman, and one is to be elected by votes of 5 men. In how many ways can the vote be given?

Answer : Let suppose 4 candidates be C_1, C_2, C_3, C_4 and 5 men be M_1, M_2, M_3, M_4, M_5

Now M_1 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_2 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_3 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

Similarly, M_4 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

And M_5 choose any one candidates from four (C_1, C_2, C_3, C_4) and give the vote to him by any 4 ways

So total numbers of ways are $4 \times 4 \times 4 \times 4 \times 4 = 1024$

Exercise 8G

Q. 1. In how many ways can 6 persons be arranged in

(i) a line, (ii) a circle?

Answer : (i) Let choose 1 person from 6 by ${}^6C_1 = 6$ and arranged it in line

Now choose another person from remaining 5 by ${}^{5}C_{1}=5$ and arranged it in line

Similarly, choose another person from remaining 4 by ${}^{4}C_{1}=4$ and arranged it in line

Similarly, choose another person from remaining 3 by ${}^{3}C_{1}=3$ and arranged it in line

Similarly, choose another person from remaining 2 by ${}^{2}C_{1}=2$ and arranged it in line

And choose another person from remaining 1 by ${}^{1}C_{1}=1$ and arranged it in line

So total number of ways is 6! =720

(ii) It is the same as above, by converting line arrangement into the circle but you need to remove some arrangement

Let suppose 6 persons as A, B, C, D ,E, F you need to arrange this 6 persons into a circle.

First, we arranged 6 persons in line(number of ways = 6!)

NOTE: A, B, C, D ,E, F and B, C, D, E, F, A consider as a different line, but when we arranged this 2 combination in circle then it becomes same,

i.e. Let takes us an example we need to arrange A, N, O, D, E.

We arrange it as shown. When we rotate first one, then 1st and 2nd became identical and so on that's why all 5 are identical, and we count it as 1

Now come back to our questions

So total number of arrangement is (6-1)! = 5! = 120

NOTE: When you want to arrange n persons in circle then a total number of ways is n!/n,

i.e. Total number of ways = (n-1)!

Q. 2. There are 5 men and 5 ladies to dine at a round table. In how many ways can they sit so that no ladies are together?

Answer : Let first arranged 5 men in the round table by 4! (By using the formula (n-1)! Mention above)

Now there are 5 gaps created between 5 men (check the figure)

So we arrange 5 ladies in this gap by 5!

A total number of ways to arrange 5 men and 5 ladies is 5! × 4! = 2880

Q. 3. In how many ways can 11 members of a committee sit at a round table so that the secretary and the joint secretary are always the neighbour of the president?

Answer : First assume the president(P), Joint secretary(JS) and secretary(S) to be 1 members(as shown below)

JS	P	S

So there are 9 members, a number of ways to arrange this 9 people is 8! (The formula used (n-1)!)

Now we need to look at the internal arrangement. There are 2 arrangement possible

JS	P	S

S	P	JS

<u>So total number of arrangement are (8!) × 2 = 80,640</u>

Q. 4. In how many ways can 8 persons be seated at a round table so that all shall not have the same neighbour in any two arrangement?

Answer : By using the formula (n-1)! (Mention in Solution-1)

So 8 persons can be arranged by 7!

Now each person have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (7!)/2 = 2520

Q. 5. In how many different ways can 20 different pearls be arranged to form a necklace?

Answer : We know that necklace in the form of a circle, So we need to arrange 20 pearls in Circle

20 pearls can be arranged by 19!

Now each pearl have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (19!)/2

Q. 6. In how many different ways can a garland of 16 different flowers be made?

Answer : It is also in the form of a circle, So we need to arrange 16flowers in Circle

16 flowers can be arranged by 15!

Now each flower have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (15!)/2

Exercise 8H

Q. 1. If (n + 1)! = 12 × [(n – 1)!], find the value of n.

Answer : To Find: Value of n

Given: (n+1)! = 12× [(n-1)!]

Formula Used: n! = (n) × (n-1) × (n-2) × (n-3) ………. 3 × 2 × 1

Now, $(n+1)! = 12\times [(n-1)!]$

$\Rightarrow (n+1) \times (n) \times [(n-1)!] = 12 \times [(n-1)!]$

$\Rightarrow (n+1) \times (n) = 12$

$\Rightarrow n^2+n = 12$

$\Rightarrow n^2+n-12 = 0$

$\Rightarrow (n-3)\ (n+4) = 0$

$\Rightarrow n = 3$ or, $n = -4$

But, n=-4 is not possible because in case of factorial (!) n cannot be negative.

Hence, n=3 is the correct answer.

Q. 2. If $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$, find the value of x.

Answer : To Find: Value of n

Given: $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

Formula Used: $n! = (n) \times (n-1) \times (n-2) \times (n-3) \ldots\ldots\ldots 3 \times 2 \times 1$

Now, $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

$\Rightarrow \frac{1}{24}+\frac{1}{120}=\frac{x}{720}$ (4! = 24, 5! = 120)

$\Rightarrow \frac{5+1}{120}=\frac{x}{720}$

$\Rightarrow \frac{6}{120}=\frac{x}{720}$

$\Rightarrow x = 36$

Q. 3. How many 3-digit numbers are there with no digit repeated?

Answer : Given: We have 10 numbers i.e. 0,1,2,3,4,5,6,7,8,9

To Find: Number of 3-digit numbers formed with no repetition of digits.

Conditions: No digit is repeated

Let us represent the 3-digit number

9 ways	9 ways	8 ways

First place can be filled with 9 numbers, i.e. 1,2,3,4,5,6,7,8,9 (0 cannot be placed as it will make it a 2-digit number) = 9 ways

Second place can be filled with remaining 9 numbers (as one number is used already) = 9 ways

Similarly, third place can be filled with 8 numbers = 8 ways

Total number of 3-digit numbers which can be formed

$= 9 \times 9 \times 8 = 648$

Q. 4. How many 3-digit numbers above 600 can be formed by using the digits 2, 3, 4, 5, 6, if repetition of digits is allowed?

Answer : Given: We have 5 digits i.e. 2,3,4,5,6

To Find: Number of 3-digit numbers

Condition: (i) Number should be greater than 600

(ii) Repetition of digits is allowed

For forming a 3 digit number, we have to fill 3 vacant spaces.

But as the number should be above 600, hence the first place must be occupied with 6 only because no other number is greater than 6.

Let us represent the 3-digit number

6	2,3,4,5,6	2,3,4,5,6

So the first place is filled with 6 = 1 ways

Second place can be filled with 5 numbers = 5 ways

Third place can be filled with 5 numbers = 5 ways

Total number of ways = 1 × 5 × 5 = 25

Total number of 3-digit numbers above 600 which can be formed by using the digits 2, 3, 4, 5, 6 with repetition allowed is 25

Q. 5. How many numbers divisible by 5 and lying between 4000 and 5000 can be formed from the digits 4, 5, 6, 7, 8 if repetition of digits is allowed?

Answer : Given: We have 5 digits, i.e. 4,5,6,7,8

To Find: Number of numbers divisible by 5

Condition: (i) Number should be between 4000 and 5000

(ii) Repetition of digits is allowed

Here as the number is lying between 4000 and 5000, we can conclude that the number is of 4-digits and the number must be starting with 4.

Now, for a number to be divisible by 5 must ends with 5

Let us represent the 4-digit number

4	4,5,6,7,8	4,5,6,7,8	5

Therefore,

The first place is occupied by 4 = 1 way

The fourth (last) place is occupied by 5 = 1 way

The second place can be filled by 5 numbers = 5 ways

The third place can be filled by 5 numbers = 5 ways

Total numbers formed = 1 × 5 × 5 × 1 = 25

There are 25 numbers which are divisible by 5 and lying between 4000 and 5000 and can be formed from the digits 4, 5, 6, 7, 8 with repetition of digits.

Q. 6. In how many ways can the letters of the word 'CHEESE' be arranged?

Answer : Given: We have 6 letters

To Find: Number of words formed with Letter of the word 'CHEESE.'

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots\ldots p_k!}$

Suppose we have these words – C,H,E_1,E_2,S,E_3

Now if someone makes two words as $CHE_1E_3SE_2$ and $CHE_2E_3SE_1$

<u>These two words are different because E_1, E2 and E_3 are different but we have three similar E's hence, in our case these arrangements will be a repetition of same words.</u>

In the word CHEESE, 3 E's are similar

$\therefore$ n = 6, p_1 = 3

$$\Rightarrow \frac{6!}{3!} = \frac{720}{6} = 120$$

In 120 ways the letters of the word 'CHEESE' can be arranged.

Q. 7. In how many ways can the letters of the word 'PERMUTATIONS' be arranged if each word starts with P and ends with S?

Answer : Given: We have 12 letters

To Find: Number of words formed with Letter of the word 'PERMUTATIONS.'

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots\ldots p_k!}$

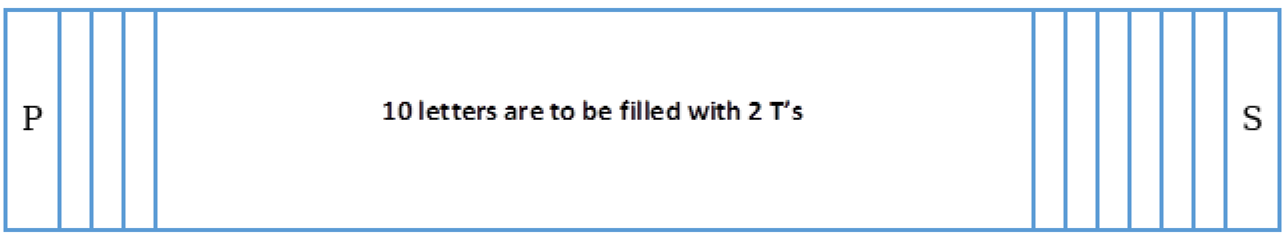
In the word 'PERMUTATIONS' we have 2 T's.

We have to start the word with P and end it with S, hence the first and last position is occupied with P and S respectively.

As two positions are occupied the remaining 10 positions are to be filled with 10 letters in which we have 2 T's.

NOTE:- Unless specified , assume that repetition is not allowed.

Let us represent the arrangement



Hence,

The first place is occupied by P = 1 way

The last place (12^{th}) is occupied by S = 1 way

For the remaining 10 places:

Using the above formula

Where,

n=10

p_1=2

$\Rightarrow \frac{10!}{2!} = 1814400$

Total number of ways are $1 \times 1814400 \times 1 = 1814400$ ways.

In 1814400 ways the letters of the word ‘PERMUTATIONS’ can be arranged if each word starts with P and ends with S.

Q. 8. How many different words can be formed by using all the letters of the word ‘ALLAHABAD’?

Answer : Given: We have 9 letters

To Find: Number of words formed with Letter of the word ‘ALLAHABAD.’

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots p_k!}$

‘ALLAHABAD’ consist of 9 letters out of which we have 4 A’s and 2 L’s.

Using the above formula

Where,

n=9

p_1=4

p_2=2

$$\Rightarrow \frac{9!}{4!2!} = 7560$$

7560 different words can be formed by using all the letters of the word ‘ALLAHABAD.’

Q. 9. How many permutations of the letters of the word ‘APPLE’ are there?

Answer : Given: We have 5 letters

To Find: Number of words formed with Letter of the word ‘APPLE.’

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots p_k!}$

‘APPLE’ consists of 5 letters out of which we have 2 Ps.

Using the above formula

Where,

n=5

$p_1=2$

$$\Rightarrow \frac{5!}{2!}=60$$

There are 60 permutations of the letters of the word ‘APPLE’.

Q. 10. How many words can be formed by the letters of the word ‘SUNDAY’?

Answer : Given: We have 6 letters

To Find: Number of words formed with Letter of the word ‘SUNDAY.’

‘SUNDAY’ consist of 6 letters.

NOTE: - Unless specified, assume that repetition is not allowed.

Let us represent the arrangement with an example

U (s,u,n,d,a,y)	N (s,n,d,a,y)	D (s,d,a,y)	A (s,a,y)	S (s,y)	Y

6 ways 5 ways 4 ways 3 ways 2 ways 1 way

We have 6 places

First place can be filled with 6 letters, i.e. S,U,N,D,A,Y = 6 ways

Second place can be filled with 5 letters (as one letter is already used in the first place) = 5 ways

Similarly,

Third place can be filled with 4 letters = 4 ways

The fourth place can be filled with 3 letters = 3 ways

The fifth place can be filled with 2 letters = 2 ways

The sixth place can be filled with 1 letters = 1 ways

Total number of letters = 6 × 5 × 4 × 3 × 2 × 1 = 720

720 words can be formed by the letters of the word 'SUNDAY.'

Q. 11. In how many ways can 4 letters be posted in 5 letter boxes?

Answer : Given: We have 4 letters and 5 letter boxes

To Find: Number of ways of posting letters.

One letter can be posted in any of 5 letter boxes.

We have to assume that all the letters are different.

So for first letter i.e. L_1, we have 5 ways

Similarly for,

L_2=5 ways

L_3=5 ways

L_4=5 ways

Total number of ways = 5 × 5 × 5 × 5 = 625

In 625 ways 4 letters can be posted in 5 letter boxes.

Q. 12. In how many ways can 4 women draw water from 4 taps if no tap remains unused?

Answer : Given: We have 4 women and 4 taps

To Find: Number of ways of drawing water

Condition: No tap remains unused

Let us represent the arrangement

4 ways	3 ways	2 ways	1 way

The first woman can use any of the four taps = 4 Ways

The second woman can use the remaining three taps = 3 ways

The third woman can use the remaining two taps = 2 ways

The fourth woman can use the remaining one tap = 1 way

Total number of ways = 4 × 3 × 2 × 1 = 24

There is 24 number of ways in which 4 women can draw water from 4 taps such that no tap remains unused.

Q. 13. How many 5-digit numbers can be formed by using the digits 0, 1 and 2?

Answer : Given: We have 3 digits, i.e. 0, 1 and 2

To Find: Number of 5-digit numbers formed

Let us represent the arrangement

2 ways, i.e. 1,2	3 ways	3 ways	3 ways	3 ways

For forming a 5-digit number, we have to fill 5 vacant spaces.

But the first place cannot be filled with 0, hence for filling first place, we have only 1 and 2

First place can be filled with 2 numbers, i.e. 1, 2 = 2 ways

Second place can be filled with 3 numbers = 3 ways

Third place can be filled with 3 numbers = 3 ways

The fourth place can be filled with 3 numbers = 3 ways

The fifth place can be filled with 3 numbers = 3 ways

Total number of ways = 2 × 3 × 3 × 3 × 3 = 162

162 5-digit numbers can be formed by using the digits 0, 1 and 2.

Q. 14. In how many ways can 5 boys and 3 girls be seated in a row so that each girl is between 2 boys?

Answer : Given: We have 5 boys and 3 girls

To Find: Number of ways of seating so that 5 boys and 3 girls are seated in a row and each girl is between 2 boys

The formula used: The number of permutations of n different objects taken r at a time (object does not repeat) is $^nP_r = \frac{n!}{(n-r)!}$

The only arrangement possible is

B__B__B__B__B

Number of ways for boys $= {}^nP_r$

$= {}^5P_5$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

=120

There are 3 girls, and they have 4 vacant positions

Number of ways for girls $= {}^4P_3 = 24$ ways

$$= \frac{4!}{(4-3)!}$$

$$= \frac{4!}{1!}$$

= 24

Total number of ways = 24 × 120 = 2880

In 2880 ways 5 boys and 3 girls can be seated in a row so that each girl is between 2 boys.

Q. 15. A child has plastic toys bearing the digits 4, 4 and 5. How many 3-digit numbers can he make using them?

Answer : Given: We have toys with bearing 4, 4 and 5

To Find: Number of 3-digit numbers he can make

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest, if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots\ldots p_k!}$

The child has to form a 3-digit number.

Here the child has two 4's.

We have to use the above formula

Where,

n=3

p_1=2

$$\Rightarrow \frac{3!}{2!} = 3 \text{ ways}$$

The numbers are 544, 454 and 445.

He can make 3 3-digit numbers.

Q. 16. In how many ways can the letters of the word ‘PENCIL’ be arranged so that N is always next to E?

Answer : Given: We have 6 letters

To Find: Number of ways to arrange letters P,E,N,C,I,L

Condition: N is always next to E

Here we need EN together in all arrangements.

So, we will consider EN as a single letter.

Now, we have 5 letters, i.e. P,C,I,L and ‘EN’.

5 letters can be arranged in ${}^5P_5$ways

$\Rightarrow {}^5P_5$

$\Rightarrow \frac{5!}{(5-5)!}$

$\Rightarrow \frac{5!}{0!}$

$\Rightarrow 120$

In 120 ways we can arrange the letters of the word ‘PENCIL’ so that N is always next to E.