Arithmetic Progression

Exercise 11A

Q. 1. Write first 4 terms in each of the sequences:

(i) $a_n = (5n + 2)$

(ii) $a_n = \frac{(2n-3)}{4}$

(iii) $a_n = (-1)^{n-1} \times 2^{n+1}$

Answer : To Find: First four terms of given series.

(i) Given: n^{th} term of series is (5n + 2)

Put n=1, 2, 3, 4 in n^{th} term, we get first (a1), Second (a2), Third (a3) & Fourth (a4) term

$a_1 = (5 \times 1 + 2) = 7$

$a_2 = (5 \times 2 + 2) = 12$

$a_3 = (5 \times 3 + 2) = 17$

$a_4 = (5 \times 4 + 2) = 22$

First four terms of given series is 7, 12,17,22

<u>ALTER</u>: When you find or you have first term (a or a_1) and second term (a_2) then find the difference (a_2 - a_1)

Now add this difference in last term to get the next term

For example a_1= 7 and a_2= 12, so difference is 12 – 5 = 7

Now a_3 = 12 + 5 = 17, a_4 = 17 + 5 = 22

(This method is only for A.P)

<u>NOTE</u>: When you have nth term in the form of (a × n + b)

Then common difference of this series is equal to a.

This type of series is called A.P (Arithmetic Progression)

(Where a, b are constant, and n is number of terms)

(ii) Given: n^{th} term of series is $\frac{(2n-3)}{4}$

Put n=1, 2, 3, 4 in n^{th} term, we get first (a1), Second (a2), Third (a3) & Fourth (a4) term.

$$a_1 = \frac{(2 \times 1 - 3)}{4} = \frac{-1}{4}$$

$$a_2 = \frac{(2 \times 2 - 3)}{4} = \frac{1}{4}$$

$$a_3 = \frac{(2 \times 3 - 3)}{4} = \frac{3}{4}$$

$$a_4 = \frac{(2 \times 4 - 3)}{4} = \frac{5}{4}$$

First four terms of given series are $\frac{-1}{4}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$

(iii) Given: n^{th} term of series is $(-1)^{n-1} \times 2^{n+1}$

Put n=1, 2, 3, 4 in n^{th} term, we get first (a1), Second (a2), Third (a3) & Fourth (a4) term.

$a_1 = (-1)^{1-1} \times 2^{1+1} = (-1)^0 \times 2^2 = 1 \times 4 = 4$

$a_2 = (-1)^{2-1} \times 2^{2+1} = (-1)^1 \times 2^3 = (-1) \times 8 = (-8)$

$a_3 = (-1)^{3-1} \times 2^{3+1} = (-1)^2 \times 2^4 = 1 \times 16 = 16$

$a_4 = (-1)^{4-1} \times 2^{4+1} = (-1)^3 \times 2^5 = (-1) \times 32 = (-32)$

First four terms of given series are 4, –8 , 16 ,–32

Q. 2. Find the first five terms of the sequence, defined by

$a_1 = 1$, $a_n = a_{n-1} + 3$ for $n \geq 2$.

Answer : To Find: First five terms of a given sequence.

Condition: $n \geq 2$

Given: $a_1 = 1$, $a_n = a_{n-1} + 3$ for $n \geq 2$

Put n= 2 in n^{th} term (i.e. a_n), we have

$a_2 = a_{2-1} + 3 = a_1 + 3 = 1 + 3 = 4$ (as $a_1 = 1$)

Put n= 3 in n^{th} term (i.e. a_n), we have

$a_3 = a_{3-1} + 3 = a_2 + 3 = 4 + 3 = 7$ (as $a_2 = 4$)

Put n= 4 in n^{th} term (i.e. a_n), we have

$a_4 = a_{4-1} + 3 = a_3 + 3 = 7 + 3 = 10$ (as $a_3 = 7$)

Put n= 5 in n^{th} term (i.e. a_n), we have

$a_5 = a_{5-1} + 3 = a_4 + 3 = 10 + 3 = 13$ (as $a_2 = 10$)

First five terms of a given sequence is 1, 4, 7, 10, 13

Q. 3. Find the first 5 terms of the sequence, defined by

$a_1 = -1$, $a_n = \frac{a_{n-1}}{n}$ for $n \geq 2$.

Answer : To Find: First five terms of a given sequence.

Condition: $n \geq 2$

$$a_1 = -1,\ a_n = \frac{a_{n-1}}{n} \text{ for } n \geq 2.$$

Put n= 2 in n^{th} term (i.e. a_n), we have

$a_2 = \frac{(-1)}{2}$ (as $a_1 = -1$)

Put n= 3 in n^{th} term (i.e. a_n), we have

$a_3 = \frac{(-1)}{6}$ (as $a_2 = \frac{(-1)}{2}$)

Put n= 4 in n^{th} term (i.e. a_n), we have

$a_4 = \frac{(-1)}{24}$ (as $a_3 = \frac{(-1)}{6}$)

Put n= 5 in n^{th} term (i.e. a_n), we have

$a_5 = \frac{(-1)}{120}$ (as $a_3 = \frac{(-1)}{24}$)

First five terms of a given sequence are –1, $\frac{(-1)}{2}$, $\frac{(-1)}{6}$, $\frac{(-1)}{24}$, $\frac{(-1)}{120}$

Q. 4. Find the 23^{rd} term of the AP 7, 3, 1, –1, –3, …

Answer : To Find: 23^{rd} term of the AP

Given: The series is 7, 5, 3, 1, –1, –3, …

a_1= 7, a_2= 5 and d= 3–5= –2

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

So put n =23 in above formula, we have

$a_{23} = a_1 + (23 - 1)(-2) = 7 - 44 = -37$

So 23^{rd} term of AP is equal to –37.

Q. 5. Find the 20^{th} term of the AP $\sqrt{2}$, $3\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$, ….

Answer : To Find: 20^{th} term of the AP

Given: The series is √2, 3√2, 5√2, 7√2, ….

$a_1=\sqrt{2}$, $a_2= 3\sqrt{2}$ and d= $3\sqrt{2}-\sqrt{2}= 2\sqrt{2}$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$a_{20}= a_1 + (20 - 1)(2\sqrt{2})= \sqrt{2} + 38\sqrt{2}= 39\sqrt{2}$

So 20[rd] term of AP is equal to $39\sqrt{2}$.

Q. 6. Find the n^{th} term of the AP 8, 3, –2, –7, –12, ….

Answer : To Find: n^{th} term of the AP

Given: The series is 8, 3, –2, –7, –12, ….

$a_1=8$, $a_2= 3$ and $d= 3–8= –5$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$a_n= a_1 + (n - 1)(–5) = 8– (5n–5) = 8– 5n + 5 = 13– 5n$

So the n^{th} term of AP is equal to $13– 5n$

Q. 7. Find the n^{th} term of the AP 1 ,$\frac{5}{6}, \frac{2}{3}, \frac{1}{2}$, ….

Answer : To Find: n^{th} term of the AP

Given: The series is 1, $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}$, , …

$a_1=1$, $a_2= \frac{5}{6}$ and $d= \frac{5}{6} –1= \frac{-1}{6}$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$$a_n= a_1 + (n - 1)\left(\frac{-1}{6}\right) = 1– \left(\frac{n-1}{6}\right) = \frac{6-n+1}{6} =\left(\frac{7-n}{6}\right)$$

So the n^{th} term of AP is equal to ($\frac{7-n}{6}$)

Q. 8. Which term of the AP 9, 14, 19,24, 29, …. is 379?

Answer : To Find: we need to find n when $a_n = 379$

Given: The series is 9, 14, 19,24, 29, …. and a_n=379

a_1=9, a_2= 14 and d=14–9 = 5

(Where a=a_1 is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$a_n = 379 = a_1 + (n–1)5$

379 – 9 = (n–1)5 [subtract 9 on both side]

370 = (n–1 � � �)5

74 = (n–1) [Divide both side by 5]

n = 75^{th}

The 75^{th} term of this AP is equal to 379.

Q. 9. Which term of the AP 64, 60, 56, 52, 48, …. is 0?

Answer : To Find: we need to find n when $a_n = 0$

Given: The series is 64, 60, 56, 52, 48, … and a_n= 0

a_1=64, a_2= 60 and d=60–64 = –4

(Where a=a_1 is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$a_n = 0 = a_1 + (n–1)(–4)$

0– 64 = (n–1)(–4) [subtract 64 on both sides]

– 64 = (n–1)(–4)

64 = (n–1)4 [Divide both side by ‘–‘]

16 = (n–1) [Divide both side by 4]

n = 17th [add 1 on both sides]

The 17th term of this AP is equal to 0.

Q. 10. How many terms are there in the AP 11, 18, 25, 32, 39, …. 207?

Answer : To Find: we need to find a number of terms in the given AP.

Given: The series is 11, 18, 25, 32, 39, …. 207

a_1=11, a_2= 18,d=18–11 = 7 and a_n=207

(Where a=a_1 is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: a_n = a + (n–1)d

a_n= 207 = a_1 + (n–1)(7)

207– 11 = (n–1)(7) [subtract 11 on both sides]

196 = (n–1)(7)

28 = (n–1) [Divide both side by 7]

n = 29 [add 1 on both sides]

So there are 29 terms in this AP.

Q. 11. How many terms are there in the AP $1\frac{5}{6}, 1\frac{1}{6}, 1\frac{1}{6}, \frac{-1}{6}, \frac{-5}{6}$, …., –16 $\frac{1}{6}$?

Answer : To Find: we need to find number of terms in the given AP.

Given: The series is $1\frac{5}{6}, 1\frac{1}{6}, 1\frac{1}{6}, \frac{-1}{6}, \frac{-5}{6}$, …., –16 $\frac{1}{6}$.

$$a_1=1\frac{5}{6}=\frac{11}{6},\ a_2=1\frac{1}{6}=\frac{7}{6},\ d=\left(\frac{7}{6}\right)-\left(\frac{11}{6}\right)=\frac{-4}{6} \text{ and } a_n=-16\ \frac{1}{6}\ =\frac{-95}{6}$$

(Where a=a_1 is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$$a_n = \frac{-97}{6} = a_1 + (n-1)\left(\frac{-2}{3}\right)$$

$$\frac{-97}{6} - \frac{11}{6} = (n-1)\left(\frac{-2}{3}\right) \text{ [subtract } \frac{11}{6} \text{ on both sides]}$$

$$\frac{-108}{6} = (n-1)\left(\frac{-2}{3}\right) \text{ [Multiply both side by } \frac{-3}{2}\text{] or [Divide both side by } \frac{-2}{3}\text{]}$$

27 = (n–1) [add 1 on both sides]

n = 28

So there are 28 terms in this AP.

Q. 12. Is - 47 a term of the AP 5, 2, –1, –4, –7, ….?

Answer : To Find: –47 is a term of the AP or not.

Given: The series is 5, 2, –1, –4, –7, ….

a_1=5, a_2= 2, and d=2–5 = –3 (Let suppose an = –47)

NOTE: n is a natural number.

(Where a=a_1 is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

a_n = –47 = a + (n - 1)d

–47 = 5 + (n - 1)(–3)

–47–5 = (n - 1)(–3) [subtract 5 on both sides]

52 = (n - 1)(3) [Divide both side by ‘–‘]

17.33 = (n - 1) [Divide both side by 3]

18.33 = n [add 1 on both sides]

As n is not come out to be a natural number, So –47 is not the term of this AP.

Q. 13. The 5th and 13th terms of an AP are 5 and –3 respectively. Find the AP and its 30th term.

Answer : To Find: AP and its 30th term (i.e. a_{30}=?)

Given: $a_5 = 5$ and $a_{13} = -3$

Formula Used: $a_n = a + (n - 1)d$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

By using the above formula, we have

$a_5 = 5 = a + (5 - 1)d$, and $a_{13} = -3 = a + (13 - 1)d$

$a + 4d = 5$ and $a + 12d = -3$

On solving above 2 equation, we and a + 12d = –3get

a = 9 and d= (–1)

So $a_{30} = 9 + 29(-1) = -20$

AP is (9,8,7,6,5,4……) and 30th term = –20

Q. 14. The 2nd, 31st and the last terms of an AP are $7\frac{3}{4}, \frac{1}{2}$ and $-6\frac{1}{2}$ respectively. Find the first term and the number of terms.

Answer : To Find: First term and number of terms.

Given: $a_2 = \frac{31}{4}$, $a_{31} = \frac{1}{2}$, and an = $\frac{-13}{2}$

Formula Used: $a_n = a + (n - 1)d$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

By using above formula, we have

$a_2 = \frac{31}{4} = a + d$ and $a_{31} = \frac{1}{2} = a + (31 - 1)d$

On solving both equation, we get

a = 8 and d = –0.25

Now an = $\frac{-13}{2}$ = 8 + (n - 1)(–0.25)

On solving the above equation, we get

N = 59

So the First term is equal to 8 and the number of terms is equal to 59.

Q. 15. If the 9^{th} term of an AP is 0, prove that its 29^{th} term is double the 19^{th}term.

Answer : Prove that: 29^{th} term is double the 19^{th} term (i.e. $a_{29} = 2a_{19}$)

Given: $a_9 = 0$

(Where $a=a_1$ is first term, a_2 is second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

So $a_9 = 0 \rightarrow a + (9 - 1)d = 0$

a + 8d = 0

a = (–8d) ….equation (i)

Now $a_{29} = a + (29 - 1)d$ and $a_{19} = a + (19 - 1)d$

$a_{29} = a + 28d$ and $a_{19} = a + 18d$ ….equation (ii)

By using equation (i) in equation (ii), we have

$a_{29} = –8d + 28d$ and $a_{19} = –8d + 18d$

$a_{29} = 20d$ and $a_{19} = 10d$

So $a_{29} = 2a_{19}$

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Q. 16. The 4^{th} term of an AP is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.

Answer : To Find: First term (a) and common difference (d)

Given: $a_4 = 3a_1$ and $a_7 = 2a_3 + 1$

(Where $a = a_1$ is first term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$a_4 = 3a_1 \rightarrow a + 3d = 3a \rightarrow 3d = 2a$ ….equation (i) and

$a_7 = 2a_3 + 1 \rightarrow a + 6d = 2(a + 2d) + 1 \rightarrow 2d = a + 1$ ….equation (ii)

On solving both equation (i) & (ii), we get

$a = 3$ and $d = 2$

So the first term is equal to 3, and the common difference is equal to 2.

Q. 17. If 7 times the 7^th^ term of an AP is equal to 11 times its 11^th^ term, show that the 18^th^ term of the AP is zero.

Answer : Show that: 18^{th} term of the AP is zero.

Given: $7a_7 = 11a_{11}$

(Where a_7 is Seventh term, a_{11} is Eleventh term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$7(a + 6d) = 11(a + 10d)$

$7a + 42d = 11a + 110d \rightarrow 68d = (-4a)$

$a + 17d = 0$ ….equation (i)

Now $a_{18} = a + (18 - 1)d$

So $a + 17d = 0$ [by using equation (i)]

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[NOTE: If n times the n^{th} term of AP is equal to m times the m^{th} term of same AP then its $(m + n)^{th}$ term is equal to zero]

Q. 18. Find the 28th term from the end of the AP 6, 9, 12, 15, 18, …., 102.

Answer : To Find 28th term from the end of the AP.

Given: The AP is 6, 9, 12, 15, 18, …., 102

$a_1 = 6$, $a_2 = 9$, $d = 9–6 = 3$ and $l = 102$

Formula Used: nth term from the end = l– (n–1)d

(Where $l$is last term and d is common difference of given AP)

By using nth term from the end = l– (n–1)d formula

28th term from the end = 102– 27d $\rightarrow$ 102– 27 $\times$ 3 = 21

So 28th term from the end is equal to 21.

Q. 19. Find the 16th term from the end of the AP 7, 2, –3, –8, –13, …., –113

Answer : To Find : 28th term from the end of the AP.

Given: The AP is 7, 2, –3, –8, –13, …., –113

$a_1 = 7$, $a_2 = 2$, $d = 2–7 = –5$ and $l = –113$

Formula Used: nth term from the end = l– (n–1)d

(Where l is last term and d is common difference of given AP)

By using nth term from the end = l– (n–1)d formula

16th term from the end = (–113) – 15d → (–113)–15 × (–5) = –38

So 16th term from the end is equal to –38.

Q. 20. How many 3 - digit numbers are divisible by 7?

Answer : To Find : 3 - digit numbers divisible by 7.

First 3 - digit number divisible by 7 is 105

Second 3 - digit number divisible by 7 is 112 and

Last 3 - digit number divisible by 7 is 994.

Given: The AP is 105, 112, 119,…………..,994

$a_1 = 105$, $a_2 = 112$, $d = 112–105 = 7$ and $a_n = 994$

(Where $a=a_1$ is First term, a_2 is Second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n - 1)d$

$994 = 105 + (n - 1)7$

$889 = (n - 1)7$

$127 = (n - 1)$

$n = 128$

So, There are total of 128 three - digit number which is divisible by 7.

Q. 21. How many 2 - digit numbers are divisible by 3?

Answer : To Find : 2 - digit numbers divisible by 3.

First 2 - digit number divisible by 3 is 12

Second 2 - digit number divisible by 3 is 15 and

Last 2 - digit number divisible by is 99.

Given: The AP is 12, 15, 18,…………..,99

$a_1 = 12$, $a_2 = 15$, $d = 15–12 = 3$ and $a_n = 99$

(Where $a=a_1$ is First term, a_2 is Second term, a_n is nth term and d is common difference of given AP)

Formula Used: $a_n = a + (n – 1)d$

$99 = 12 + (n - 1)3$

$87 = (n – 1)3$

$29 = (n – 1)$

$n = 30$

So, There are total of 30 two - digit number which is divisible by 3.

Q. 22. If θ_1, θ_2, θ_3, …., θ_n are in AP whose common difference is d, show that

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta n = \frac{(\tan\theta_n - \tan\theta_1)}{\sin d}.$$

Answer :

Show that: $\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta n = \frac{(\tan\theta_n - \tan\theta_1)}{\sin d}$

Given: Given AP is $\theta_1, \theta_2, \theta_3, ..., \theta_n$

$a = \theta_1,\ a_2 = \theta_2$ and $d = \theta_2 - \theta_1 = \theta_3 - \theta_2 = \theta_4 - \theta_3 = ... = \theta_n - \theta_{n-1}$

$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta n = \frac{1}{\cos\theta 1} \times \frac{1}{\cos\theta 2} + \frac{1}{\cos\theta 2} \times \frac{1}{\cos\theta 3} + ... + \frac{1}{\cos\theta n-1} \times \frac{1}{\cos\theta n}$

Multiply both side by sin d

$\sin d\,(\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta n) = \frac{\sin(\theta 2 - \theta 1)}{\cos\theta 1} \times \frac{1}{\cos\theta 2} + \frac{\sin(\theta 3 - \theta 2)}{\cos\theta 2} \times \frac{1}{\cos\theta 3} + ... + \frac{\sin(\theta n - \theta n-1)}{\cos\theta n-1} \times \frac{1}{\cos\theta n}$

[NOTE: $\sin(x - y) = \sin x \cos y - \cos x \sin y$, & $\sec\theta \times \cos\theta = 1$]

By using above formula on R.H.S. , we get

R.H.S. = $\tan\theta_2 - \tan\theta_1 + \tan\theta_3 - \tan\theta_2 + \tan\theta_4 - \tan\theta_3$ .... $+ \tan\theta_n - \tan\theta_{n-1}$

R.H.S. = $\tan\theta_n - \tan\theta_1$ (All the remaining term cancle out)

$\sin d\,(\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta n) = \tan\theta_n - \tan\theta_1$ (Divide sin d on both sides), we get

$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta n = \dfrac{(\tan\theta_n - \tan\theta_1)}{\sin d}$.

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Q. 23. In an AP, it is being given that $\frac{T_4}{T_7} = \frac{2}{3}$. Find $\frac{T_7}{T_{10}}$.

Answer : To Find: $\frac{T7}{T10}$

Given: $\frac{T_4}{T_7} = \frac{2}{3}$

(Where T_n is nth term and d is common difference of given AP)

Formula Used: $T_n = a + (n - 1)d$

$$\frac{T_4}{T_7} = \frac{2}{3} \rightarrow \frac{a+3d}{a+6d} = \frac{2}{3} \text{ (cross multiply)}$$

3a + 9d = 2a + 12d → a = 3d ....equation (i)

$$\text{Now } \frac{T_7}{T_{10}} = \frac{a+6d}{a+9d} \rightarrow \frac{T_7}{T_{10}} = \frac{3d+6d}{3d+9d} = \frac{9d}{12d}$$

$$\frac{T_7}{T_{10}} = \frac{3}{4}$$

$$\text{So } \frac{T_7}{T_{10}} = \frac{3}{4}$$

Q. 24. Three numbers are in AP. If their sum is 27 and their product is 648, find the numbers.

Answer : To Find: The three numbers which are in AP.

Given: Sum and product of three numbers are 27 and 648 respectively.

Let required number be (a - d), (a), (a + d). Then,

(a - d) + a + (a + d) = 27 ⇒ 3a = 27 ⇒ a = 9

Thus, the numbers are (9 - d), 9 and (9 + d).

But their product is 648.

∴ (9 - d) × 9 × (9 + d)= 648

⇒ (9 - d)(9 + d)= 72

⇒ $81 - d^2 = 72 \Rightarrow d^2 = 9 \Rightarrow d = \pm 3$

When $d = 3$ numbers are 6, 9, 12

When $d = (-3)$ numbers are 12, 9, 6

So, Numbers are 6, 9, 12 or 12, 9, 6.

Q. 25. The sum of three consecutive terms of an AP is 21, and the sum of the squares of these terms is 165. Find these terms

Answer : To Find: The three numbers which are in AP.

Given: Sum and sum of the squares of three numbers are 21 and 165 respectively.

Let required number be $(a - d), (a), (a + d)$. Then,

$(a - d) + a + (a + d) = 21 \Rightarrow 3a = 21 \Rightarrow a = 7$

Thus, the numbers are $(7 - d)$, 7 and $(7 + d)$.

But their sum of the squares of three numbers is 165.

$\therefore (7 - d)^2 + 7^2 + (7 + d)^2 = 165$

$\Rightarrow 49 + d^2 - 14d + 49 + d^2 + 14d = 116$

$\Rightarrow 2d^2 = 18 \Rightarrow d^2 = 9 \Rightarrow d = \pm 3$

When $d=3$ numbers are 4, 7, 10

When $d= (-3)$ numbers are 10, 7, 4

So, Numbers are 4, 7, 10 or 10, 7, 4.

Q. 26. The angles of a quadrilateral are in AP whose common difference is 10°. Find the angles.

Answer : To Find: The angles of a quadrilateral.

Given: Angles of a quadrilateral are in AP with common difference = 10°.

Let the required angles be a, $(a + 10°)$, $(a + 20°)$ and $(a + 30°)$.

Then, $a + (a + 10°) + (a + 20°) + (a + 30°) = 360° \Rightarrow 4a + 60° = 360° \Rightarrow a = 75°$

NOTE: Sum of angles of quadrilateral is equal to 360°

So Angles of a quadrilateral are 75°, 85°, 95° and 105°.

Q. 27. The digits of a 3 - digit number are in AP, and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Answer : To Find: The number

Given: The digits of a 3 – digit number are in AP, and their sum is 15.

Let required digit of 3 - digit number be (a - d), (a), (a + d). Then,

$(a - d) + (a) + (a + d) = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$

(Figure show 3 digit number original number)

5 - d	5	5 + d

(Figure show 3 digit number in reversing form)

5 + d	5	5 - d

So, $(5 + d) \times 100 + 5 \times 10 + (5 - d) \times 1 = \{(5 - d) \times 100 + 5 \times 10 + (5 + d) \times 1\} - 594$

$200d - 2d = -594 \Rightarrow d = -3$ and $a = 5$

<u>So the original number is 852</u>

Q. 28. Find the number of terms common to the two arithmetic progressions 5, 9, 13, 17, …., 217 and 3, 9, 15, 21, …., 321.

Answer : To Find: The number of terms common to both AP

Given: The 2 AP's are 5, 9, 13, 17, …., 217 and 3, 9, 15, 21, …., 321

As we find that first common term of both AP is 9 and the second common term of both AP is 21

Let suppose the new AP whose first term is 9, the second term is 21, and the common difference is 21 – 9 = 12

NOTE: As first AP the last term is 217 and second AP last term is 321. So last term of supposing AP should be less than or equal to 217 because after that there are no common terms

Formula Used: $T_n = a + (n - 1)d$

(Where T_n is nth term and d is common difference of given AP)

$217 \geq a + (n - 1)d \Rightarrow 9 + (n - 1)12 \leq 217$

$\therefore (n - 1)12 \leq 208 \Rightarrow (n - 1) \leq 17.33 \Rightarrow n \leq 18.33$

So, Number of terms common to both AP is 18.

Q. 29. We know that the sum of the interior angles of a triangle is 180°. Show that the sum of the interior angles of polygons with 3, 4, 5, 6, …. sides form an arithmetic progression. Find the sum of the interior angles for a 21 - sided polygon.

Answer : Show that: the sum of the interior angles of polygons with 3, 4, 5, 6, …. sides form an arithmetic progression.

To Find: The sum of the interior angles for a 21 - sided polygon.

Given: That the sum of the interior angles of a triangle is 180°.

NOTE: We know that sum of interior angles of a polygon of side n is (n – 2) x 180°.

Let $a_n = (n - 2) \times 180° \Rightarrow$ Since a_n is linear in n. So it forms AP with 3, 4, 5, 6,……sides

{a_n is the sum of interior angles of a polygon of side n}

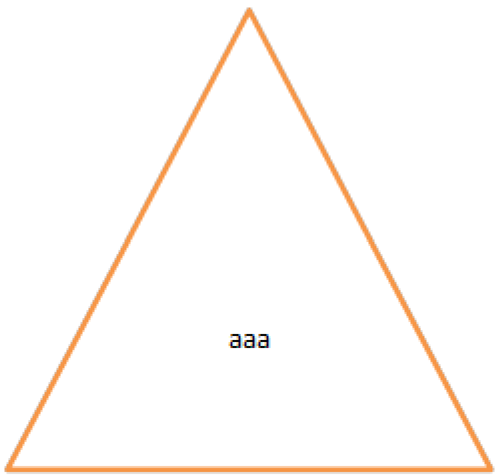
By using the above formula, we have

$a_{21} = (21 - 2) \times 180°$

$a_{21} = 3420°$

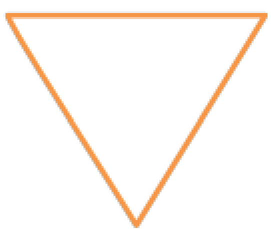
So, the Sum of the interior angles for a 21 - sided polygon is equal to 3420°.

Q. 30. A side of an equilateral triangle is 24 cm long. A second equilateral triangle is inscribed in it by joining the midpoints of the sides of the first triangle; the process is continued. Find the perimeter of the sixth inscribed equilateral triangle.



Answer : To Find: The perimeter of the sixth inscribed equilateral triangle.

1st Given: Side of an equilateral triangle is 24 cm long.



As 2nd triangle is formed by joining the midpoints of the sides of the first triangle whose side is equal to 24cm

$\mathbf{2^n}$ [As shown in the figure]

So Side of a 2nd equilateral triangle is 12 cm long [half of the first triangle side]

∴ Side of 2nd equilateral triangle = half of side of a 1st equilateral triangle

∴ Side of 3rd equilateral triangle = half of side of a 2nd equilateral triangle

∴ …………. and So on

Therefore, Side of 6th equilateral triangle = half of side of a 5th equilateral triangle

equilateral triangle	Length of side (in cm)
1^{st}	24
2^{nd}	12
3^{rd}	6
4^{th}	3
5^{th}	1.5
6^{th}	0.75

So, Perimeter of a 6^{th} equilateral triangle is 3 times the side of a 6^{th} equilateral triangle

[**NOTE:** Perimeter of the triangle is equal to the sum of all three sides of the triangle, and in case of an equilateral triangle all sides are equal]

So, Perimeter of 6^{th} equilateral triangle = $3 \times 0.75 = 2.25$ cm

Q. 31. A man starts repaying a loan as the first instalment of 10000. If he increases the instalment by 500 every month, what amount will he pay in 30thinstalment?

Answer : To Find: what amount will he pay in the 30th instalment.

Given: first instalment =10000 and it increases the instalment by 500 every month.

∴ So it form an AP with first term is 10000, common difference 500 and number of instalment is 30

Formula Used: $T_n = a + (n - 1)d$

(Where a is first term, T_n is nth term and d is common difference of given AP)

∴ $T_n = a + (n - 1)d \Rightarrow T_n = 10000 + (30 - 1)500 \Rightarrow T_n = 10000 + 29 \times 500$

∴ $T_n = 10000 + 14500 \Rightarrow T_n = 24{,}500$

So, he will pay 24,500 in the 30th instalment.

Exercise 11B

Q. 1. Find the sum of 23 terms of the AP 17, 12, 7, 2, –3, ….

Answer : To Find: The sum of 25 terms of the given AP series.

Sum of n terms of an AP with first term a and common difference d is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here, a = 17, n = 23 and d = - 5

$$S = \frac{23}{2}[34 + 22(-5)]$$

$$\Rightarrow S = \frac{23}{2}[34 - 110] = \frac{23}{2} \times (-76)$$

= – 874

Sum of 23 terms of the AP IS – 874.

Q. 2. Find the sum of 16 terms of the AP 6, 5 $\frac{1}{3}$, 4 $\frac{2}{3}$, 4, ….

Answer : To find: Sum of 16 terms of the AP

Given:

First term = 6

Common difference = $-\frac{2}{3}$

$$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_n = \frac{16}{2}\left[2 \times 6 + 15 \times \left(-\frac{2}{3}\right)\right]$$

$$\Rightarrow S_n = \frac{16}{2}[12 - 10]$$ $S_n = 16$

The sum of first 16 terms of the series is 16

Q. 3. Find the sum of 25 terms of the AP $\sqrt{2}$, 2$\sqrt{2}$, 3$\sqrt{2}$, 4$\sqrt{2}$, ….

Answer : To Find: The sum of 25 terms of the given AP series.

Sum of n terms of an AP with first term a and common difference d is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here,

a = √2, n = 25, d = √2 $\Rightarrow S = \frac{25}{2}\left[2\sqrt{2} + 24\sqrt{2}\right]$

= 25 × 13 × √2 = 325√2

Sum of 25 terms is 325√2.

Q. 4. Find the sum of 100 term of the AP 0.6, 0.61, 0.62, 0.63, ….

Answer : To Find: The sum of 100 terms of the given AP series.

Sum of n terms of an AP with first term a and common difference d is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here a = 0.6, n = 100, d = 0.01

$$\Rightarrow S = \frac{100}{2}[1.2 + 99 \times 0.01]$$

= 50[1.2 + 0.99]

= 50 × 2.19

109.5

Sum of the series is 109.5

Q. 5. Find the sum of 20 terms of the AP (x + y), (x – y), (x – 3y), ….

Answer : To Find: The sum of 20 terms of the given AP.

Sum of n terms of an AP with first term a and common difference d is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here a = x + y, n = 20, d = - 2y

⇒ S = 10[2x + 2y + 19(- 2y)] = 10[2x + 2y - 38y] = 10[2x - 36y]

⇒ S = 20[x - 18y]

Sum of the series is 20(x - 18y).

Q. 6. Find the sum of n term of the AP $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, ...$

Answer : To Find: The sum of n terms of the given AP.

Sum of n terms of an AP with first term a and common difference d is given by

$$S = \frac{n}{2}[2a + (n-1)d]$$

Here a = x - y, d = 2x - y

$$\Rightarrow S = \frac{1}{x + y} \times \frac{n}{2} \times [2x - 2y + (n-1)(2x - y)]$$

$$\Rightarrow S = \frac{n}{2(x + y)}[2x - 2y + n(2x - y) - 2x + y]$$

$$\Rightarrow S = \frac{n}{2(x + y)}[n(2x - y) - y]$$

The sum of the series is $\frac{n}{2(x+y)}[n(2x - y) - y]$

Q. 7. Find the sum of the series 2 + 5 + 8 + 11 + …. + 191.

Answer : To Find: The sum of the given series.

The nth term of an AP series is given by

$t_n = a + (n – 1)d$

⇒191 = 2 + (n - 1)3

⇒3(n – 1) = 189

⇒ n – 1 = 63

⇒ n = 64

Therefore,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{64}{2}[4 + 63 \times 3]$$

= 32 × 193 = 6176

The sum of the series is 6176.

Q. 8. Find the sum of the series 101 + 99 + 97 + 95 + …. + 43.

Answer : To Find: The sum of the given series.

Sum of the series is given by

$$S = \frac{n}{2}(a + l)$$

Where n is the number of terms, a is the first term and l is the last term

Here a = 101, l = 43, n = 30

$$S = \frac{30}{2}[101 + 43]$$

= 15 × 144 = 2160

The sum of the series is 2160.

Q. 9. Find the sum of the series 1 + 4 + 7 + 10 + …. + x = 715.

Answer : Note: The sum of the series is already provided in the question. The solution to find x is given below.

Let there be n terms in the series.

x = 1 + (n – 1)3

= 3n – 2

Let S be the sum of the series

$$S = \frac{n}{2}[1 + x] = 715$$

⇒ n[1 + 3n – 2] = 1430

⇒ n + $3n^2$ – 2n = 1430

⇒ $3n^2$ – n – 1430 = 0

Applying Sri Dhar Acharya formula, we get

$$n = \frac{1 \pm 131}{2 \times 3}$$

$$n = \frac{132}{6} \text{ or } \frac{130}{6}$$

⇒ n = 22 as n cannot be a fraction

Therefore x = 3 × 22 – 2 = 64

The value of x is 64

Q. 10. Find the value of x such that 25 + 22 + 19 + 16 + …. + x = 112.

Answer : To Find: The value of x, i.e. the last term.

Given: The series and its sum.

The series can be written as x, (x + 3), …, 16, 19, 22, 25

Let there be n terms in the series

25 = x + (n - 1)3

3(n – 1) = 25 – xx = 25 – 3(n – 1) = 28 – 3n

Let S be the sum of the series

$$S = \frac{n}{2}[x + 25] = 112$$

⇒ n[28 - 3n + 25] = 224

⇒ n(53 - 3n) = 224

⇒ $3n^2$ - 53n + 224 = 0

$$\Rightarrow (n-7)\left(n-\frac{32}{3}\right) = 0$$

⇒ n = 7 as n cannot be a fraction.

Therefore, x = 28 - 3n = 28 - 3(7) = 28 - 21 = 7

The value of x is 7.

Q. 11. Find the r^{th} term of the AP, the sum of whose first n terms is ($3n^2$ + 2n).

Answer : Given: The sum of first n terms.

To Find: The r^{th} term.

Let the first term be a and common difference be d

Put n = 1 to get the first term

$a = S_1 = 3 + 2 = 5$

Put n = 2 to get a + (a + d)2a + d = 12 + 4 = 1610 + d = 16d = $6t_r = a + (r - 1)d$
t �$_r$ = 5 + (r - 1)6 = 5 + 6r - 6 = 6r - 1

The r^{th} term is given by 6r - 1.

Q. 12. Find the sum of n term of an AP whose r^{th} term is (5r + 1).

Answer : To Find: The sum of n terms of an AP

Given: The r^{th} term.

The r^{th} term of the series is given by

$$t_r = 5r + 1$$

Sum of the series is given by sum upto n terms of t_r

$$S_r = \sum_{i=1}^{n} t_r = \sum_{i=1}^{n} 5r + 1 = \frac{5n(n + 1)}{2} + n$$

Q. 13. If the sum of a certain number of terms of the AP 27, 24, 21, 18, …. is –30, find the last term.

Answer : To Find: Last term of the AP.

Let the number of terms be n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{n}{2}[54 + (n-1)(-3)] = -30$$

⇒ n[54 - 3n + 3] = - 60

$\Rightarrow 3n^2 - 57n - 60 = 0$

$$\Rightarrow n = \frac{57 \pm 63}{6}$$

Either n = 20 or n = - 1 (n cannot be negative)

Therefore n = 20

Also,

$S = \frac{n}{2}(a + l)$, where l is the last term.

$$\Rightarrow -30 = \frac{20}{2}(27 + l)$$

$\Rightarrow$ - 30 = 270 + 10l

$$\Rightarrow -\frac{300}{10} = l$$

$\Rightarrow$ l = - 30

The last term is - 30.

Q. 14. How many terms of the AP 26, 21 16, 11, …. are needed to give the sum 11?

Answer : To Find: Number of terms required

Let the number of terms be n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{n}{2}[52 + (n-1)(-5)] = 11 \Rightarrow n[52 - 5n + 5] = 22$$

$$\Rightarrow n(57 - 5n) = 11 \times 2 = 11[57 - 5(11)]$$

$$\Rightarrow n = 11$$

11 terms are required to give the sum 11.

Q. 15. How many terms of the AP 18 16, 14, 12, …. are needed to give the sum 78? Explain the double answer.

Answer : To Find: Number of terms required to make the sum 78.

Here a = 18, d = - 2

Let n be the number of terms required to make the sum 78.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$78 = \frac{n}{2}[2 \times 18 + (n-1)(-2)]$$

⇒ $78 \times 2 = 36n - 2n^2 + 2n$

⇒ $n^2 - 19n + 78 = 0$

⇒ $n^2 - 6n - 13n + 78 = 0$

⇒ $n(n - 6) - 13(n - 6) = 0$

⇒ $(n - 13)(n - 6) = 0$

Either n = 13 or n = 6

Explanation: Since the given AP is a decreasing progression where $a_{n-1} > a_n$,it is bound to have negative values in the series. S_n is maximum for n = 9 or n = 10 since T_{10} is 0($S_{10} = S_9 = S_{max} = 90$). The sum of 78 can be attained by either adding 6 terms or 13 terms so that negative terms from T_{11} onward decrease the maximum sum to 78.

Q. 16. How many terms of the AP 20, $19\frac{1}{3}, 18\,\frac{2}{3}$ must be taken to make the sum 300? Explain the double answer.

Answer : To Find: Number of terms required to make the sum of the AP 300.

Let the first term of the AP be a and the common difference be d

Here a = 20, $d = -\frac{2}{3}$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$300 = \frac{n}{2}\left[2 \times 20 + (n-1)\left(-\frac{2}{3}\right)\right]$$

$\Rightarrow 300 \times 6 = n[120 - 2(n - 1)]$

$\Rightarrow n[- 2n + 122] = 6 \times 300$

$\Rightarrow n(- n + 61) = 3 \times 300$

$\Rightarrow n = 36$ or 25

Explanation: Since the given AP is a decreasing progression where $a_{n-1} > a_n$,it is bound to have negative values in the series. S_n is maximum for n = 30 or n = 31($S_{30} = S_{31} = S_{max} = 310$). The sum of 300 can be attained by either adding 25 terms or 36 terms so that negative terms decrease the maximum sum to 300.

Q. 17. The sums of an terms of two arithmetic progressions are in the ratio (7n – 5) : (5n + 17). Show that their 6th terms are equal.

Answer : Wrong question. It will be 7n + 5 instead of 7n – 5.

Given: Ratio of sum of n terms of 2 AP's

To Prove: 6th terms of both AP'S are equal

Let us consider 2 AP series AP_1 and AP_2.

Putting n = 1, 2, 3… we get AP_1 as 12,19,26… and AP_2 as 22,27,32….

So, $a_1 = 12$, $d_1 = 7$ and $a_2 = 22$, $d_2 = 5$

For AP_1

$S_6 = 12 + (6 - 1)7 = 47$

For AP_2

$S_6 = 22 + (6 - 1)5 = 47$

Therefore their 6th terms are equal.

Hence proved.

Q. 18. If the ratio between the sums of n terms of two arithmetic progressions is (7n + 1) : (4n + 27), find the ratio of their 11th terms.

Answer : Given: Ratio of sum of nth terms of 2 AP's

To Find: Ratio of their 11th terms

Let us consider 2 AP series AP_1 and AP_2.

Putting n = 1, 2, 3… we get AP_1 as 8, 15 22… and AP_2 as 31, 35, 39….

So, $a_1 = 8$, $d_1 = 7$ and $a_2 = 31$, $d_2 = 4$

For AP_1

$S_6 = 8 + (11 - 1)7 = 87$

For AP_2

$S_6 = 31 + (11 - 1)4 = 81$

Required ratio $= \frac{87}{81} = \frac{29}{27}$

Q. 19. Find the sum of all odd integers from 1 to 201.

Answer : To Find: The sum of all odd integers from 1 to 201.

The odd integers form the following AP series:

1,3,5….201

First term = a = 1

Common difference = d = 2

Last term = 201

Let the number of terms be n

⇒ 1 + 2(n - 1) = 201

⇒ n – 1 = 100

⇒ n = 101

$$Sum\ of\ AP\ series = \frac{n}{2}(First\ term + Last\ term)$$

$$= \frac{101}{2}(1 + 201)$$

= 101 × 101 = 10201

The sum of all odd integers from 1 to 201 is 10201.

Q. 20. Find the sum of all even integers between 101 and 199.

Answer : To Find: The sum of all even integers between 101 and 199.

The even integers form the following AP series -

102, 104, …, 198

It is and AP series with a = 102 and l = 198.

198 = 102 + (n - 1)2

⇒ 96 = (n - 1)2

⇒ 48 = n – 1

⇒ n = 49

Now, $S = \frac{49}{2}[102 + 198] = 49 \times 150 = 7350$

The sum of all even integers between 101 and 199 is 7350.

Q. 21. Find the sum of all integers between 101 and 500, which are divisible by 9.

Answer : To Find: Sum of all integers between 101 and 500 divisible by 9

The integers between 101 and 500 divisible by 9 are 108, 117, 126,…, 495(Add 9 to 108 to get 117, 9 to 117 to get 126 and so on).

Let a be the first term and d be the common difference and n be the number of terms of the AP

Here a = 108, d = 9, l = 495

⇒ a + (n - 1)d = 495

⇒ 108 + 9(n - 1) = 495

⇒ 12 + (n - 1) = 55

⇒ n = 55 - 11 = 44

Now, $S = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S = \frac{44}{2}[2 \times 108 + (44-1)9]$

⇒ S = 22[216 + 387] = 22[603] = 13266

Sum of all integers divisible by 9 between 100 and 500 is 13266.

Q. 22. Find the sum of all integers between 100 and 600, each of which when divided by 5 leaves 2 as remainder.

Answer : The integers between 100 and 600 divisible by 5 and leaves remainder 2 are 102, 107, 112, 117,…, 597.

To Find: Sum of the above AP

Here a = 102, d = 5, l = 597

a + (n - 1)d = 597

⇒102 + 5(n - 1) = 597

⇒ (n - 1) = 99

⇒ n = 100

Now, $S = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S = \frac{100}{2}[2 \times 102 + 5(100-1)]$

⇒ S = 50[204 + 495] = 50 × 699 = 34950

The sum of all such integers is 34950.

Q. 23. The sum of first 7 terms of an AP is 10 and that of next 7 terms is 17. Find the AP.

Answer : To Find: AP

Given: Sum of first 7 terms = 10

Sum of next 7 terms = 17

According to the problem,

Sum of first 14 terms of the given AP is 10 + 17 = 27.

So we can say $10 = \frac{7}{2}(2a + 6d)$ and $27 = \frac{14}{2}(2a + 13d)$

Solving the equations we get 14a + 42d = 20…(i) and

14a + 91d = 27… (ii)

Subtracting (i) from (ii)we get 49d = 7

$$\Rightarrow d = \frac{1}{7}$$

$$Therefore\ from\ (i), 14a = 20 - 42 \times \frac{1}{7}$$

⇒ a = 1

$$The\ series\ is\ 1, 1\frac{1}{7}, 1\frac{2}{7}, 1\frac{3}{7} \ldots.$$

Q. 24. If the sum of n terms of an AP is ($3n^2$ + 5n) and its m^{th} term is 164, find the value of m.

Answer : To Find: m

Given: Sum of n terms, m^{th} term

Put n = 1 to get the first term

So a_1 = 3 + 5 = 8

Put n = 2 to get the sum of first and second term

So $a_1 + a_2$ = 12 + 10 = 22

So a_2 = 14

Common difference = 14 - 8 = 6

$T_n = a + (n - 1)d = 8 + (n - 1)6 = 6n + 2$

Now $6m + 2 = 164$

Or $m = 27$

The value of m is 27.

Q. 25. Find the sum of all natural numbers from 1 and 100 which are divisible by 4 or 5.

Answer : To Find: The sum of all natural numbers from 1 to 100 which are divisible by 4 or 5.

A number divisible by both 4 and 5 should be divisible by 20
which is the LCM of 4 and 5.

Sum of numbers divisible by 4 OR 5 = Sum of numbers divisible by 4 + Sum of numbers divisible by 5 - Sum of numbers divisible by both 4 and 5.

Sum of numbers divisible by 4 = 4 + 8 + 12 + ...100

$= 4(1 + 2 + 3 + ...25) = 4 \times \frac{25}{2}[2 + 24] = 50 \times 26 = 1800$ Sum of numbers divisible by 5 = 5 + 10 + 15 + 20 + ...100

$= 5(1 + 2 + 3 + ..20)$

$= 5 \times \frac{20}{2}[2 + 19] = 50 \times 21 = 1050$ Sum of numbers divisible by 20 = 20 + 40 + 60...100

$= 20(1 + 2 + 3 + 4 + 5) = 20 \times 15 = 300$

Required sum = 1800 + 1050 - 300 = 2550

Sum of numbers which are divisible by 4 or 5 is 2550

Q. 26. If the sum of n terms of an AP is $\left\{nP + \frac{1}{2}n(n - 1)Q\right\}$, where P and Q are constants then find the common difference.

Answer : Let the first term be a and common difference be d

To Find: d

Given: Sum of n terms of AP = $nP + \frac{n}{2}(n-1)Q$

$$\Rightarrow \frac{n}{2}[2a + (n-1)d] = nP + \frac{n}{2}(n-1)Q$$

⇒ 2a + (n - 1)d = 2P + (n - 1)Q

⇒ 2(a - P) = (n - 1)(Q - d)

Put n = 1 to get the first term as sum of 1 term of an AP is the term itself.

⇒ P = a

⇒ (n - 1)(Q - d) = 0

For n not equal to 1 Q = d

Common difference is Q.

Q. 27. If $S_m = m^2p$ and $S_n = n^2p$, where m ≠ n in an AP then prove that $S_p = p^3$.

Answer : Let the first term of the AP be a and the common difference be d

Given: $S_m = m^2p$ and $S_n = n^2p$

To prove: $S_p = p^3$

According to the problem

$$\frac{m}{2}[2a + (m-1)d] = m^2p \Rightarrow 2a + (m - 1)d = 2mp$$

$$and\ \frac{n}{2}[2a + (n-1)d] = n^2p \Rightarrow 2a + (n - 1)d = 2np$$

Subtracting the equations we get,

(m - n)d = 2p(m - n)

Now m is not equal to n

So d = 2p

Substituting in 1st equation we get

$$2a + (m - 1)(2p) = 2mp$$
$$\Rightarrow a = mp - mp + p = p$$
$$\Rightarrow S_p = \frac{p}{2}[2p + (p-1)(2p)]$$
$$\Rightarrow S_p = \frac{p}{2}[2p + 2p^2 - 2p] = p^3$$

Hence proved.

Q. 28. A carpenter was hired to build 192 window frames. The first day he made 5 frames and each day, thereafter he made 2 more frames than he made the day before. How many days did he take to finish the job?

Answer : Let the carpenter take n days to finish the job.

To Find: n

He builds 5 frames on day 1, 7 on day 2, 9 on day 3 and so on.

So it forms an AP 5, 7, 9, 11,… and so on.

We need to find the number of terms in this AP such that the sum of the AP will be equal to 192

Given: Sum of AP = 192

$$\frac{n}{2}[10 + (n-1)2] = 192$$

$$\Rightarrow \frac{n}{2}[n + 8] = 192 \Rightarrow n(n + 8) = 192 \times 2 = 16 \times 24$$
$$\Rightarrow n = 16$$

He finishes the job in 16 days.

Exercise 11C

Q. 1. The interior angles of a polygon are in AP. The smallest angle is $52^{0,}$ and the common difference is 8^0. Find the number of sides of the polygon.

Answer : Given:

Interior angles of a polygon are in A.P

Smallest angle = a = $52°$

Common difference = d = $8°$

Let the number of sides of a polygon = n

Angles are in the following order

$52°, 52° + d, 52° + 2d, \ldots\ldots\ldots, 52° + (n - 1) \times d$

Sum of n terms in A.P = s = $\frac{n}{2}\{2a + (n - 1)d\}$.

Sum of angles of the given polygon is $\frac{n}{2}\{(2 \times 52°) + (n - 1) \times 8°\}$.

Hint:

Sum of interior angles of a polygon of n sides is $(n - 2) \times 180°$

Therefore,

$$(n - 2) \times 180° = \frac{n}{2}\{104° + (n - 1) \times 8°\}$$

$180n - 360 = 52n + n(n - 1) \times 4$

$4n^2 + 48n = 180n - 360$

$4n^2 - 132n + 360 = 0$

$n^2 - 33n + 90 = 0$

$(n - 3)(n - 30) = 0$

$n = 3$ &$n = 30$

∴ It can be a triangle or a 30 sided polygon.

The number of sides of the polygon is 3 or 30.

Q. 2. A circle is completely divided into n sectors in such a way that the angles of the sectors are in AP. If the smallest of these angles is 8^0 and the largest is 72^0, calculate n and the angle in the fifth sector.

Answer : A circle is divided into n sectors.

Given,

Angles are in A.P

Smallest angle = a = 8°

Largest angle = l = 72°

Final term of last term of an A.P series is l = a + (n - 1)×d

So,

72° = 8° + (n - 1) ×d

(n - 1) ×d = 64° $\longrightarrow$ (1)

Sum of all angles of all divided sectors is $360°$

Sum of n terms of A.P whose first term and the last term are known is $\frac{n}{2}\{a + l\}$

Where n is the number of terms in A.P.

So,

$$\frac{n}{2}\{8° + 72°\} = 360°$$

n(40°) = 360°

n $= \frac{360°}{40°}$

n = 9 $\longrightarrow$ (2)

From equations (1) & (2) we get,

(9 - 1) ×d = 64°

8×d = 64°

$d = \frac{64°}{8}$

d = 8°

The circle is divided into nine sectors whose angles are in A.P with a common difference of 8°.

Angle in fifth sector is a + (5 - 1) ×d = 40°

∴ n = 9

The angle in the fifth sector = 40°.

Q. 3. There are 30 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A Gardner waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the Gardner will cover in order to water all the trees.

Answer : Hint:

Distances between trees and well are in A.P.

Given:

The distance of well from its nearest tree is 10 metres

Distance between each tree is 5 metres.

So,

In A.P

The first term is 10 metres and the common difference is 5 metres.

a = 10 & d = 5

The distances are in the following order

10, 15, 20… (30 terms)

The farthest tree is at a distance of a + (30 - 1) ×d

l = 10 + (29) ×5

L = 155 metres.

Total distance travelled by the Gardner = 2×Sum of all the distances of 30 trees from the well.

Sum of distances of all the 30 trees is $\frac{n}{2}\{a + l\}$

Sum = $\frac{30}{2}\{10 + 155\}$metres

= 15 × 165 metres

= 2475 metres.

Total distance travelled by the Gardner is 2 × 2475 metres.

∴The total distance travelled by the Gardner is 4950 metres.

Q. 4. Two cars start together from the same place in the same direction. The first go with a uniform speed of 60 km/hr. The second goes at a speed of 48 km/hr in the first hour and increases the speed by 1 km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non - stop?

Answer : Given :

Two cars start together from the same place and move in the same direction.

The first car moves with a uniform speed of 60km/hr.

The second car moves with 48km/hr in the first hour and increases the speed by 1 km each succeeding hour.

Let the cars meet at n hours.

Distance travelled the first car in n hours = 60×n

Distance travelled by the second car in n hours is

$$= \frac{n}{2}\{2 \times 48 + (n - 1) \times 1\}$$

Tip: -

When the cars meet the distances travelled by cars are equal.

$$\frac{n}{2}\{2 \times 48 + (n - 1) \times 1\} = 60 \times n$$

96 + (n - 1) = 120

n = 25

∴ The two cars meet after 25 hours from their start and overtake the first car.

Q. 5. Arun buys a scooter for ₹44000. He pays ₹8000 in cash and agrees to pay the balance in annual instalments of ₹4000 each plus 10% interest on the unpaid amount. How much did he pay for it?

Answer : Given:

The amount that is to be paid to buy a scooter = 44000

The amount that he paid by cash = ₹8000

Remaining balance = ₹36000

Annual instalment = ₹4000 + interest@10% on the unpaid amount

	UNPAID AMOUNT	Interest on the unpaid amount	Amount of the instalment
1^{st} instalment	36000	$= \frac{10}{100} \times 36000 = 3600$	$= 4000 + 3600 = 7600$
2^{nd} instalment	32000	$= \frac{10}{100} \times 32000 = 3200$	$= 4000 + 3200 = 7200$

Thus, our instalments are 7600, 7200, 6800…….

Total number of instalments $= \frac{\text{The remaining balance left}}{\text{balance that is cleared per instalment}}$

$$= \frac{36000}{4000}$$

= 9

So our instalments are 7600, 7200, 6800 ... up to 9 terms.

Hint: - All our instalments are in A.P with a common difference of 400.

Here

First term, a = 7200

Common difference = d = 7200 - 7600

d = - 400

Number of terms = 9

Sum of all instalments $= s_n = \frac{n}{2}\{2 \times a + (n - 1) \times d\}$

$= \frac{9}{2}\{2 \times 7600 + (9 - 1) \times (-400)\}$

= 54000

Hence,

The total cost of the scooter = amount that is paid earlier + amount paid in 9 instalments.

= 8000 + 54000

= 62000

∴The total cost paid by Arun = 62000

Q. 6. A man accepts a position with an initial salary of ₹26000 per month. It is understood that he will receive an automatic increase of ₹250 in the very next month and each month thereafter.

Find this (i) salary for the 10th month, (ii) total earnings during the first year.

Answer : Given: -

An initial salary that will be given = ₹26000

There will be an automatic increase of ₹250 per month from the very next month and thereafter.

Hint: - In the given information the salaries he receives are in A.P.

Let the number of the month is n.

Initial salary = a = ₹26000

Increase in salary = common difference = d = ₹250

i. Salary for the 10^{th} month,

n = 10,

Salary = a + (n - 1)×d

= 26000 + (10 - 1)×250

= 28250

∴ Salary for the 10^{th} month = ₹28250

ii. Total earnings during the first year = sum off all salaries received per month.

Total earnings = $= \frac{n}{2}[2 \times a + (n - 1)\times d]$

Here n = 12.

Total earnings $= \frac{12}{2}[2 \times 26000 + (12 - 1) \times 250]$

= 6×(42000 + 2750)

= 268500

Total earnings during the first year = ₹268500

Q. 7. A man saved ₹660000 in 20 years. In each succeeding year after the first year, he saved ₹2000 more than what he saved in the previous year. How much did he save in the first year?

Answer : Given: -

Amount saved by a man in 20 years is Rs.660000.

Let the amount saved by him in the first year be a.

In every succeeding year, he saves Rs.2000 more than what he saved in the previous year.

Increment of saving of the year when compared last year is Rs.2000

Hint: - The above information looks like the savings are in Arithmetic Progression.

Amount saved in first year = a

Common difference = d = ₹2000

Total number of years = n = 20

The total amount saved in 20 years is ₹660000

$$\text{Sum of n terms in an A.P} = \frac{n}{2}[2 \times a + (n-1) \times d]$$

$$660000 = \frac{20}{2}[2 \times a + (20 - 1) \times 2000]$$

a = 14000

∴ In the first year, he saved ₹14000.

Q. 8. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day, and so on. It takes 8 more days to finish work now. Find the number of days in which the work was completed.

Answer : Given: -

Initially let the work can be completed in n days when 150 workers work on every day.

However every day 4 workers are being dropped from the work so that work took 8 more days to be finished.

Finally, it takes (n + 8) days to finish the works.

Work equivalent when 150 workers work without being dropped = 150 × n

Work equivalent when workers are dropped day by day = 150 + (150 - 4) + (150 - 8) + ... + (150 - 4(n + 8)).

So,

$150 \times n = 150 + (150 - 4) + ... + (150 - 4 \times (n + 8))$

$150 \times n = 150 \times n + 150 \times 8 - 4 \times (1 + 2 + 3 + ... + (n + 8))$

$(n + 8)(n + 9) = 600$

$n^2 + 17n - 528 = 0$

$n = - 33$ or $n = 16$

Since the number of days cannot be negative, n = 16.

∴ In 24 days the work is completed.

Q. 9. A man saves ₹4000 during the first year, ₹5000 during the second year and in this way he increases his savings by ₹1000 every year. Find in what time his savings will be ₹85000.

Answer : A Man saves some amount of money every year.

In the first year, he saves Rs.4000.

In the next year, he saves Rs.5000.

Like this, he increases his savings by Rs.1000 ever year.

Given a total amount of Rs. 85000 is saved in some ‘n’ years.

According to the above information the savings in every year are in Arithmetic Progression.

First year savings = a = Rs.4000

Increase in every year savings = d = Rs.1000

Total savings (s_n) = Rs.85000

$$\text{Sum of n terms in A.P} = \frac{n}{2}[2 \times a + (n - 1) \times d]$$

$$s_n = \frac{n}{2}[2 \times 4000 + (n - 1) \times 1000]$$

$$85000 = \frac{n}{2}[8000 + (n - 1) \times 1000]$$

$n^2 + 7 \times n - 170 = 0$

$(n + 17) \times (n - 10) = 0$

$n = -17$ or $n = 10$

Since the number of years cannot be negative, n = 10.

After 10 years his savings will become Rs.85000.

Q. 10. A man arranges to pay off a debt of ₹36000 by 40 annual instalments which form an AP. When 30 of the instalments are paid, he dies, leaving one - third of the debt unpaid. Find the value of the first instalment.

Answer : Given: -

Total debt = Rs.36000

A man pays this debt in 40 annual instalments that forms an A.P.

After annual instalments, that man dies leaving one - third of the debt unpaid.

So,

Within 30 instalments he pays two - thirds of his debt.

Sum of n terms in an Arithmetic Progression = $\frac{n}{2}[2 \times a + (n - 1) \times d]$

He has to pay 36000 in 40 annual instalments,

$$36000 = \frac{40}{2}[2 \times a + (40 - 1) \times d] \rightarrow (1)$$

Where,

a = amount paid in the first instalment,

d = difference between two Consecutive instalments.

He paid two – a third of the debt in 30 instalments,

$$\frac{2}{3}(36000) = \frac{30}{2}[2 \times a + (30 - 1) \times d] \longrightarrow (2)$$

From equations (1) & (2) we get,

a = 510 & d = 20

∴The value of the first instalment is Rs.510.

Q. 11. A manufacturer of TV sets produced 6000 units in the third year and 7000 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production

(i) in the first year,
(ii) in the 10th year,
(iii) in 7 years.

Answer : Hint: - In the question it is mentioned that the production increases by a fixed number every year.

So it is an A.P. (a_1, a_2, a_3, a_4, ...a_{n-1}, a_n).

Given: -

The 3rd year production is 6000 units

So,

a_3 = 6000 $a_3 = 6000$

We know that a_n = a + (n - 1) ×d

a_3 = a + (3 - 1)×d

6000 = a + 2d $\longrightarrow (1)$

The 7th year production is 7000 units

So,

a_7 = 7000

a_7 = a + (7 - 1)×d

$7000 = a + 6d \longrightarrow (2)$

From equations (1)&(2) we get,

$6000 - 2d = 7000 - 6d$

$4 \times d = 1000$

$d = 250 \longrightarrow (3)$

From equations (1)&(2) we get,

$a = 5500$

i. Production in the first year = a = 5500

∴ 5500 units were produced by the manufacturer of TV sets in the first year.

ii. Production in the 10^{th} year = $a_{10} = a + (10 - 1) \times d$

$a_{10} = 5500 + (9) \times 250$

$= 7750$

∴ 7750 units were produced by the manufacturer of TV sets in the 10^{th} year.

iii. Total production in seven years = $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$

$$s_7 = \frac{7}{2}[2 \times a + (n - 1) \times d]$$

$$s_7 = \frac{7}{2}[2 \times 5500 + (6) \times 250]$$

$s_7 = 43750$

∴ A total of 16, 250 units was produced by the manufacturer in 7 years.

Q. 12. A farmer buys a used for ₹180000. He pays ₹90000 in cash and agrees to pay the balance in annual instalments of ₹9000 plus 12% interest on the unpaid amount. How much did the tractor cost him?

Answer : Given: -

The amount that is to be paid to buy a tractor = ₹180000.

An amount that he paid by cash = ₹90000.

Remaining balance = ₹90000

Annual instalment = ₹9000 + interest @12% on unpaid amount.

	UNPAID AMOUNT	Interest on the unpaid amount	Amount of the instalment
1^{st} instalment	90000	$= \frac{12}{100} \times 90000 = 10800$	$= 9000 + 10800 = 19800$
2^{nd} instalment	81000	$= \frac{12}{100} \times 81000 = 9720$	$= 9000 + 9720 = 18720$

Thus, our instalments are 19800, 18720, 17640…….

Total number of instalments = $\frac{\text{The remaining balance left}}{\text{balance that is cleared per instalment}}$

$= \frac{90000}{9000}$

= 10

So our instalments are 19800, 18720, 17640 ... upto 10 terms.

All our instalments are in A.P with a common difference d.

Here

First term(a) = 19800

Common difference = d = 18720 - 19800

d = - 1080

Number of terms is 10

Sum of all instalments $= s_n = \frac{n}{2}\{2 \times a + (n - 1) \times d\}$

$= \frac{10}{2}\{2 \times 19800 + (10 - 1) \times (-1080)\}$

= 149400

Hence,

The total cost of the scooter = amount that is paid earlier + amount paid in 10 instalments.

= 90000 + 149400

∴The total cost paid by the farmer = ₹239400

Exercise 11D

Q. 1. Find the arithmetic mean between:

(i) 9 and 19
(ii) 15 and -7
(iii) -16 and -8

Answer : (i) 9 and 19

To find: Arithmetic mean between 9 and 19

The formula used: Arithmetic mean between a and b = $\frac{a+b}{2}$

We have 9 and 19

$$\text{A.M.} = \frac{9+19}{2}$$

$$= \frac{28}{2}$$

$$= 14$$

(ii) 15 and -7

To find: Arithmetic mean between 15 and -7

The formula used: Arithmetic mean between a and b = $\frac{a+b}{2}$

We have 15 and -7

$$A.M. = \frac{(15) + (-7)}{2}$$

$$= \frac{15-7}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

(iii) -16 and -8

To find: Arithmetic mean between -16 and -8

The formula used: Arithmetic mean between = a and b = $\frac{a+b}{2}$

We have -16 and -8

$$A.M. = \frac{(-16) + (-8)}{2}$$

$$= \frac{-16-8}{2}$$

$$= \frac{-24}{2}$$

$$= -12$$

Q. 2. Insert four arithmetic means between 4 and 29.

Answer : To find: Four arithmetic means between 4 and 29

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) $A_n = a + nd$

We have 4 and 29

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{29 - 4}{4 + 1}$$

$$d = \frac{25}{5}$$

$$d = 5$$

Using Formula, $A_n = a + nd$

First arithmetic mean, $A_1 = a + d$

= 4 + 5

= 9

Second arithmetic mean, $A_2 = a + 2d$

= 4 + 2(5)

= 4 + 10

= 14

Third arithmetic mean, $A_3 = a + 3d$

= 4 + 3(5)

= 4 + 15

= 19

Fourth arithmetic mean, A_4 = a + 4d

= 4 + 4(5)

= 4 + 20

= 24

Ans) The four arithmetic means between 4 and 29 are 9, 14, 19 and 24

Q. 3. Insert three arithmetic means between 23 and 7.

Answer : To find: Three arithmetic means between 23 and 7

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) A_n = a + nd

We have 23 and 7

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{7 - 23}{3 + 1}$$

$$d = \frac{-16}{4}$$

$$d = -4$$

Using Formula, A_n = a + nd

First arithmetic mean, A_1 = a + d

= 23 + (-4)

= 19

Second arithmetic mean, $A_2 = a + 2d$

= 23 + 2(-4)

= 23 + (-8)

= 15

Third arithmetic mean, $A_3 = a + 3d$

= 23 + 3(-4)

= 23 + (-12)

= 11

Ans) The three arithmetic means between 23 and 7 are 19, 15 and 11

Q. 4. Insert six arithmetic means between 11 and -10.

Answer : To find: Six arithmetic means between 11 and -10

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) $A_n = a + nd$

We have 11 and -10

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{-10 - (11)}{6 + 1}$$

$$d = \frac{-21}{7}$$

$$d = -3$$

Using Formula, $A_n = a + nd$

First arithmetic mean, $A_1 = a + d$

$= 11 + (-3)$

$= 8$

Second arithmetic mean, $A_2 = a + 2d$

$= 11 + 2(-3)$

$= 11 + (-6)$

$= 5$

Third arithmetic mean, $A_3 = a + 3d$

$= 11 + 3(-3)$

$= 11 + (-9)$

$= 2$

Fourth arithmetic mean, $A_4 = a + 4d$

$= 11 + 4(-3)$

$= 11 + (-12)$

$= -1$

Fifth arithmetic mean, $A_5 = a + 5d$

$= 11 + 5(-3)$

$= 11 + (-15)$

$= -4$

Sixth arithmetic mean, $A_6 = a + 6d$

$= 11 + 6(-3)$

$= 11 + (-18)$

$= -7$

Ans) The six arithmetic means between 11 and -10 are 8, 5, 2, - 1, -4 and -7.

Q. 5. There is n arithmetic means between 9 and 27. If the ratio of the last mean to the first mean is 2 : 1, find the value of n.

Answer : To find: The value of n

Given: (i) The numbers are 9 and 27

(ii) The ratio of the last mean to the first mean is 2 : 1

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) $A_n = a + nd$

We have 9 and 27,

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{27 - 9}{n + 1}$$

$$d = \frac{18}{n + 1}$$

Using Formula, $A_n = a + nd$

First mean i.e., $A_1 = 9 + (1)\left(\frac{18}{n+1}\right)$

$$= 9 + \frac{18}{n+1}$$

$$= \frac{9n+9+18}{n+1}$$

$A_1 = \frac{9n+27}{n+1}$ … (i)

Last mean i.e., $A_n = 9 + (n)\left(\frac{18}{n+1}\right)$

$= 9 + \frac{18n}{n+1}$

$= \frac{9n+9+18n}{n+1}$

$A_n = \frac{27n + 9}{n+1}$ … (ii)

The ratio of the last mean to the first mean is 2 : 1

$\Rightarrow \frac{A_n}{A_1} = \frac{2}{1}$

Substituting the value of A_1 and A_n from eqn. (i) and (ii)

$\Rightarrow \frac{\frac{27n + 9}{n+1}}{\frac{9n+27}{n+1}} = \frac{2}{1}$

$\Rightarrow \frac{27n + 9}{9n + 27} = \frac{2}{1}$

⇒ 27n + 9 = 18n + 54

⇒ 9n = 45

⇒ n = 5

Ans) The value of n is 5

Q. 6. Insert arithmetic means between 16 and 65 such that the 5th AM is 51. Find the number of arithmetic means.

Answer : To find: The number of arithmetic means

Given: (i) The numbers are 16 and 65

(ii) 5th arithmetic mean is 51

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) $A_n = a + nd$

We have 16 and 65,

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{65 - 16}{n + 1}$$

$$d = \frac{49}{n + 1}$$

Using Formula, $A_n = a + nd$

Fifth arithmetic mean, $A_5 = a + 5d$

$$= 16 + 5\left(\frac{49}{n+1}\right)$$

$$A_5 = 16 + \left(\frac{245}{n+1}\right)$$

$A_5 = 51$ (Given)

Therefore, $A_5 = 16 + \left(\frac{245}{n+1}\right) = 51$

$$\Rightarrow 16 + \left(\frac{245}{n+1}\right) = 51$$

$$\Rightarrow \left(\frac{245}{n+1}\right) = 51 - 16$$

$$\Rightarrow \left(\frac{245}{n+1}\right) = 35$$

$\Rightarrow 245 = 35n + 35$

$\Rightarrow 210 = 35n$

$\Rightarrow n = 6$

The number of arithmetic means are 6.

Using Formula, $d = \frac{b - a}{n + 1}$

$$d = \frac{65 - 16}{6 + 1}$$

$$d = \frac{49}{7}$$

$d = 7$

Using Formula, $A_n = a + nd$

First arithmetic mean, $A_1 = a + d$

$= 16 + 7$

$= 23$

Second arithmetic mean, $A_2 = a + 2d$

$= 16 + 2(7)$

$= 16 + 14$

$= 30$

Third arithmetic mean, $A_3 = a + 3d$

$= 16 + 3(7)$

$= 16 + 21$

= 37

Fourth arithmetic mean, $A_4 = a + 4d$

= 16 + 4(7)

= 16 + 28

= 44

Fifth arithmetic mean, $A_5 = a + 5d$

= 16 + 5(7)

= 16 + 35

= 51

Sixth arithmetic mean, $A_6 = a + 6d$

= 16 + 6(7)

= 16 + 42

= 58

Ans) The six arithmetic means between 1 and 65 are 23, 30, 37, 44, 51 and 58.

Q. 7. Insert five numbers between 11 and 29 such that the resulting sequence is an AP.

Answer : To find: Five numbers between 11 and 29, which are in A.P.

Given: (i) The numbers are 11 and 29

Formula used: (i) $A_n = a + (n-1)d$

Let the five numbers be A_1, A_2, A_3, A_4 and A_5

According to question 11, A_1, A_2, A_3, A_4, A_5 and 29 are in A.P.

We can see that the number of terms in this series is 7

For the above series:-

a = 11, n=7

$A_7 = 29$

Using formula, $A_n = a + (n-1)d$

$\Rightarrow A_7 = 11 + (7-1)d = 29$

$\Rightarrow 6d = 29 – 11$

$\Rightarrow 6d = 18$

$\Rightarrow d = 3$

We can see from the definition that A_1, A_2, A_3, A_4 and A_5 are five arithmetic mean between 11 and 29, where d = 3, a = 11

Therefore, Using formula of arithmetic mean i.e. $A_n = a + nd$

$A_1 = a + nd$

$= 11 + 3$

$= 14$

$A_2 = a + nd$

$= 11 + (2)3$

$= 17$

$A_3 = a + nd$

$= 11 + (3)3$

$= 20$

$A_4 = a + nd$

$= 11 + (4)3$

$= 23$

$A_5 = a + nd$

$= 11 + (5)3$

$= 26$

Ans) 14, 17, 20, 23 and 26 are the required numbers.

Q. 8. Prove that the ratio of sum of m arithmetic means between the two numbers to the sum of n arithmetic means between them is m:n.

Answer : To prove: ratio of sum of m arithmetic means between the two numbers to the sum of n arithmetic means between them is m:n

Formula used: (i) $d = \frac{b - a}{n + 1}$, where, d is the common difference

n is the number of arithmetic means

(ii) $S_n = \frac{n}{2}[a+l]$, Where n = Number of terms

a = First term

l = Last term

Let the first series of arithmetic mean having m arithmetic means be,

a, A_1, A_2, A_3 … A_m, l

In the above series we have (m + 2) terms

⇒ l = a + (m + 2 – 1)d

⇒ l = a + (m + 1)d … (i)

In the above series A_1 is second term

⇒ A_1 = a + (2-1)d

= a + d

In the above series A_m is the $(m+1)^{th}$ term

⇒ A_m = a + (m+1-1)d

= a + md

Now, $A_1 + A_m$ = a + d + a + md

= a + a + (m+1)d

$= a + l$ [From eqn (i)]

Therefore, $A_1 + A_m = a + l$ … (ii)

For the sum of arithmetic means in the above series:-

First term = A_1, Last term = A_m, No. of terms = m

Using Formula, $S_n = \frac{n}{2}[a+l]$

$$S_m = \frac{m}{2}[\ A_1 + A_m\ ]$$

From eqn. (ii)

$$S_m = \frac{m}{2}[\ a + l\ ]$$

Let the second series of arithmetic mean having n arithmetic means be,

$a, A_1, A_2, A_3 \dots A_n, l$

In the above series we have (n + 2) terms

$\Rightarrow l = a + (n + 2 – 1)d$

$\Rightarrow l = a + (n + 1)d$ … (iii)

In the above series A_1 is second term

$\Rightarrow A_1 = a + (2-1)d$

$= a + d$

In the above series A_n is the $(n+1)^{th}$ term

$\Rightarrow A_n = a + (n+1-1)d$

$= a + nd$

Now, $A_1 + A_n = a + d + a + nd$

$= a + a + (n+1)d$

$= a + l$ [From eqn (iii)]

Therefore, $A_1 + A_n = a + l$ … (iv)

For the sum of arithmetic means in the above series:-

First term = A_1, Last term = A_n, No. of terms = n

Using Formula, $S_n = \frac{n}{2}[a+l]$

$$S_n = \frac{n}{2}[A_1 + A_n]$$

From eqn. (iv)

$$S_n = \frac{n}{2}[a + l]$$

There, $\frac{S_m}{S_n} = \frac{\frac{m}{2}[a+l]}{\frac{n}{2}[a+l]} = \frac{m}{n}$

Hence Proved

Exercise 11E

Q. 1. If a, b, c are in AP, prove that

(i) $(a - c)^2 = 4(a - b)(b - c)$
(ii) $a^2 + c^2 + 4ac = 2(ab + bc + ca)$
(iii) $a^3 + c^3 + 6abc = 8b^3$

Answer : (i) $(a - c)^2 = 4(a - b)(b - c)$

To prove: $(a - c)^2 = 4(a - b)(b - c)$

Given: a, b, c are in A.P.

Proof: Since a, b, c are in A.P.

⇒ $c - b = b - a$ = common difference

⇒ $b - c = a - b$ … (i)

And, $2b = a + c$ (a, b, c are in A.P.)

$\Rightarrow 2b - c = a$ … (ii)

Taking LHS = $(a - c)^2$

= $(2b - c - c)^2$ [from eqn. (ii)]

= $(2b - 2c)^2$

= $4(b - c)^2$

= $4(b - c) (b - c)$

= $4(a - b) (b - c)$ [b–c = a–b from eqn. (i)]

= RHS

Hence Proved

(ii) $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

To prove: $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Given: a, b, c are in A.P.

Proof: Since a, b, c are in A.P.

$\Rightarrow 2b = a + c$

$\Rightarrow b = \frac{a+c}{2}$ … (i)

Taking RHS = 2(ab + bc + ca)

Substituting value of b from eqn. (i)

$$=2\left[\left\{a\left(\frac{a+c}{2}\right)\right\}+\left\{\left(\frac{a+c}{2}\right)c\right\}+\{ca\}\right]$$

$$=2\left[\left\{\frac{a^2+ac}{2}\right\}+\left\{\frac{ac+c^2}{2}\right\}+\{ca\}\right]$$

$$=2\left[\frac{a^2+ac+ac+c^2+2ac}{2}\right]$$

$$=2\left[\frac{a^2+c^2+4ac}{2}\right]$$

= $a^2 + c^2 + 4ac$

= LHS

Hence Proved

(iii) $a^3 + c^3 + 6abc = 8b^3$

To prove: $a^3 + c^3 + 6abc = 8b^3$

Given: a, b, c are in A.P.

Formula used: $(a+b)^3 = a^3 + 3ab(a+b) + b^3$

Proof: Since a, b, c are in A.P.

$\Rightarrow 2b = a + c$ … (i)

Cubing both side,

$\Rightarrow (2b)^3 = (a+c)^3$

$\Rightarrow 8b^3 = a^3 + 3ac(a+c) + c^3$

$\Rightarrow 8b^3 = a^3 + 3ac(2b) + c^3$ [a+c = 2b from eqn. (i)]

$\Rightarrow 8b^3 = a^3 + 6abc + c^3$

On rearranging,

$a^3 + c^3 + 6abc = 8b^3$

Hence Proved

Q. 2. If a, b, c are in AP, show that

(a + 2b – c)(2b + c – a)(c + a – b) = 4abc.

Answer : To prove: (a + 2b – c)(2b + c – a)(c + a – b) = 4abc.

Given: a, b, c are in A.P.

Proof: Since a, b, c are in A.P.

⇒ 2b = a + c … (i)

Taking LHS = (a + 2b – c) (2b + c – a) (c + a – b)

Substituting the value of 2b from eqn. (i)

= (a + a + c – c) (a + c + c – a) (c + a – b)

= (2a) (2c) (c + a – b)

Substituting the value of (a + c) from eqn. (i)

= (2a) (2c) (2b – b)

= (2a) (2c) (b)

= 4abc

= RHS

Hence Proved

Q. 3. If a, b, c are in AP, show that

(i) (b + c – a), (c + a – b), (a + b – c) are in AP.
(ii) (bc – a^2), (ca – b^2), (ab – c^2) are in AP.

Answer : (i) (b + c – a), (c + a – b), (a + b – c) are in AP.

To prove: (b + c – a), (c + a – b), (a + b – c) are in AP.

Given: a, b, c are in A.P.

Proof: Let d be the common difference for the A.P. a,b,c

Since a, b, c are in A.P.

⇒ b – a = c – b = common difference

⇒ a – b = b – c = d

⇒ 2(a – b) = 2(b – c) = 2d … (i)

Considering series (b + c – a), (c + a – b), (a + b – c)

For numbers to be in A.P. there must be a common difference between them

Taking (b + c – a) and (c + a – b)

Common Difference = (c + a – b) - (b + c – a)

= c + a – b – b – c + a

= 2a – 2b

= 2(a – b)

= 2d [from eqn. (i)]

Taking (c + a – b) and (a + b – c)

Common Difference = (a + b – c) - (c + a – b)

= a + b – c – c – a + b

= 2b – 2c

= 2(b – c)

= 2d [from eqn. (i)]

Here we can see that we have obtained a common difference between numbers i.e. 2d

Hence, (b + c – a), (c + a – b), (a + b – c) are in AP.

(ii) $(bc – a^2)$, $(ca – b^2)$, $(ab – c^2)$ are in AP.

To prove: $(bc – a^2)$, $(ca – b^2)$, $(ab – c^2)$ are in AP.

Given: a, b, c are in A.P.

Proof: Let d be the common difference for the A.P. a,b,c

Since a, b, c are in A.P.

⇒ b – a = c – b = common difference

⇒ a – b = b – c = d … (i)

Considering series $(bc – a^2)$, $(ca – b^2)$, $(ab – c^2)$

For numbers to be in A.P. there must be a common difference between them

Taking $(bc - a^2)$ and $(ca - b^2)$

Common Difference = $(ca - b^2) - (bc - a^2)$

$= [ca - b^2 - bc + a^2]$

$= [ca - bc + a^2 - b^2]$

$= [c (a - b) + (a + b) (a - b)]$

$= [(a - b) (a + b + c)]$

$a - b = d$, from eqn. (i)

$\Rightarrow [(d) (a + b + c)]$

Taking $(ca - b^2)$ and $(ab - c^2)$

Common Difference = $(ab - c^2) - (ca - b^2)$

$= [ab - c^2 - ca + b^2]$

$= [ab - ca + b^2 - c^2]$

$= [a (b - c) + (b - c) (b + c)]$

$= [(b - c) (a + b + c)]$

$b - c = d$, from eqn. (i)

$\Rightarrow [(d) (a + b + c)]$

Here we can see that we have obtained a common difference between numbers i.e. $[(d) (a + b + c)]$

Hence, $(bc - a^2)$, $(ca - b^2)$, $(ab - c^2)$ are in AP.

Q. 4. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, prove that

(i) $\frac{(b+c)}{a}, \frac{(c+a)}{b}, \frac{(a+b)}{c}$ **are in AP.**

(ii) $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ **are in AP.**

Answer :

(i) $\frac{(b+c)}{a}, \frac{(c+a)}{b}, \frac{(a+b)}{c}$ are in A.P.

To prove: $\frac{(b+c)}{a}, \frac{(c+a)}{b}, \frac{(a+b)}{c}$ are in A.P.

Given: $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Proof: $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

Multiplying the A.P. with (a + b + c)

$\Rightarrow \frac{(a+b+c)}{a}, \frac{(a+b+c)}{b}, \frac{(a+b+c)}{c}$ are also in A.P.

If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

Substracting the above A.P. with 1

$\Rightarrow \frac{(a+b+c)}{a} - 1, \frac{(a+b+c)}{b} - 1, \frac{(a+b+c)}{c} - 1$, are also in A.P.

$\Rightarrow \frac{a+b+c-a}{a}, \frac{a+b+c-b}{b}, \frac{a+b+c-c}{c}$, are also in A.P.

$\Rightarrow \frac{b+c}{a}, \frac{a+c}{b}, \frac{a+b}{c}$, are also in A.P.

Hence Proved

(ii) $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in A.P.

To prove: $\frac{(b+c-a)}{a}, \frac{(c+a-b)}{b}, \frac{(a+b-c)}{c}$ are in A.P.

Given: $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Proof: $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

Multiplying the A.P. with (a + b + c)

$\Rightarrow \frac{(a+b+c)}{a}, \frac{(a+b+c)}{b}, \frac{(a+b+c)}{c}$ are also in A.P.

If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

Substracting the above A.P. with 2

$$\Rightarrow \frac{(a+b+c)}{a}-2, \frac{(a+b+c)}{b}-2, \frac{(a+b+c)}{c}-2$$, are also in A.P.

$$\Rightarrow \frac{a+b+c-2a}{a}, \frac{a+b+c-2b}{b}, \frac{a+b+c-2c}{c}$$, are also in A.P.

$$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{c}$$, are also in A.P.

Hence Proved

Q. 5. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP, prove that $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in AP.

Answer : To prove: $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.

Given: $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

Proof: $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

$\Rightarrow \left(\frac{a}{b}+\frac{a}{c}\right), \left(\frac{b}{c}+\frac{b}{a}\right), \left(\frac{c}{a}+\frac{c}{b}\right)$ are in A.P.

$\Rightarrow \left(\frac{ac+ab}{bc}\right), \left(\frac{ab+bc}{ca}\right), \left(\frac{cb+ac}{ab}\right)$ are in A.P.

If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

Multiplying the A.P. with (abc)

$\Rightarrow \left(\frac{ac+ab}{bc}\right)(abc), \left(\frac{ab+bc}{ca}\right)(abc), \left(\frac{cb+ac}{ab}\right)(abc)$, are in A.P.

$\Rightarrow [(ac + ab)(a)], [(ab + bc)(b)], [(cb + ac)(c)]$ are in A.P.

$\Rightarrow$ [$(a^2c + a^2b)$], [$ab^2 + b^2c$], [$c^2b + ac^2$] are in A.P.

On rearranging,

$\Rightarrow$ [$a^2(b + c)$], [$b^2(c + a)$] , [$c^2(a + b)$] are in A.P.

Hence Proved

Q. 6. If a, b, c are in AP, show that $\frac{a(b+c)}{bc}, \frac{b(c+a)}{ca}, \frac{c(a+b)}{ab}$ **are also in AP.**

Answer : To prove: $\frac{a(b+c)}{bc}, \frac{b(c+a)}{ca}, \frac{c(a+b)}{ab}$ are in A.P.

Given: a, b, c are in A.P.

Proof: a, b, c are in A.P.

If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.

Multiplying the A.P. with (ab + bc + ac)

$\Rightarrow (a)(ab+bc+ac), (b)(ab+bc+ac), (c)(ab+bc+ac)$, are in A.P.

Multiplying the A.P. with $\left(\frac{1}{abc}\right)$

$\Rightarrow \left[\frac{(a)(ab+bc+ac)}{abc}\right], \left[\frac{(b)(ab+bc+ac)}{abc}\right], \left[\frac{(c)(ab+bc+ac)}{abc}\right]$, are in A.P.

$\Rightarrow \left[\frac{(ab+bc+ac)}{bc}\right], \left[\frac{(ab+bc+ac)}{ac}\right], \left[\frac{(ab+bc+ac)}{ab}\right]$, are in A.P.

If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.

Substracting the A.P. with 1

$\Rightarrow \left[\frac{(ab+bc+ac)}{bc} - 1\right], \left[\frac{(ab+bc+ac)}{ac} - 1\right], \left[\frac{(ab+bc+ac)}{ab} - 1\right]$, are in A.P.

$\Rightarrow \left[\frac{(ab+ac)}{bc}\right], \left[\frac{(ab+bc)}{ac}\right], \left[\frac{(bc+ac)}{ab}\right]$, are in A.P.

On rearranging

$\Rightarrow \left[\frac{a(b+c)}{bc}\right], \left[\frac{b(c+a)}{ac}\right], \left[\frac{c(a+b)}{ab}\right]$, are in A.P.

Hence Proved

Exercise 11F

Q. 1. If the sum of n terms of an AP is given by $S_n = (2n^2 + 3n)$, then find its common difference.

Answer : Given: $S_n = (2n^2 + 3n)$

To find: find common difference

Put n = 1 we get

$S_1 = 5$ OR we can write

a = 5 …equation 1

Similarly put n = 2 we get

$S_2 = 14$ OR we can write

2a + d = 14

Using equation 1 we get

d = 4

Q. 2. If 9 times the 9th term of an AP is equal to 13 times the 13th term, show that its 22nd term is 0.

Answer : Given : 9 × (9th term) = 13 × (13th term)

To prove: 22nd term is 0

9 × (a + 8d) = 13 × (a + 12d)

9a + 72d = 13a + 156d

$- 4a = 84d$

$a = - 21d$ …..Equation 1

Also 22^{nd} term is given by

$a + 21d$

Using equation 1 we get

$- 21d + 21d = 0$

Hence proved 22^{nd} term is 0.

Q. 3. In an AP it is given that $S_n = qn^2$ and $S_m = qm^2$. Prove that $S_q = q^3$.

Answer : Given: $S_n = qn^2$, $S_m = qm^2$

To prove: $S_q = q^3$

Put n = 1 we get

$a = q$ …… equation 1

Put n = 2

$2a + d = 4q$ ……equation 2

Using equation 1 and 2 we get

$d = 2q$

$$\text{So } S_q = \frac{q}{2}(2q + (q-1) \times 2q)$$

$S_q = q^3$

Hence proved.

Q. 4. Find three arithmetic means between 6 and - 6.

Answer : let the three AM be x_1, x_2, x_3.

So new AP will be

$6, x_1, x_2, x_3, -6$

Also $-6 = 6 + 4d$

$d = -3$

$x_1 = 3$

$x_2 = 0$

$x_3 = -3$

Q. 5. The 9th term of an AP is 0. Prove that its 29th term is double the 19th term.

Answer : Given :9th term is 0

To prove: 29th term is double the 19th term

$a + 8d = 0$

$a = -8d$

29th term is

$a + 28d$

$\Longrightarrow 20d$

19th term is

$a + 18d$

$\Longrightarrow 10d$

Hence proved 29th term is double the 19th term

Q. 6. How many terms are there in the AP 13, 16, 19, …., 43?

Answer : To find: number of terms in AP

Also

$d = 16 – 13$

$d = 3$

Also

$43 = 13 + n \times 3 - 3$

So

$n = 11$

Q. 7. Find the 8^{th} term from the end of the AP 7, 9, 11, …., 201.

Answer : To find: 8^{th} term from the end

$d = 9 - 7$

$d = 2$

Also

$201 = 7 + n \times 2 - 2$

$n = 98$

So 8^{th} term from end will be

$7 + 90 \times 2$

$\Rightarrow 187$

Q. 8. How many 2 - digit numbers are divisible by 7?

Answer : The first 2 digit number divisible by 7 is 14, and the last 2 digit number divisible by 7 is 98, so it forms AP with common difference 7

14,…,98

$98 = 14 + (n - 1) \times 7$

$n = 22$

Q. 9. If 7^{th} and 13^{th} terms of an AP be 34 and 64 respectively then find its 18^{th}term.

Answer : Given: 7^{th} term is 34 and 8^{th} term is 64

To find: find its 18^{th} term

$34 = a + 6d$ …………equation1

64 = a + 12d ………… equation2

Subtract equation1 from equation2 we get

d = 5

Put in equation1 we get

a = 4

So 18^{th} term is

4 + 17 × 5 = 89

Q. 10. What is the 10^{th} common term between the APs 3, 7, 11, 15, 19, … and 1, 6, 11, 16, …?

Answer : To find: 10^{th} common term between the APs

Common difference of 1^{st} series = 4

Common difference of 2^{nd} series = 5

LCM of common difference will give us a common difference of new series

⟹ 5 × 4

⟹ 20

The first term of new AP will be 11, so the 10^{th} = term of this series is

⟹ 11 + 20 × 9

⟹ 191

Q. 11. The first and last terms of an AP are 1 and 11 respectively. If the sum of its terms is 36, find the number of terms.

Answer : Given: the sum of its terms is 36, the first and last terms of an AP are 1 and 11.

To find: the number of terms

Sum of AP using first and last terms is given by

$$S_n = \frac{n}{2}(a + l)$$

36 × 2 = n (1 + 11)

n = 6

Q. 12. In an AP, the p^{th} term is q and $(p + q)^{th}$ term is 0. Show that its q^{th} term is p.

Answer : Given: p^{th} term is q and $(p + q)^{th}$ term is 0.

To prove: q^{th} term is p.

p^{th} term is given by

q = a + (p - 1) × d……equation1

$(p + q)^{th}$ term is given by

0 = a + (p + q - 1) × d

0 = a + (p - 1) × d + q × d

Using equation1

0 = q + q × d

d = - 1

Put in equation1 we get

a = q + p - 1

q^{th} term is

⟹ q + p - 1 + (q - 1) × (- 1)

⟹ p

Hence proved.

Q. 13. If $\frac{3+5+7+9+\ldots\text{up to 35 terms}}{5+8+11+\ldots\text{up to n terms}} = 7$, find the value of n.

Answer : To find: the value of n.

We can write it as

$$\frac{\frac{35}{2}(6 + 34(5-3))}{\frac{n}{2}(10 + 3(n-1))} = 7$$

$3n^2 + 7 \times n - 370 = 0$

Therefore n = 37/3, 10

Rejecting 37/3 we get n = 10

Q. 14. Write the sum of first n even natural numbers.

Answer : Even natural numbers are

2, 4, 6, 8…..

$$S = \frac{n}{2} \times (4 + 2 \times n - 2)$$

$$S = n^2 + 2n$$

Q. 15. Write the sum of first n odd natural numbers.

Answer : n odd natural numbers are given by

3,5,7,9,…….

$$S = \frac{n}{2} \times (6 + 2 \times n - 2)$$

$$S = \frac{n}{2} \times (4 + 2 \times n)$$

$$S = n^2 + 2n$$

Q. 16. The sum of n terms of an AP is $\frac{1}{2}$ an^2 + bn. Find the common difference.

Answer : Given: the sum of n terms of an AP is $\frac{1}{2}$ an^2 + bn

To find: common difference.

Put n = 1 we get

First term = $\frac{1}{2} + b$

Put n = 2 we get

First term + second term = 2 × a + 2 × b

Second term = $\frac{3}{2}$ a + b

Therefore common difference will be

Second term – first term

Common difference = 2a

Q. 17. If the sums of n terms of two APs are in ratio (2n + 3) : (3n + 2), find the ratio of their 10th terms.

Answer : Given: sums of n terms of two APs are in ratio (2n + 3) : (3n + 2)

To find: find the ratio of their 10th terms.

For the sum of n terms of two APs is given by

$$S_1 = \frac{n}{2}(2a_1 + (n-1) \times d_1)$$

$$S_2 = \frac{n}{2}(2a_2 + (n-1) \times d_2)$$

$$\frac{S_1}{S_2} = \frac{2n + 3}{3n + 2}$$

$$= \frac{(2a_1 + (n-1) \times d_1}{(2a_2 + (n-1) \times d_2)}$$

Or we can write it as

$$= \frac{(a_1 + \frac{(n-1) \times d_1}{2})}{(a_2 + \frac{(n-1) \times d_2}{2})}$$

For 10th term put $\frac{(n-1)}{2} = 10$

n = 19

Therefore the ratio of the 10th term will be

$$= \frac{2 \times 19 + 3}{3 \times 19 + 2}$$

$$\Longrightarrow 41:57$$