Geometrical Progression

Exercise 12A

Q. 1. Find the 6^{th} and nth terms of the GP 2, 6, 18, 54….

Answer : Given: GP is 2, 6, 18, 54….

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{6}{2} = 3$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 6^{th} term in the GP,

$a_6 = ar^5$

$= 2 \times 3^5$

$= 486$

n^{th} term in the GP,

$a_n = ar^{n-1}$

$= 2.3^{n-1}$

Hence, 6^{th} term = 486 and n^{th} term = 2.3^{n-1}

Q. 2. Find the 17^{th} and nth terms of the GP 2, $2\sqrt{2}$, 4, $8\sqrt{2}$….

Answer : Given GP is 2, $2\sqrt{2}$, 4, $8\sqrt{2}$ …..

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 2\sqrt{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 17^{th} term in the GP,

$a_{17} = ar^{16}$

$= 2 \times (\sqrt{2})^{16}$

$= 512$

n^{th} term in the GP,

$a_n = ar^{n-1}$

$= 2(\sqrt{2})^{n-1}$

$= (\sqrt{2})^{n+1}$

Hence, 17^{th} term = 512 and n^{th} term = $(\sqrt{2})^{n+1}$

Q. 3. Find the 7^{th} and nth terms of the GP 0.4, 0.8, 1.6….

Answer : Given GP is 0.4, 0.8, 1.6….

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 0.4$

Second term in GP, $a_2 = 0.8$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{0.8}{0.4} = 2$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 7^{th} term in the GP,

$a_7 = ar^6$

$= 0.4 \times 2^6$

$= 25.6$

n^{th} term in the GP,

$a_n = ar^{n-1}$

$= (0.4)(2)^{n-1}$

$= (0.2)2^n$

Hence, 7^{th} term = 25.6 and n^{th} term = $(0.2)2^n$

Q. 4. Find the 10^{th} and nth terms of the GP $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9} \ldots.$

Answer : Given GP is $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9} \ldots.$.

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

The first term in the given GP, $a = a_1 = -\frac{3}{4}$

The second term in GP, $a_2 = \frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$$

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

So, the 10^{th} term, $a_{10} = ar^9$

$$a_{10} = ar^9 = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^9 = \frac{128}{6561}$$

Now, the required n^{th} term, $a_n = ar^{n-1}$

$$a_n = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^{n-1} = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$$

Hence, the 10^{th} term, $a_{10} = \frac{128}{6561}$ and n^{th} term, $a_n = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$.

Q. 5. Which term of the GP 3, 6, 12, 24…. Is 3072?

Answer : Given GP is 3, 6, 12, 24….

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 3$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{6}{3} = 2$$

Let us consider 3072 as the n^{th} term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$3072 = 3.2^{n-1}$

$$\frac{3072 \times 2}{3} = 2^n$$

$2^n = 2^{11}$

n = 11

So, 3072 is the 11^{th} term in GP.

Q. 6. Which term of the GP $\frac{1}{4}, \frac{-1}{2}, 1 \ldots$.. is -128?

Answer : Given GP is $\frac{1}{4}, \frac{-1}{2}, 1 \ldots$..

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

The first term in the given GP, $a = a_1 = \frac{1}{4}$

The second term in GP, $a_2 = -\frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{4}{2} = -2$$

Let us consider -128 as the n^th^ term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$$-128 = \left(\frac{1}{4}\right)(-2)_{n-1}$$

$(-2)^n = 1024 = (-2)^{10}$

n = 10

So, -128 is the 10^{th} term in GP.

Q. 7. Which term of the GP√3, 3, 3√3… is 729?

Answer : Given GP is √3, 3, 3√3….

The given GP is of the form, a, ar, ar^2, ar^3….

Where r is the common ratio.

First term in the given GP, $a_1 = a = \sqrt{3}$

Second term in GP, $a_2 = 3$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Let us consider 729 as the n^{th} term of the GP.

Now, n^{th} term of GP is, $a_n = ar^{n-1}$

$729 = \sqrt{3}\,(\sqrt{3})^{n-1}$

$\sqrt{3}^n = \sqrt{3}^{12}$

n = 12

So, 729 is the 12^{th} term in GP.

Q. 8. Find the geometric series whose 5^{th} and 8^{th} terms are 80 and 640 respectively.

Answer : The n^{th} term of a GP is $a_n = ar^{n-1}$

It's given in the question that 5^{th} term of the GP is 80 and 8^{th} term of GP is 640.

So, $a_5 = ar^4 = 80 \rightarrow (1)$

$a_8 = ar^7 = 640 \rightarrow (2)$

$$\frac{(2)}{(1)} \rightarrow \frac{ar^7}{ar^4} = r^3 = \frac{640}{80} = 8$$

Common ratio, r = 2,

$ar^4 = 80$

16a = 80

a = 5

The required GP is of the form a, ar, ar^2, ar^3, ar^4….

First term of GP, a = 5

Second term of GP, ar = 5 x 2 =10

Third term of GP, $ar^2 = 5 \times 2^2 = 20$

Fourth term of GP, $ar^3 = 5 \times 2^3 = 40$

Fifth term of GP, $ar^4 = 5 \times 2^4 = 80$

And so on...

The required GP is 5, 10, 20, 40, 80…

Q. 9. Find the GP whose 4^{th} and 7^{th} terms are $\frac{1}{18}$ and $\frac{-1}{486}$ respectively.

Answer : The n^{th} term of a GP is $a_n = ar^{n-1}$

It's given in the question that 4^{th} term of the GP is $\frac{1}{18}$ and 7^{th} term of GP is $-\frac{1}{486}$.

$$\text{So, } a_4 = ar^3 = \frac{1}{18} \rightarrow (1)$$

$$a^7 = ar^6 = -\frac{1}{486} \rightarrow (2)$$

$$\frac{(2)}{(1)} \rightarrow \frac{ar^6}{ar^3} = r^3 = -\frac{1}{27}$$

$$\text{Common ratio, } r = -\frac{1}{3}$$

$$ar^3 = \frac{1}{18}$$

$$a = -\frac{3}{2}$$

The required GP is of form a, ar, ar^2, ar^3, ar^4….

The first term of GP, a= $-\frac{3}{2}$

The second term of GP, $ar = -\frac{3}{2} \times -\frac{1}{3} = \frac{1}{2}$

The third term of GP, $ar^2 = \frac{1}{2} \times -\frac{1}{3} = -\frac{1}{6}$

The fourth term of GP, $ar^3 = -\frac{1}{6} \times -\frac{1}{3} = \frac{1}{18}$

The fifth term of GP, $ar^4 = \frac{1}{18} \times -\frac{1}{3} = -\frac{1}{54}$

And so on...

The required GP is $-\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}$ ....

Q. 10. The 5th, 8th and 11th terms of a GP are a, b, c respectively. Show that $b^2 = ac$

Answer : It is given in the question that 5th, 8th and 11th terms of GP are a, b and c respectively.

Let us assume the GP is A, AR, AR^2, and AR3….

So, the nth term of this GP is $a_n = AR^{n-1}$

Now, 5th term, $a_5 = AR^4 = a \rightarrow (1)$

8th term, $a_8 = AR^7 = b \rightarrow (2)$

11th term, $a_{11} = AR^{10} = c \rightarrow (3)$

Dividing equation (3) by (2) and (2) by (1),

$$\frac{(3)}{(2)} \rightarrow \frac{AR^{10}}{AR^{7}} = R^3 = \frac{c}{b} \rightarrow (4)$$

$$\frac{(2)}{(1)} \rightarrow \frac{AR^{7}}{AR^{4}} = R^3 = \frac{b}{a} \rightarrow (5)$$

So, both equation (4) and (5) gives the value of R^3. So we can equate them.

$$\frac{c}{b} = \frac{b}{a} = R^3,$$

$\therefore b^2 = ac,$

Hence proved.

Q. 11. The first term of a GP is -3 and the square of the second term is equal to its 4^{th} term. Find its 7^{th} term.

Answer : It is given that the first term of GP is -3.

So, a = -3

It is also given that the square of the second term is equal to its 4^{th} term.

$\therefore (a_2)^2 = a_4$

n^{th} term of GP, $a_n = ar^{n-1}$

So, $a_2 = ar$; $a_4 = ar^3$

$(ar)^2 = ar^3 \rightarrow a = r = -3$

Now, the 7^{th} term in the GP, $a_7 = ar^6$

$a_7 = (-3)^7 = -2187$

Hence, the 7^{th} term of GP is -2187.

Q. 12. Find the 6^{th} term from the end of GP 8, 4, 2…$\frac{1}{1024}$.

Answer : The given GP is 8, 4, 2…$\frac{1}{1024}$.$\rightarrow$ (1)

First term in the GP, $a_1 = a = 8$

Second term in the GP, $a_2 = ar = 4$

The common ratio, $r = \frac{4}{8} = \frac{1}{2}$

The last term in the given GP is $\frac{1}{1024}$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3} \ldots ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $\frac{1}{r}$.

So, common ratio = 2

$$a = \frac{1}{1024}$$

So, 6th term of the GP (2),

$a_6 = ar^5$

$= \frac{1}{1024} \times 2^5 = \frac{1}{32}$

Hence, the 6th term from the end of the given GP is $\frac{1}{32}$.

Q. 13. Find the 4th term from the end of the GP $\frac{2}{27}.\frac{2}{9},\frac{2}{3},\ldots,162$.

Answer : The given GP is $\frac{2}{27}.\frac{2}{9},\frac{2}{3},\ldots,162. \rightarrow (1)$

The first term in the GP, $a_1 = a = \frac{2}{27}$

The second term in the GP, $a_2 = \frac{2}{9}$

The common ratio, $r = 3$

The last term in the given GP is $a_n = 162$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3}....ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $r' = \frac{1}{r}$.

So, $r' = \frac{1}{3}$

So, 4th term of the GP (2),

$a_4 = ar^3$

$$= 162 \times \frac{1}{3^3} = 6$$

Hence, the 4th term from the end of the given GP is 6.

Q. 14. If a, b, c are the pth, qth and rth terms of a GP, show that

$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$.

Answer : As per the question, a, b and c are the pth, qth and rth term of GP.

Let us assume the required GP as $A, AR, AR^2, AR^3...$

Now, the nth term in the GP, $a_n = AR^{n-1}$

pth term, $a_p = AR^{p-1} = a \rightarrow (1)$

qth term, $a_q = AR^{q-1} = b \rightarrow (2)$

rth term, $a_r = AR^{r-1} = c \rightarrow (3)$

$$\frac{(1)}{(2)} \to \frac{R^{p-1}}{R^{q-1}} = R^{p-q} = \frac{a}{b} \longrightarrow \text{(i)}$$

$$\frac{(2)}{(3)} \to \frac{R^{q-1}}{R^{r-1}} = R^{q-r} = \frac{b}{c} \longrightarrow \text{(ii)}$$

$$\frac{(3)}{(1)} \to \frac{R^{r-1}}{R^{p-1}} = R^{r-p} = \frac{c}{a} \longrightarrow \text{(iii)}$$

Taking logarithm on both sides of equation (i), (ii) and (iii).

$(p - q) \log R = \log a - \log b,$

$$\therefore (p - q) = \frac{\log a - \log b}{\log R} \longrightarrow (4)$$

$(q - r) \log R = \log b - \log c$

$$\therefore (q - r) = \frac{\log b - \log c}{\log R} \longrightarrow (5)$$

$(r - p) \log R = \log c - \log a$

$$\therefore (r - p) = \frac{\log c - \log a}{\log R} \longrightarrow (6)$$

Now, multiply equation (4) with log c,

$$(p-q)\log c = \left(\frac{\log a - \log b}{\log R}\right)\log c \rightarrow (7)$$

Now, multiply equation (5) with log a,

$$(q-r)\log a = \left(\frac{\log b - \log c}{\log R}\right)\log a \rightarrow (8)$$

Now, multiply equation (6) with log b,

$$(r-p)\log b = \left(\frac{\log c - \log a}{\log R}\right)\log b \rightarrow (9)$$

Now, add equations (7), (8) and (9).

$$(p-q)\log c + (q-r)\log a + (r-p)\log b = \left(\frac{\log a - \log b}{\log R}\right)\log c + \left(\frac{\log b - \log c}{\log R}\right)\log a + \left(\frac{\log c - \log a}{\log R}\right)\log b$$

On solving the above equation, we will get,

(p – q) log c + (q – r) log a + (r – p) log b = 0

Hence proved.

Q. 15. The third term of a GP is 4; Find the product of its five terms.

Answer : Given that the third term of the GP, $a_3 = 4$

Let us assume the GP mentioned in the question be,

$\frac{A}{R^2}, \frac{A}{R}, A, AR, AR^2 \ldots$

With the first term $\frac{A}{R^2}$ and common ratio R.

Now, the third term in the assumed GP is A.

So, A = 4 (given data)

Now,

$$\text{Product of the first five terms of GP} = \frac{A}{R^2} \times \frac{A}{R} \times A \times AR \times AR^2 = A^5$$

So, the required product = $A^5 = 4^5 = 1024$

∴ The product of first five terms of a GP with its third term 4 is 1024.

Q. 16. In a finite GP, prove that the product of the terms equidistant from the beginning and end is the product of first and last terms.

Answer : We need to prove that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

Let us first consider a finite GP.

A, AR, AR^2….AR^{n-1}, AR^n.

Where n is finite.

Product of first and last terms in the given GP = $A.AR^n$

= A^2R^n → (a)

Now, n^{th} term of the GP from the beginning = AR^{n-1} → (1)

Now, starting from the end,

First term = AR^n

Last term = A

$$\frac{1}{R} = \text{Common Ratio}$$

So, an n^{th} term from the end of GP, $A_n = (AR^n)\left(\frac{1}{R^{n-1}}\right) = AR$ → (2)

So, the product of n^{th} terms from the beginning and end of the considered GP from (1) and (2) = $(AR^{n-1})(AR)$

$= A^2R^n \rightarrow$ (b)

So, from (a) and (b) its proved that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

Q. 17. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx} (x \neq 0)$ **then show that a, b, c, d are in GP.**

Answer : $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$ (Given data in the question) $\rightarrow$ (1)

Cross multiplying (1) and expanding,

(a + bx)(b – cx) = (b + cx)(a-bx)

$ab - acx + b^2x - bcx^2 = ba - b^2x + acx - bcx^2$

$2b^2x = 2acx$

$b^2 = ac \rightarrow$ (i)

If three terms are in GP, then the middle term is the Geometric Mean of first term and last term.

$\rightarrow b^2 = ac$

So, from (i) b, is the geometric mean of a and b.

So, a, b, c are in GP.

$\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ (Given data in the question) $\rightarrow$ (2)

Cross multiplying (2) and expanding,

(b + cx)(c – dx) = (c + dx)(b – cx)

$bc - bdx + c^2x - cdx^2 = cb - c^2x + bdx - dcx^2$

$2c^2x = 2bdx$

$c^2 = bd \rightarrow$ (ii)

So, from (ii), c is the geometric mean of b and d.

So, b, c, d is in GP.

∴ a, b, c, d are in GP.

Q. 18. If a and b are the roots of $x^2 – 3x + p = 0$ and c and d are the roots of $x^2 – 12x + q = 0$, where a, b, c, d from a GP, prove that (q + p): (q – p) = 17: 15.

Answer : Given data is,

$x^2 – 3x + p = 0 \rightarrow (1)$

a and b are roots of (1)

So, (x + a)(x + b) = 0

$x^2 - (a + b)x + ab = 0$

So, a + b = 3 and ab = p → (2)

Given data is,

$x^2 – 12x + q = 0 \rightarrow (3)$

c and d are roots of (1)

So, (x + c)(x + d) = 0

$x^2 - (c + d)x + cd = 0$

So, c + d = 12 and cd = q → (4)

a, b, c, d are in GP.(Given data)

Similarly A, AR, AR^2, AR^3 also forms a GP, with common ratio R.

From (2),

a + b = 3

A + AR = 3

$$\frac{3}{A} = 1 + R \rightarrow (5)$$

From (4),

c + d = 12

$AR^2 + AR^3 = 12$

$AR^2 (1 + R) = 12 \rightarrow (6)$

Substituting value of (1 + R) in (6).

R = 2

Now, substitute value of R in (5) to get value of A,

A = 1

Now, the GP required is A, AR, AR^2, and AR^3

1, 2, 4, 8…is the required GP.

So,

a = 1, b = 2, c = 4, d = 8

From (2) and (4),

ab = p and cd = q

So, p = 2, and q = 32.

$$\frac{q+p}{q-p} = \frac{cd+ab}{cd-ab} = \frac{34}{30} = \frac{17}{15}$$

So, (q + p): (q – p) = 17: 15.

Exercise 12C

Q. 1. A. Find the sum of the GP :

1 + 3 + 9 + 27 + …. To 7 terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) 3 ÷ 1 = 3

n = 7 terms

$$\therefore S_n = 1\frac{3^7 - 1}{3 - 1}$$

$$\Rightarrow S_n = \frac{2187 - 1}{3 - 1}$$

$$\Rightarrow S_n = \frac{2186}{2}$$

$$\Rightarrow S_n = 1093$$

Q. 1. B. Find the sum of the GP :

1 + $\sqrt{3}$ + 3 + 3$\sqrt{3}$ +….. to 10 terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $\sqrt{3}$ ÷ 1 = $\sqrt{3}$ = 1.732

n = 10 terms

$$\therefore S_n = 1.\frac{\sqrt{3}^{10}-1}{\sqrt{3}-1}$$

$$\Rightarrow S_n = \frac{1.732^{10}-1}{1.732-1}$$

$$\Rightarrow S_n = \frac{242.929-1}{0.732}$$

$$\Rightarrow S_n = \frac{241.929}{0.732}$$

$$\Rightarrow S_n = 330.504$$

Q. 1. C. Find the sum of the GP :

0.15 + 0.015 + 0.0015 + …. To 6 terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 0.15

r = (ratio between the n term and n-1 term) 0.015 ÷ 0.15 = 0.1

n = 6 terms

$$\Rightarrow S_n = 0.15 \times \frac{1-0.1^6}{1-0.1}$$

$$\Rightarrow S_n = 0.15 \times \frac{1-0.000001}{0.9}$$

$$\Rightarrow S_n = 0.15 \times \frac{0.999999}{0.9}$$

$$\therefore S_n = 16.67$$

Q. 1. D. Find the sum of the GP :

$1 - \frac{1}{2} + \frac{1}{4} = \frac{1}{8} + \ldots$ to 9 terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-\frac{1}{2} \div 1 = -\frac{1}{2}$

n = 9 terms

$\therefore S_n = 1 \times \frac{1-\frac{-1}{2}^9}{1-(\frac{-1}{2})}$

$\Rightarrow S_n = \frac{1+\frac{1}{512}}{1+\frac{1}{2}}$

$\Rightarrow S_n = \frac{\frac{513}{2}}{\frac{3}{2}}$

$\therefore S_n = 171$

Q. 1. E. Find the sum of the GP :

$\sqrt{2} + \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} +$**……to 8 terms**

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\sqrt{2}$

r = (ratio between the n term and n-1 term) $\frac{1}{\sqrt{2}} \div \sqrt{2} = \frac{1}{2}$

n = 8 terms

$$\therefore S_n = \sqrt{2} \times \frac{1-\frac{1}{2}^8}{1-\frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{1-\frac{1}{256}}{\frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{\frac{255}{256}}{\frac{1}{2}}$$

$$\Rightarrow S_n = \sqrt{2} \times \frac{255}{128}$$

$$\therefore S_n = \frac{255\sqrt{2}}{128}$$

Q. 1. F. Find the sum of the GP :

$\frac{2}{9} - \frac{1}{3} - \frac{1}{2} - \frac{3}{4} + \dots$ **To 6 terms**

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\frac{2}{9}$

r = (ratio between the n term and n-1 term) $-\frac{1}{3} \div \frac{2}{9} = -\frac{3}{2} = 1.5$

n = 6 terms

$$\therefore S_n = \frac{2}{9} \times \frac{1.5^6 - 1}{1.5 - 1}$$

$$\Rightarrow S_n = \frac{2}{9} \times \frac{10.39}{0.5}$$

$$\therefore S_n = 4.62$$

Q. 2. A. Find the sum of the GP :

$\sqrt{7} + \sqrt{21} + 3\sqrt{7}$ **+ … to n terms**

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r - 1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\sqrt{7}$

r = (ratio between the n term and n-1 term) $\sqrt{7} \div \sqrt{21} = \sqrt{3}$

n terms

$$\therefore S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1} \text{ [Rationalizing the denominator]}$$

$$\Rightarrow S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow S_n = \sqrt{7} \times \frac{\left(\sqrt{3}^n - 1\right)\left(\sqrt{3} + 1\right)}{3 - 1}$$

$$\therefore S_n = \frac{\sqrt{7}\left(\sqrt{3}^n - 1\right)\left(\sqrt{3} + 1\right)}{2}$$

Q. 2. B. Find the sum of the GP :

$1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+$ **… to n terms**

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-\frac{1}{3} \div 1 = -\frac{1}{3}$

n terms

$$\therefore S_n = 1 \times \frac{1-\frac{-1}{3}^n}{1-\frac{1}{3}}$$

$$\Rightarrow S_n = \frac{1-\frac{1}{3}^n}{\frac{2}{3}}$$

$$\therefore S_n = \frac{3-\frac{1}{3}^{n-1}}{2}$$

Q. 2. C. Find the sum of the GP :

1 – a + a^2 – a^3 + …to n terms (a ≠ 1)

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-a \div 1 = -a$

n terms

$\therefore S_n = 1 \times \frac{(-a)^n - 1}{-a-1}$

[Multiplying both numerator and denominator by -1]

$\Rightarrow S_n = \frac{1-(-a)^n}{1+a}$

Q. 2. D. Find the sum of the GP :

$x^3 + x^5 + x^7 + \ldots$ To n terms

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$a = x^3$

r = (ratio between the n term and n-1 term) $x^5 \div x^3 = x^2$

n terms

$\therefore S_n = x^3 \times \frac{x^{2n} - 1}{x^2 - 1}$

$\Rightarrow S_n = \frac{x^3 (x^n - 1)(x^n + 1)}{(x-1)(x+1)}$

Q. 2. E. Find the sum of the GP :

$x(x + y) + x^2(x^2 + y^2) + x^3(x^3 + y^3) + \ldots$ To n terms

Answer : The given expression can be written as

$= (x^2 + xy) + (x^4 + x^2y^2) + (x^6 + x^3y^3) + \ldots$ To n terms

$= (x^2 + x^4 + x^6 + \ldots$ to n terms $) + (xy + x^2y^2 + x^3y^3 + \ldots$ to n terms $)$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

a= x^2 first part and xy for the second part

r = (ratio between the n term and n-1 term) x^2 for the first part and xy for the second part

n terms

$$\therefore S_n = x^2 \times \frac{x^{2n} - 1}{x^2 - 1} + xy \times \frac{x^n y^n - 1}{xy - 1}$$

$$\Rightarrow S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x+1)(x-1)} + \frac{x^{n+1}y^{n+1} - 1}{xy - 1}$$

Q. 3. Find the sum to n terms of the sequence :

(i) $\left(x + \frac{1}{x}\right)^2, \left(x^2 + \frac{1}{x^2}\right)^2, \left(x^3 + \frac{1}{x^3}\right)^2$ **,….. to n terms**

(ii) (x + y), 9x^2 + xy + y^2), (x^3 + x2y + xy^2 + y^3), …. to n terms

Answer : This can also be written as

$$= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 2\right) + \left(x^6 + \frac{1}{x^6} + 2\right) + \ldots \ldots \text{to n term}$$

$$= (x^2 + x^4 + x^6 + \ldots.\text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots.\text{to n terms}\right) + (2 + 2 + 2 + \ldots.\text{to n terms})$$

$$= (x^2 + x^4 + x^6 + \ldots.\text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots.\text{to n terms}\right) + 2n$$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

a = $x^2, \frac{1}{x^2}$

r = (ratio between the n term and n-1 term) $x^2, \frac{1}{x^2}$

n terms

∴ $S_n = x^2 \times \frac{x^{2n} - 1}{x^2 - 1} + \frac{1}{x^2} \times \frac{(\frac{1}{x^2})^n - 1}{\frac{1}{x^2} - 1} + 2n$

⇒ $S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{1}{x^2} \times \frac{\frac{1}{x^2}^n - 1}{\frac{x^2 - 1}{x^2}} + 2n$

⇒ $S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{x^2 - 1} + 2n$

⇒ $S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$

∴ $S_n = \frac{x^2(x^n - 1)(x^n + 1) + \frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$

(ii) If we divide and multiply the terms by (x-y)

= $\frac{(x-y)(x+y) + (x-y)(x^2 + xy + y^2) + (x-y)(x^3 + x^2y + xy^2 + y^3) + \ldots\text{to n terms}}{(x-y)}$

= $\frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \ldots\text{to n terms}}{(x-y)}$

= $\frac{(x^2 + x^3 + x^4 + \ldots\text{to n terms}) + (y^2 + y^3 + y^4 + \ldots\text{to n terms})}{(x-y)}$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r - 1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = x^2, y^2

r = (ratio between the n term and n-1 term) x, y n terms

$\therefore S_n = \frac{x^2 \times \frac{x^n - 1}{x-1} + y^2 \times \frac{y^n - 1}{y-1}}{(x-y)}$

$\Rightarrow S_n = \frac{\frac{x^2(x^n - 1)}{x-1} + \frac{y^2(y^n - 1)}{y-1}}{(x-y)}$

Q. 4. Find the sum :

$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} +$ **…. To 2n terms**

Answer : We can split the above expression into 2 parts. We will split 2n terms into 2 parts also which will leave it as n terms and another n terms.

$= \left(\frac{3}{5} + \frac{3}{5^3} + \text{... to n terms}\right) + \left(\frac{4}{5} + \frac{4}{5^2} + \text{... to n terms}\right)$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when $|r|<1$. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\frac{3}{5}, \frac{4}{5}$

r = (ratio between the n term and n-1 term) $\frac{3}{5^3} \div \frac{3}{5}, \frac{4}{5^2} \div \frac{4}{5} = \frac{1}{5^2}, \frac{1}{5}$

n terms

$\therefore S_n = \frac{3}{5} \times \frac{1 - \frac{1}{5^2}^n}{1 - \frac{1}{5^2}} + \frac{4}{5} \times \frac{1 - \frac{1}{5}^n}{1 - \frac{1}{5}}$

$\Rightarrow S_n = \frac{3}{5} \times \frac{1 - \frac{1}{5^2}^n}{\frac{24}{5^2}} + \frac{4}{5} \times \frac{1 - \frac{1}{5}^n}{\frac{4}{5}}$

$\Rightarrow S_n = \frac{5\left(1-\frac{1}{5^2}^n\right)}{8} + \left(1 - \frac{1}{5}^n\right)$

$\Rightarrow S_n = \frac{\left(5-\frac{5}{5^{2n}}\right)}{8} + \left(1 - \frac{1}{5}^n\right)$

$\therefore S_n = \frac{\left(5-\frac{1}{5^{2n-1}}\right)}{8} + \left(1 - \frac{1}{5}^n\right)$

Q. 5. Evaluate :

NOTE: In an expression like this ⇒ $\sum_{i=1}^{n} x$, n represents the upper limit, 1 represents the lower limit , x is the variable expression which we are finding out the sum of and i represents the index of summarization.

(i) $\sum_{n=1}^{10}(2+3^n)$

(ii) $\sum_{k=1}^{n}[2^k + 3^{(k-1)}]$

(iii) $\sum_{n=1}^{8} 5^n$

Answer : We can write this as $(2 + 3^1)+(2+3^2) + (2 +3^3)+\ldots$ to 10 terms

= (2+2+2+… to 10 terms) + ($3+3^2+3^3+\ldots$ to 10 terms)

= 2×10 + ($3+3^2+3^3+\ldots$ to 10 terms)

= 20 + ($3+3^2+3^3+\ldots$ to 10 terms)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 3

r = (ratio between the n term and n-1 term) 3

n = 10 terms

$$S_n = 3 \times \frac{3^{10} - 1}{3 - 1}$$

$\Rightarrow S_n = 3 \times \frac{59049-1}{2}$

$\Rightarrow S_n = 3 \times \frac{59048}{2}$

$\Rightarrow S_n = 88572$

Thus, sum of the given expression is

= 20 + ($3+3^2+3^3+$… to 10 terms)

= 20 + 88572

= 88592

(ii) The given expression can be written as,

$(2^1 + 3^{1-1}) + (2^2 + 3^{2-1})$ + …to n terms

= $(2 + 3^0) + (2^2 + 3^1)$ + …to n terms

= $(2 + 1) + (2^2 + 3)$ + …to n terms

= $(2 + 2^2$ + …to $\frac{n}{2}$ terms) + (1 + 3 + … to $\frac{n}{2}$ terms)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 2, 1

r = (ratio between the n term and n-1 term) 2, 3

$\frac{n}{2}$ terms

$$S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{2 - 1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{3 - 1}$$

$$\Rightarrow S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{2}$$

$$\Rightarrow S_n = 2^{\frac{n}{2}+1} - 2 + \frac{3^{\frac{n}{2}} - 1}{2}$$

(iii) We can rewrite the given expression as

($5^1 + 5^2 + 5^3 +$ …to 8 terms)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r - 1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 5

r = (ratio between the n term and n-1 term) 5

n = 8 terms

$$S_n = 5 \times \frac{5^8 - 1}{5 - 1}$$

$$\Rightarrow S_n = 5 \times \frac{390625 - 1}{4}$$

$$\Rightarrow S_n = 5 \times \frac{390624}{4}$$

$$\Rightarrow S_n = 488280$$

Q. 6. Find the sum of the series :

NOTE: The following terms are not G.P. series, but we can convert them to form one.
(i) 8 + 88 + 888 + …. To n terms
(ii) 3 + 33 + 333 + …. To n terms
(iii) 0.7 + 0.77 + 0.777 + …. To n terms

Answer : The expression can be rewritten as

[Taking 8 as a common factor]

8(1+ 11 + 111+ … to n terms)

[Multiplying and dividing the expression by 9]

$= \frac{8}{9}$ (9 + 99+ 999 + … to n terms)

$= \frac{8}{9}$ ((10-1) + (100-1) + (1000-1) + … to n terms)

$= \frac{8}{9}$ ((10 + 100 + 1000 + … to n terms) – (1+1+1+ … to n terms)

$= \frac{8}{9}$ ((10 + 100 + 1000 + … to n terms) – n)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 10

r = (ratio between the n term and n-1 term) 10

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$$\Rightarrow S_n = 10 \times \frac{10^n - 1}{9}$$

$$\Rightarrow S_n = \frac{10^{n+1} - 10}{9}$$

∴ The sum of the given expression is

$= \frac{8}{9}$ ((10 + 100 + 1000 + … to n terms) – n)

$= \frac{8}{9} (\frac{10^{n+1} - 10}{9} - n)$

(ii) The given expression can be rewritten as

[taking 3 common]

= 3(1+11+111+ …to n terms)

[Multiplying and dividing the expression by 9]

$= \frac{3}{9}$ (9+99+999+ … to n terms)

$= \frac{3}{9}$ ((10-1) + (100-1) + (1000-1) + … to n terms)

$= \frac{3}{9}$ ((10+100+1000+ …to n terms) – (1+1+1+ … to n terms))

$= \frac{3}{9}$ ((10+100+1000+ to n terms) – n)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 10

r = (ratio between the n term and n-1 term) 10

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$$\Rightarrow S_n = 10 \times \frac{10^n - 1}{9}$$

$$\Rightarrow S_n = \frac{10^{n+1} - 10}{9}$$

∴ The sum of the given expression is

$= \frac{3}{9}$ ((10+100+1000+ to n terms) – n)

$= \frac{3}{9} \left(\frac{10^{n+1} - 10}{9} - n \right)$

(iii) We can rewrite the expression as

[taking 7 as a common factor]

= 7(0.1+0.11+0.111+ … to n terms)

[multiplying and dividing by 9]

$= \frac{7}{9}$ (0.9+0.99+0.999+ … to n terms)

$= \frac{7}{9}$ ((1-0.1)+(1-0.01)+(1-0.001)+ … to n terms)

$= \frac{7}{9}$ ((1+1+1+ … to n terms)–(0.1+0.01+0.001+… to n terms))

$= \frac{7}{9}$ (n – (0.1+0.01+0.001+ … to n terms))

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 0.1

r = (ratio between the n term and n-1 term) 0.1

n terms

$$S_n = 0.1 \times \frac{1 - 0.1^n}{1 - 0.1}$$

$$\Rightarrow S_n = 0.1 \times \frac{1-0.1^n}{0.9}$$

[multiplying both numerator and denominator by 10]

$\Rightarrow S_n = \frac{1-0.1^n}{9}$

∴ The sum of the given expression is

$= \frac{7}{9}$ (n – (0.1+0.01+0.001+ … to n terms))

$= \frac{7}{9}\left(n - \left(\frac{1-0.1^n}{9} \right) \right)$

Q. 7. The sum of n terms of a progression is (2^n – 1). Show that it is a GP and find its common ratio.

Answer : In this question, we will try to rewrite the given sum of the progression like the formula for the sum a G.P. series.

It is given that $S_n = (2^n - 1)$

The formula for the sum of a G.P. series is,

$$S_n = a\frac{r^n - 1}{r - 1}$$

By solving the 2 equations together, we can say that

$$(2^n - 1) = a\frac{r^n - 1}{r - 1}$$

$$\Rightarrow 1 \times \frac{(2^n - 1)}{2-1} = a\frac{r^n - 1}{r-1}$$

By corresponding the numbers with the variables, we can conclude

a = 1

r = 2

The G.P. series will therefore look like ⇒ 1,2,4,8,16,……to n terms

∴ The given progression is a G.P. series with the common ration being 2.

Q. 8. In a GP, the ratio of the sum of the first three terms is to first six terms is 125 : 152. Find the common ratio.

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Sum of first 3 terms = $a \times \frac{r^3-1}{r-1}$

Sum of first 6 terms = $a \times \frac{r^6-1}{r-1}$

$\therefore \frac{a \times \frac{r^3-1}{r-1}}{a \times \frac{r^6-1}{r-1}} = \frac{125}{152}$

$\Rightarrow \frac{(r^3-1)}{(r^6-1)} = \frac{125}{152}$

$\Rightarrow 152r^3 - 152 = 125r^6 - 125$

$\Rightarrow 125r^6 - 152r^3 - 125 + 152 = 0$

$\Rightarrow 125r^6 - 152r^3 + 27 = 0$

$\Rightarrow 125r^6 - 125r^3 - 27r^3 + 27 = 0$

$\Rightarrow (125r^3 - 27)(r^3 - 1) = 0$

Either $125r^3 - 27 = 0$ or $r^3 - 1 = 0$

Either $125r^3 = 27$ or $r^3 = 1$

Either $r^3 = \frac{27}{125}$ or r=1

Either $r = \frac{3}{5}$ or r=1

Since r ≠ 1 [if r is 1, all the terms will be equal which destroys the purpose]

$\therefore r = \frac{3}{5}$

Q. 9. Find the sum of the geometric series 3 + 6 + 12 + … + 1536.

Answer : Tn represents the n^{th} term of a G.P. series.

$r = 6 \div 3 = 2$

$T_n = ar^{n-1}$

$\Rightarrow 1536 = 3 \times 2^{n-1}$

$\Rightarrow 1536 \div 3 = 2^n \div 2$

$\Rightarrow 1536 \div 3 \times 2 = 2^n$

$\Rightarrow 1024 = 2^n$

$\Rightarrow 2^{10} = 2^n$

$\therefore n = 10$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 3

r = 2

n = 10 terms

$\therefore S_n = 3 \times \frac{2^{10}-1}{2-1}$

$\Rightarrow S_n = 3 \times (1024 - 1)$

$\Rightarrow S_n = 3 \times 1023$

$\therefore S_n = 3069$

Q. 10. How many terms of the series 2 + 6 + 18 + …. + must be taken to make the sum equal to 728?

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 2

r = (ratio between the n term and n-1 term) 6 ÷ 2 = 3

S_n = 728

∴ $728 = 2 \times \frac{3^n - 1}{3 - 1}$

⇒ $728 = 2 \times \frac{3^n - 1}{2}$

⇒ $728 = 3^n - 1$

⇒ $728 + 1 = 3^n$

⇒ $729 = 3^n$

⇒ $3^6 = 3^n$

∴ $n = 6$

∴ 6 terms must be taken to reach the desired answer.

Q. 11. The common ratio of a finite GP is 3, and its last term is 486. If the sum of these terms is 728, find the first term.

Answer : 'T_n' represents the n^{th} term of a G.P. series.

$T_n = ar^{n-1}$

⇒ $486 = a(3)^{n-1}$

⇒ $486 = a(\ 3^n \div 3)\ )$

⇒ $486 \times 3 = a(3^n)$

⇒ $1458 = a(3^n\ )$ ………(i)

Sum of a G.P. series is represented by the formula, $Sn = a\frac{r^n - 1}{r-1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

∴ $728 = a \times \frac{3^n - 1}{3-1}$

⇒$728 = a \times \frac{3^n - 1}{2}$

⇒728 × 2 = a (3^n)-a …… [Putting a(3^n) = 1458 from (i)]

⇒1456 = 1458 -a

⇒1456-1458 = -a

⇒-2=-a [Multipying both sides by -1]

⇒a = 2

Q. 12. The first term of a GP is 27, and its 8^{th} term is $\frac{1}{81}$. Find the sum of its first 10 terms.

Answer : 'T_n' represents the n^{th} term of a G.P. series.

$T_n = ar^{n-1}$

⇒$\frac{1}{81} = 27 \times r^{8-1}$

⇒$\frac{1}{81} = 27 \times r^7$

⇒$\frac{1}{81} \div \frac{1}{27} = r^7$

⇒$\frac{1}{2187} = r^7$

⇒$\left(\frac{1}{3}\right)^7 = r^7$

∴ $r = \frac{1}{3}$

Sum of a G.P. series is represented by the formula, $Sn = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 27

r = (ratio between the n term and n-1 term) $\frac{1}{3}$

n = 10 terms

$$\therefore S_n = 27 \times \frac{1-\frac{1}{3}^{10}}{1-\frac{1}{3}}$$

$$\Rightarrow S_n = 27 \times \frac{1-\frac{1}{59049}}{\frac{2}{3}}$$

$$\Rightarrow S_n = 27 \times \frac{\frac{59048}{59049}}{\frac{2}{3}}$$

$$\Rightarrow S_n = 27 \times \frac{39524}{19683}$$

$$\therefore S_n = \frac{39524}{729}$$

Q. 13. The 2nd and 5th terms of a GP are $\frac{-1}{2}$ and $\frac{1}{16}$ respectively. Find the sum of n terms GP up to 8 terms.

Answer : 2nd term = $ar^{2-1} = ar^1$

5th term = $ar^{5-1} = ar^4$

Dividing the 5th term using the 3rd term

$$\frac{ar^4}{ar} = \frac{\frac{1}{16}}{\frac{-1}{2}}$$

$r^3 = -\frac{1}{8}$

$\therefore r = -\frac{-1}{2}$

$\therefore a = 1$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

n = 8 terms

$$S_n = 1 \times \frac{1-\frac{-1}{2}^8}{1-\frac{-1}{2}}$$

$$\Rightarrow S_n = \frac{1-\frac{1}{256}}{\frac{3}{2}}$$

$$\Rightarrow S_n = \frac{\frac{255}{256}}{\frac{3}{2}}$$

$$\therefore S_n = \frac{170}{256}$$

Q. 14. The 4^{th} and 7^{th} terms of a GP are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of the GP.

Answer : 4^{th} term = $ar^{4-1} = ar^3 = \frac{1}{27}$

7^{th} term = $ar^{7-1} = ar^6 = \frac{1}{729}$

Dividing the 7^{th} term by the 4^{th} term,

$$\frac{ar^6}{ar^3} = \frac{\frac{1}{729}}{\frac{1}{27}}$$

$\Rightarrow r^3 = \frac{1}{27}$ ……(i)

$\therefore r = \frac{1}{3}$

$ar^3 = \frac{1}{27}$ [putting from eqn (i)]

$a\frac{1}{27} = \frac{1}{27}$

$\therefore a = 1$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

$r = \frac{1}{3}$

n terms

$\therefore S_n = 1 \times \frac{1-\frac{1}{3}^n}{1-\frac{1}{3}}$

$\Rightarrow S_n = \frac{1-\frac{1}{3}^n}{\frac{2}{3}}$

$\Rightarrow S_n = \frac{3\left(1-\frac{1}{3^n}\right)}{2}$

$\therefore S_n = \frac{3-\frac{1}{3^{n-1}}}{2}$

Q. 15. A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places, find the common ratio of the GP.

Answer : Let the terms of the G.P. be a, ar, ar^2, ar^3, … , ar^{n-2}, ar^{n-1}

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Thus, the sum of this G.P. series is $S_n = a\frac{r^n-1}{r-1}$

The odd terms of this series are a, ar^2, ar^4, … , ar^{n-2}

{Since the number of terms of the G.P. series is even; the 2^{nd} last term will be an odd term.}

Here,

No. of terms will be $\frac{n}{2}$ as we are splitting up the n terms into 2 equal parts of odd and even terms. {since the no. of terms is even, we have 2 equal groups of odd and even terms }

Sum of the odd terms ⇒

$$S_n = a \times \frac{r^{2(\frac{n}{2})} - 1}{r^2 - 1}$$

$$\Rightarrow S_n = a \times \frac{r^n - 1}{(r-1)(r+1)}$$

By the problem,

$$a\frac{r^n - 1}{r - 1} = 5 \times a \times \frac{r^n - 1}{(r-1)(r+1)}$$

$$\Rightarrow 1 = \frac{5}{(r+1)}$$

⇒ r +1 =5

⇒∴ r = 4

Thus, the common ratio (r) = 4

Q. 16. Show that the ratio of the sum of first n terms of a GP to the sum of the terms from (n + 1)th to (2n)th term is $\frac{1}{r^n}$.

Answer : Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. 'S_n' represents the sum of the G.P. series upto n^{th} terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Thus, the sum of the first n terms of the G.P. series is, $S_n = a\frac{r^n - 1}{r-1}$

Sum of $(n+1)^{th}$ term to $2n^{th}$ term

= Sum of the first $2n^{th}$ term – the sum of 1^{st} term to n^{th} term

$= a\frac{r^{2n}-1}{r-1} - a\frac{r^n - 1}{r-1}$

$= \frac{(ar^{2n} - a) - (ar^n - a)}{r-1}$

$= \frac{ar^{2n} - a - ar^n + a}{r-1}$

$= \frac{ar^n(r^n - 1)}{r-1}$

The ratio of the sum of first n terms of the G.P. to the sum of the terms from $(n + 1)^{th}$ to $(2n)^{th}$ term

$= \frac{a\frac{r^n - 1}{r-1}}{\frac{ar^n(r^n - 1)}{r-1}}$

[Cancelling out the common factors from the numerator and denominator ⇒ a, (r-1), $(r^n – 1)$]

$= \frac{1}{r^n}$

Hence Proved.

Exercise 12D

Q. 1. What will 15625 amount to in 3 years after its deposit in a bank which pays annual interest at the rate of 8% per annum, compounded annually?

Answer : To find: The amount after three years

Given: (i) Principal – 15625

(ii) Time – 3 years

(iii) Rate – 8% per annum

Formula used: $A = P\left(1+ \frac{r}{100}\right)^t$

$$A = 15625\left(1+ \frac{8}{100}\right)^3$$

$$A=15625\left(\frac{108}{100}\right)^3$$

A = 19683

Ans) 19683

Q. 2. The value of a machine costing 80000 depreciates at the rate of 15% per annum. What will be the worth of this machine after 3 days?

Answer : To find: The amount after three days

Given: (i) Principal – 80000

(ii) Time – 3 days

(iii) Rate – 15% per annum

Deduction = P × R × T

$$= 80000 \times \frac{15}{100} \times \frac{3}{365}$$

= 98.63

The final amount after deduction = 80000 – 98.63

= 79901.37

The value of the machine after 3 days is Rs. 79901.37

Q. 3. Three years before the population of a village was 10000. If at the end of each year, 20% of the people migrated to a nearby town, what is its present population?

Answer : To find: Present population of the village

Given: (i) Three years back population - 10000

(ii) Time – 3 years

(iii) Rate – 20% per annum

Number of people migrated on the very first year is 20% of 10000

$\Rightarrow \frac{10000 \times 20}{100} = 2000$

People left after migration in the very first year = 10000 – 2000

= 8000

Number of people migrated in the second year is 20% of 8000

$\Rightarrow \frac{8000 \times 20}{100} = 1600$

People left after migration in the second year = 8000 – 1600

= 6400

Number of people migrated in the third year is 20% of 6400

$\Rightarrow \frac{6400 \times 20}{100} = 1280$

People left after migration in the third year = 6400 – 1280

= 5120

Ans) The present population is 5120

Q. 4. What will 5000 amount to in 10 years, compounded annually at 10% per annum? [Given $(1.1)^{10} = 2.594$]

Answer : To find: The amount after ten years

Given: (i) Principal – 5000

(ii) Time – 10 years

(iii) Rate – 10% per annum

Formula used: $A=P\left(1+\frac{r}{100}\right)^t$

$\Rightarrow A=5000\left(1+\frac{10}{100}\right)^{10}$

$\Rightarrow A=5000\left(\frac{110}{100}\right)^{10}$

$\Rightarrow A=5000(1.1)^{10}$

$\Rightarrow A=5000\times2.594$

$\Rightarrow A=12970$

Ans) The amount after years will be Rs.12970

Q. 5. A manufacturer reckons that the value of a machine which costs him 156250, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Answer : To find: The amount after five years

Given: (i) Principal – 156250

(ii) Time – 5 years

(iii) Rate – 20% per annum

Formula used: $A=P\left(1-\frac{r}{100}\right)^t$

$\Rightarrow A=156250\left(1-\frac{20}{100}\right)^5$

$\Rightarrow A=156250\left(\frac{80}{100}\right)^5$

$\Rightarrow A=156250(0.8)^5$

$\Rightarrow A=156250\times 0.32768$

$\Rightarrow A=51200$

Ans) The amount after five years will be Rs.51200

Q. 6. The number of bacteria in a certain culture doubles every hour. If there were 50 bacteria present in the culture originally, how many bacteria would be present at the end of (i) 2^{nd} hour, (ii) 5^{th} hour and (iii) nth hour?

Answer : To find: The number of bacteria after

(i) 2^{nd} hour

(ii) 5^{th} hour

(iii) nth hour

Given: (i) Initially, there were 50 bacteria

(ii) Rate – 100% per hour

The formula used: $A=P\left(1+\frac{r}{100}\right)^t$

(i) For 2^{nd} hour

$\Rightarrow$ No. of bacteria$=50\left(1+\frac{100}{100}\right)^2$

$\Rightarrow$ No. of bacteria$=50(1+1)^2$

$\Rightarrow$ No. of bacteria$=50(2)^2$

$\Rightarrow$ No. of bacteria $= 50 \times 4$

$\Rightarrow$ No. of bacteria $= 200$

(ii) For 5th hour

$\Rightarrow$ No. of bacteria $= 50\left(1 + \frac{100}{100}\right)^5$

$\Rightarrow$ No. of bacteria $= 50(1 + 1)^5$

$\Rightarrow$ No. of bacteria $= 50(2)^5$

$\Rightarrow$ No. of bacteria $= 50 \times 32$

$\Rightarrow$ No. of bacteria $= 1600$

(iii) For nth hour

$\Rightarrow$ No. of bacteria $= 50\left(1 + \frac{100}{100}\right)^n$

$\Rightarrow$ No. of bacteria $= 50(1 + 1)^n$

$\Rightarrow$ No. of bacteria $= 50(2)^n$

$\Rightarrow$ No. of bacteria $= 2^n 50$

Ans) Number of bacteria in a 2nd hour will be 200, the number of bacteria in a 5th hour will be 1600 and number of bacteria in an nth hour will be $2^n 50$

Exercise 12E

Q. 1. If p, q, r are in AP, then prove that pth, qth and rth terms of any GP are in GP.

Answer : To prove: pth, qth and rth terms of any GP are in GP.

Given: (i) p, q and r are in AP

The formula used: (i) General term of GP, $T_n = ar^{n-1}$

As p, q, r are in A.P.

⇒ q – p = r – q = d = common difference … (i)

Consider a G.P. with the first term as a and common difference R

Then, the p^{th} term will be ar^{p-1}

The q^{th} term will be ar^{q-1}

The r^{th} term will be ar^{r-1}

Considering p^{th} term and q^{th} term

$$\Rightarrow \frac{q^{th}\ \text{term}}{p^{th}\ \text{term}} = \frac{ar^{q-1}}{ar^{p-1}}$$

$$\Rightarrow \frac{q^{th}\ \text{term}}{p^{th}\ \text{term}} = r^{q-1-p+1}$$

$$\Rightarrow \frac{q^{th}\ \text{term}}{p^{th}\ \text{term}} = r^{q-p}$$

From eqn. (i) q – p = d

$$\Rightarrow \frac{q^{th}\ \text{term}}{p^{th}\ \text{term}} = r^{d}$$

Considering q^{th} term and r^{th} term

$$\Rightarrow \frac{r^{th}\ \text{term}}{q^{th}\ \text{term}} = \frac{ar^{r-1}}{ar^{q-1}}$$

$$\Rightarrow \frac{r^{th}\ \text{term}}{q^{th}\ \text{term}} = r^{r-1-q+1}$$

$$\Rightarrow \frac{r^{th}\ \text{term}}{q^{th}\ \text{term}} = r^{r-q}$$

From eqn. (i) r – q = d

$\Rightarrow \frac{r^{th}\ \text{term}}{q^{th}\ \text{term}} = r^{d}$

We can see that p^{th}, q^{th} and r^{th} terms have common ration i.e r^{d}

Hence they are in G.P.

Hence Proved

Q. 2. If a, b, c are in GP, then show that log a^n, log b^n, log c^n are in AP.

Answer : To prove: log a^n, log b^n, log c^n are in AP.

Given: a, b, c are in GP

Formula used: (i) log ab = log a + log b

As a, b, c are in GP

$\Rightarrow b^2 = ac$

Taking power n on both sides

$\Rightarrow b^{2n} = (ac)^n$

Taking log both side

$\Rightarrow \log b^{2n} = \log(ac)^n$

$\Rightarrow \log b^{2n} = \log(a^n c^n)$

$\Rightarrow 2\log b^n = \log(a^n) + \log(c^n)$

Whenever a,b,c are in AP then 2b = a+c, considering this and the above equation we can say that log a^n, log b^n, log c^n are in AP.

Hence Proved

Q. 3. If a, b, c are GP, then show that $\frac{1}{\log_a m}$, $\frac{1}{\log_b m}$, $\frac{1}{\log_c m}$ are in AP.

Answer : To prove: $\frac{1}{\log_a m}$, $\frac{1}{\log_b m}$, $\frac{1}{\log_c m}$ are in AP.

Given: a, b, c are in GP

Formula used: (i) $\frac{1}{\log_a m} = \log_m a = \frac{\log a}{\log m}$

As, a, b, c are in GP

⇒ $\frac{b}{a} = \frac{c}{b}$

Taking log both side $\log\frac{b}{a} = \log\frac{c}{b}$

⇒ log b – log a = log c – log b

⇒ 2log b = log a + log c

Dividing by log m

⇒ $2\left(\frac{\log b}{\log m}\right) = \frac{\log a}{\log m} + \frac{\log c}{\log m}$

⇒ $2\log_m b = \log_m a + \log_m c \quad \left(\text{As, } \log_m a = \frac{\log a}{\log m}\right)$

⇒ $2\left(\frac{1}{\log_b m}\right) = \frac{1}{\log_a m} + \frac{1}{\log_c m} \left(\text{As } \frac{1}{\log_a m} = \log_m a\right)$

Whenever any number a,b,c are in AP then 2b = a+c, considering this and the above equation we can say that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in AP

Hence proved

Q. 4. Find the values of k for which k + 12, k – 6 and 3 are in GP.

Answer : To find: Value of k

Given: k + 12, k – 6 and 3 are in GP

Formula used: (i) when a,b,c are in GP $b^2 = ac$

As, k + 12, k – 6 and 3 are in GP

⇒ $(k - 6)^2 = (k + 12)(3)$

$\Rightarrow k^2 - 12k + 36 = 3k + 36$

$\Rightarrow k^2 - 15k = 0$

$\Rightarrow k (k - 15) = 0$

$\Rightarrow k = 0$, Or $k = 15$

Ans) We have two values of k as 0 or 15

Q. 5. Three numbers are in AP, and their sum is 15. If 1, 4, 19 be added to them respectively, then they are in GP. Find the numbers.

Answer : To find: The numbers

Given: Three numbers are in A.P. Their sum is 15

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the numbers be a - d, a, a + d

According to first condition

$a + d + a + a - d = 15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Hence numbers are 5 - d, 5, 5 + d

When 1, 4, 19 be added to them respectively then the numbers become –

5 – d + 1, 5 + 4, 5 + d + 19

$\Rightarrow$ 6 – d, 9, 24 + d

The above numbers are in GP

Therefore, $9^2 = (6 - d)(24 + d)$

$\Rightarrow 81 = 144 - 24d + 6d - d^2$

$\Rightarrow 81 = 144 - 18d - d^2$

$\Rightarrow d^2 + 18d - 63 = 0$

⇒ $d^2 + 21d – 3d – 63 = 0$

⇒ d (d + 21) -3 (d + 21) = 0

⇒ (d – 3) (d + 21) = 0

⇒ d = 3, Or d = -21

Taking d = 3, the numbers are

5 - d, 5, 5 + d = 5 - 3, 5, 5 + 3

= 2, 5, 8

Taking d = -21, the numbers are

5 - d, 5, 5 + d = 5 – (-21), 5, 5 + (-21)

= 26, 5, -16

Ans) We have two sets of triplet as 2, 5, 8 and 26, 5, -16.

Q. 6. Three numbers are in AP, and their sum is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain three numbers in GP. Find the numbers.

Answer : To find: Three numbers

Given: Three numbers are in A.P. Their sum is 21

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the numbers be a - d, a, a + d

According to first condition

a + d + a +a – d = 21

⇒ 3a = 21

⇒ a = 7

Hence numbers are 7 - d, 7, 7 + d

When second number is reduced by 1 and third is increased by 1 then the numbers become –

$7 – d , 7 – 1 , 7 + d + 1$

$\Rightarrow 7 – d , 6 , 8 + d$

The above numbers are in GP

Therefore, $6^2 = (7 – d) (8 + d)$

$\Rightarrow 36 = 56 + 7d – 8d – d^2$

$\Rightarrow d^2 + d – 20 = 0$

$\Rightarrow d^2 + 5d – 4d – 20 = 0$

$\Rightarrow d (d + 5) – 4 (d + 5) = 0$

$\Rightarrow (d – 4) (d + 5) = 0$

$\Rightarrow d = 4$, Or $d = -5$

Taking $d = 4$, the numbers are

$7 - d, 7, 7 + d = 7 - 4, 7, 7 + 4$

$= 3, 7, 11$

Taking $d = -5$, the numbers are

$7 - d, 7, 7 + d = 7 – (-5), 7, 7 + (-5)$

$= 12, 7, 2$

Ans) We have two sets of triplet as 3, 7, 11 and 12, 7, 2.

Q. 7. The sum of three numbers in GP is 56. If 1, 7, 21 be subtracted from them respectively, we obtain the numbers in AP. Find the numbers

Answer : To find: Three numbers

Given: Three numbers are in G.P. Their sum is 56

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the three numbers in GP be a, ar, ar^2

According to condition :-

$a + ar + ar^2 = 56$

$a(1 + r + r^2) = 56$ … (i)

1, 7, 21 be subtracted from them respectively, we obtain the numbers as :-

$a – 1, ar – 7, ar^2 – 21$

According to question the above numbers are in AP

$\Rightarrow ar – 7 – (a – 1) = ar^2 – 21 – (ar – 7)$

$\Rightarrow ar – 7 – a + 1 = ar^2 – 21 – ar + 7$

$\Rightarrow ar – a – 6 = ar^2 – ar – 14$

$\Rightarrow 8 = ar^2 – 2ar + a$

$\Rightarrow 8 = a(r^2 – 2r + 1)$

Multiplying the above eqn. with 7

$\Rightarrow 56 = 7a(r^2 – 2r + 1)$

$\Rightarrow a(1 + r + r^2) = 7a(r^2 – 2r + 1)$

$\Rightarrow 1 + r + r^2 = 7r^2 – 14r + 7$

$\Rightarrow 6r^2 – 15r + 6 = 0$

$\Rightarrow 6r^2 – 12r – 3r + 6 = 0$

$\Rightarrow 6r(r – 2) -3 (r – 2) = 0$

$\Rightarrow (6r – 3) (r – 2) = 0$

$\Rightarrow r = \frac{3}{6} = \frac{1}{2}$ Or $r = 2$

Putting $r = \frac{1}{2}$ in eqn. (i)

$a(1 + r + r^2) = 56$

$$a\left(1+\frac{1}{2}+\ \frac{1}{2^2}\right)=56$$

$$a\left(\frac{4+2+1}{4}\right)=56$$

$$a\left(\frac{7}{4}\right)=56$$

a = 32

The numbers are a, ar, ar^2

$$\Rightarrow 32,\ 32\times\frac{1}{2}\ ,\ 32\times\frac{1}{2^2}$$

⇒ 32, 16, 8

Putting r = 2 in eqn. (i)

$a(1 + r + r^2) = 56$

$$a\left(1+2\ +\ 2^2\right)=56$$

$$a(1+2+4)=56$$

$$a(7)=56$$

a = 8

The numbers are a, ar, ar^2

$$\Rightarrow 32,\ 32\times\frac{1}{2}\ ,\ 32\times\frac{1}{2^2}$$

⇒ 8, 16, 32

Ans) We have two sets of triplet as 32, 16, 8 and 8, 16, 32.

Q. 8. If a, b, c are in GP, prove that $\frac{a^2+ab+b^2}{ab+bc+ca}=\ \frac{b+a}{c+b}$.

Answer : To prove: $\frac{a^2+ab+b^2}{ab+bc+ca} = \frac{b+a}{c+b}$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

a, b, c are in GP,

$\Rightarrow b^2 = ac$ … (i)

$\Rightarrow b = \sqrt{ac}$ … (ii)

Taking LHS = $\frac{a^2+ab+b^2}{ab+bc+ca}$

Substituting the value $b^2 = ac$ from eqn. (i)

LHS = $\frac{a^2+ab+ac}{ab+bc+b^2}$

$\Rightarrow \frac{a(a+b+c)}{b(a+b+c)}$

$\Rightarrow \frac{a}{b}$

Substituting the value b = $\sqrt{ac}$ from eqn. (ii)

$\Rightarrow \frac{a}{\sqrt{ac}}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{c}}$

Multiplying and dividing with $(\sqrt{a}+\sqrt{c})$

$\Rightarrow \frac{\sqrt{a}(\sqrt{a}+\sqrt{c})}{\sqrt{c}(\sqrt{a}+\sqrt{c})}$

$\Rightarrow \frac{(a+\sqrt{ac})}{(\sqrt{ac}+c)}$

$\Rightarrow \frac{a+b}{b+c}$ = RHS

Hence Proved

Q. 9. If (a – b), (b – c), (c – a) are in GP then prove that $(a + b + c)^2 = 3(ab + bc + ca)$.

Answer : To prove: $(a + b + c)^2 = 3(ab + bc + ca)$.

Given: (a – b), (b – c), (c – a) are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

As, (a – b), (b – c), (c – a) are in GP

$\Rightarrow (b - c)^2 = (a - b)(c - a)$

$\Rightarrow b^2 - 2cb + c^2 = ac - a^2 - bc + ab$

$\Rightarrow a^2 + b^2 + c^2 - bc - ac - ab = 0$

Adding 3(ab + bc + ac) both side

$\Rightarrow a^2 + b^2 + c^2 - bc - ac - ab + 3(ab + bc + ac) = 3(ab + bc + ac)$

$\Rightarrow a^2 + b^2 + c^2 + 2bc + 2ac + 2ab = 3(ab + bc + ac)$

$\Rightarrow (a + b + c)^2 = 3(ab + bc + ac)$

Hence Proved

Q. 10. If a, b, c are in GP, prove that

(i) $a(b^2 + c^2) = c(a^2 + b^2)$

(ii) $\frac{1}{(a^2 - b^2)} + \frac{1}{b^2} = \frac{1}{(b^2 - c^2)}$

(iii) $(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

(iv) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

Answer : (i) $a(b^2 + c^2) = c(a^2 + b^2)$

To prove: $a(b^2 + c^2) = c(a^2 + b^2)$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

When a,b,c are in GP, $b^2 = ac$

Taking LHS = $a(b^2 + c^2)$

= $a(ac + c^2)$ [$b^2 = ac$]

= $(a^2c + ac^2)$

= $c(a^2 + ac)$

= $c(a^2 + b^2)$ [$b^2 = ac$]

= RHS

Hence Proved

(ii) $\frac{1}{\left(a^2-b^2\right)}+\frac{1}{b^2}=\frac{1}{\left(b^2-c^2\right)}$

To prove: $a(b^2 + c^2) = c(a^2 + b^2)$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP, $b^2 = ac$

Taking LHS = $\frac{1}{\left(a^2-b^2\right)}+\frac{1}{b^2}$

$$\Rightarrow \frac{b^2+a^2-b^2}{(a^2-b^2)(b^2)}$$

$$\Rightarrow \frac{a^2}{(a^2-b^2)(ac)}$$

$$\Rightarrow \frac{a^2}{(a^3c-a^2c^2)}$$

$$\Rightarrow \frac{a^2}{a^2(ac-c^2)}$$

$\Rightarrow \frac{1}{(b^2-c^2)}$ $[b^2 = ac]$

Hence Proved

(iii) $(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

To prove: $(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP, $b^2 = ac$

Taking LHS = $(a + 2b + 2c)(a - 2b + 2c)$

$\Rightarrow [(a + 2c) + 2b]\,[(a + 2c) - 2b]$

$\Rightarrow [(a + 2c)^2 - (2b)^2]\,[(a + b)(a - b) = a^2 - b^2]$

$\Rightarrow [(a^2 + 4ac + 4c^2) - 4b^2]$

$\Rightarrow [(a^2 + 4ac + 4c^2) - 4b^2]\,[\,b^2 = ac]$

$\Rightarrow [(a^2 + 4ac + 4c^2 - 4ac]$

$\Rightarrow a^2 + 4c^2 = \text{RHS}$

Hence Proved

(iv) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

To prove: $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP, $b^2 = ac$

Taking LHS = $a^2b^2c^2\left(\frac{b^3c^3+a^3c^3+a^3b^3}{a^3b^3c^3}\right)$

$\Rightarrow\left(\frac{b^3c^3+a^3c^3+a^3b^3}{abc}\right)$

$\Rightarrow\left(\frac{b^2bc^3+(ac)^2ac+a^3b^2b}{abc}\right)$

$\Rightarrow\left(\frac{acbc^3+(b^2)^2ac+a^3acb}{abc}\right)$ [$b^2 = ac$]

$\Rightarrow\left(\frac{acbc^3+b^3abc+a^3acb}{abc}\right)$

$\Rightarrow\left(a^3+b^3+c^3\right)$ = RHS

Hence Proved

Q. 11. If a, b, c, d are in GP, prove that

(i) (b + c)(b + d) = (c + a)(c + a)

(ii) $\frac{ab-cd}{b^2-c^2}=\frac{a+c}{b}$

(iii) $(a+b+c+d)^2=(a+b)^2+2(b+c)^2+(c+d)^2$

Answer : (i) (b + c)(b + d) = (c + a)(c + a)

To prove: (b + c)(b + d) = (c + a)(c + a)

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

⇒ bc = ad … (i)

⇒ b^2 = ac … (ii)

⇒ c^2 = bd … (iii)

Taking LHS = (b + c)(b + d)

= b^2 + bd + bc + cd

Using eqn. (i) , (ii) and (iii)

= ac + c^2 + ad + cd

= c(a + c) + d(a + c)

= (a + c) (c + d)

Hence Proved

(ii) $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

To prove: $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

$\Rightarrow$ bc = ad … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$

$\Rightarrow d = \frac{c^2}{b}$ … (iii)

Taking LHS = $\frac{ab-cd}{b^2-c^2}$

$= \frac{ab-c\frac{c^2}{b}}{b^2-c^2}$ [From eqn. (iii)]

$= \frac{ab-\frac{c^3}{b}}{b^2-c^2}$

$= \frac{\frac{ab^2-c^3}{b}}{b^2-c^2}$

$= \frac{ab^2-c^3}{b(b^2-c^2)}$

$= \frac{a^2c-c^3}{bac-bc^2}$ [From eqn. (ii)]

$= \frac{c(a^2-c^2)}{b(ac-c^2)}$

$= \frac{c(a-c)(a+c)}{b(ac-c^2)}$

$= \frac{(ac-c^2)(a+c)}{b(ac-c^2)}$

$= \frac{(a+c)}{b}$

= RHS

Hence Proved

(iii) $(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$

To prove: $(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Taking LHS = $(a + b + c + d)^2$

$\Rightarrow (a + b + c + d)(a + b + c + d)$

$\Rightarrow a^2 + ab + ac + ad + ba + b^2 + bc + bd + ca + cb + c^2 + cd + da + db + dc + d^2$

On rearranging

$\Rightarrow a^2 + ab + ba + b^2 + ac + ad + bc + bd + ca + cb + c^2 + cd + da + db + dc + d^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ad + bc + bd + ca + cb + da + db + c^2 + cd + dc + d^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ad + bc + bd + ca + cb + da + db + (c + d)^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ca + ad + bc + cb + da + bd + db + (c + d)^2$

Using eqn. (i)

$\Rightarrow (a + b)^2 + ac + ca + bc + bc + bc + bc + bd + db + (c + d)^2$

Using eqn. (ii)

$\Rightarrow (a + b)^2 + b^2 + b^2 + bc + bc + bc + bc + bd + db + (c + d)^2$

Using eqn. (iii)

$\Rightarrow (a + b)^2 + 2b^2 + 4bc + c^2 + c^2 + (c + d)^2$

On rearranging

$\Rightarrow (a + b)^2 + 2b^2 + 4bc + 2c^2 + (c + d)^2$

$\Rightarrow (a + b)^2 + 2[b^2 + 2bc + c^2] + (c + d)^2$

$\Rightarrow (a + b)^2 + 2(b + c)^2 + (c + d)^2$

= RHS

Hence proved

Q. 12. If a, b, c are in GP, prove that $\frac{1}{(a+b)}, \frac{1}{(2b)}, \frac{1}{(b+c)}$ are in AP.

Answer : To prove: $\frac{1}{(a+b)}, \frac{1}{(2b)}, \frac{1}{(b+c)}$ are in AP

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

When a,b,c are in GP, $b^2 = ac$

Taking $\frac{1}{(a+b)}$ and $\frac{1}{(b+c)}$

$$\frac{1}{(a+b)} + \frac{1}{(b+c)}$$

$\Rightarrow \frac{b+c+a+b}{(a+b)(b+c)}$

$\Rightarrow \frac{a+c+2b}{ab+ac+b^2+bc}$

$\Rightarrow \frac{a+c+2b}{ab+b^2+b^2+bc}$ [b^2 = ac]

$\Rightarrow \frac{a+c+2b}{ab+2b^2+bc}$

$\Rightarrow \frac{a+c+2b}{b(a+c+2b)}$

$\Rightarrow \frac{1}{b}$

$\Rightarrow 2\times \frac{1}{2b}$

We can see that $\frac{1}{(a+b)} + \frac{1}{(b+c)} = 2\times \frac{1}{2b}$

Hence we can say that $\frac{1}{(a+b)}, \frac{1}{(2b)}, \frac{1}{(b+c)}$ are in AP.

Q. 13. If a, b, c are in GP, prove that a^2, b^2, c^2 are in GP.

Answer : To prove: a^2, b^2, c^2 are in GP

Given: a, b, c are in GP

Proof: As a, b, c are in GP

$\Rightarrow b^2 = ac$ … (i)

Considering b^2, c^2

$\frac{c^2}{b^2}$ = common ratio = r

$\Rightarrow \frac{c^2}{ac}$ [From eqn. (i)]

$\Rightarrow \frac{c}{a} = r$

Considering a^2, b^2

$\frac{b^2}{a^2}$ = common ratio = r

$\Rightarrow \frac{ac}{a^2}$ [From eqn. (i)]

$\Rightarrow \frac{c}{a} = r$

We can see that in both the cases we have obtained a common ratio.

Hence a^2, b^2, c^2 are in GP.

Q. 14. If a, b, c are in GP, prove that a^3, b^3, c^3 are in GP

Answer : To prove: a^3, b^3, c^3 are in GP

Given: a, b, c are in GP

Proof: As a, b, c are in GP

$\Rightarrow b^2 = ac$

Cubing both sides

$$\Rightarrow \left(b^2\right)^3 = (ac)^3$$

$$\Rightarrow b^6 = a^3c^3$$

$$\Rightarrow \frac{b^3}{a^3} = \frac{c^3}{b^3} = \text{common ratio} = r$$

From the above equation, we can say that a^3, b^3, c^3 are in GP

Q. 15. If a, b, c are in GP, prove that $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$ are in GP.

Answer : To prove: $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$ are in GP

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP,

$b^2 = ac$ … (i)

Considering $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$

$(ab + bc)^2 = (a^2b^2 + 2ab^2c + b^2c^2)$

$= (a^2b^2 + ab^2c + ab^2c + b^2c^2)$

$= (a^2b^2 + b^4 + a^2c^2 + b^2c^2)$ [From eqn. (i)]

$= [b^2 (a^2 + b^2) + c^2 (a^2 + b^2)]$

$(ab + bc)^2 = [(b^2 + c^2) (a^2 + b^2)]$

From the above equation we can say that $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$ are in GP

Q. 16. If a, b, c, d are in GP, prove that $(a^2 – b^2)$, $(b^2 – c^2)$, $(c^2 – d^2)$ are in GP.

Answer : To prove: $(a^2 – b^2)$, $(b^2 – c^2)$, $(c^2 – d^2)$ are in GP.

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Considering $(a^2 – b^2)$, $(b^2 – c^2)$, $(c^2 – d^2)$

$(a^2 – b^2) (c^2 – d^2) = a^2c^2 – a^2d^2 – b^2c^2 + b^2d^2$

$= (ac)^2 – (ad)^2 – (bc)^2 + (bd)^2$

From eqn. (i) , (ii) and (iii)

$= (b^2)^2 - (bc)^2 - (bc)^2 + (c^2)^2$

$= b^4 - 2b^2c^2 + c^4$

$(a^2 - b^2)(c^2 - d^2) = (b^2 - c^2)^2$

From the above equation we can say that $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in GP

Q. 17. If a, b, c, d are in GP, then prove that

$\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ **are in GP**

Answer : To prove: $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ are in GP.

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Considering $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$

$\frac{1}{(a^2+b^2)} \times \frac{1}{(c^2+d^2)} = \frac{1}{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

$= \frac{1}{(ac)^2+(ad)^2+(bc)^2+(bd)^2}$

From eqn. (i) , (ii) and (iii)

$= \frac{1}{\left(b^2\right)^2 + (bc)^2 + (bc)^2 + \left(c^2\right)^2}$

$= \frac{1}{b^4 + 2b^2c^2 + c^4}$

$\frac{1}{\left(a^2+b^2\right)} \times \frac{1}{\left(c^2+d^2\right)} = \frac{1}{\left(b^2+c^2\right)^2}$

From the above equation, we can say that $\frac{1}{\left(a^2+b^2\right)}, \frac{1}{\left(b^2+c^2\right)}, \frac{1}{\left(c^2+d^2\right)}$ are in GP.

Q. 18. If $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP then prove that p, q, r are in GP

Answer : To prove: p, q, r are in GP

Given: $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP,

$(pq + qr)^2 = (p^2 + q^2)(q^2 + r^2)$

$p^2q^2 + 2pq^2r + q^2r^2 = p^2q^2 + p^2r^2 + q^4 + q^2r^2$

$2pq^2r = p^2r^2 + q^4$

$pq^2r + pq^2r = p^2r^2 + q^4$

$pq^2r - q^4 = p^2r^2 - pq^2r$

$q^2(pr – q^2) = pr\,(pr – q^2)$

$q^2 = pr$

From the above equation we can say that p, q and r are in G.P.

Q. 19. If a, b, c are in AP, and a, b, d are in GP, show that a, (a – b) and (d – c) are in GP.

Answer : To prove: a, (a – b) and (d – c) are in GP.

Given: a, b, c are in AP, and a, b, d are in GP

Proof: As a,b,d are in GP then

$b^2 = ad$ … (i)

As a, b, c are in AP

$2b = (a + c)$ … (ii)

Considering a, (a – b) and (d – c)

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 - (2b)a + b^2$

From eqn. (i) and (ii)

$= a^2 - (a+c)a + ad$

$= a^2 - a^2 - ac + ad$

$= ad - ac$

$(a - b)^2 = a(d - c)$

From the above equation we can say that a, (a – b) and (d – c) are in GP.

Q. 20. If a, b, c are in AP, and a, x, b and b, y, c are in GP then show that x^2, b^2, y^2 are in AP.

Answer : To prove: x^2, b^2, y^2 are in AP.

Given: a, b, c are in AP, and a, x, b and b, y, c are in GP

Proof: As, a,b,c are in AP

$\Rightarrow 2b = a + c$ … (i)

As, a,x,b are in GP

$\Rightarrow x^2 = ab$ … (ii)

As, b,y,c are in GP

$\Rightarrow y^2 = bc$ … (iii)

Considering x^2, b^2, y^2

$x^2 + y^2 = ab + bc$ [From eqn. (ii) and (iii)]

$= b(a + c)$

$= b(2b)$ [From eqn. (i)]

$x^2 + y^2 = 2b^2$

From the above equation we can say that x^2, b^2, y^2 are in AP.

Exercise 12F

Q. 1. Find two positive numbers a and b, whose
(i) AM = 25 and GM = 20
(ii) AM = 10 and GM = 8

Answer : (i) AM = 25 and GM = 20

To find: Two positive numbers a and b

Given: AM = 25 and GM = 20

Formula used: (i) Arithmetic mean between $a \text{ and } b = \frac{a+b}{2}$

(ii) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Arithmetic mean of two numbers $= \frac{a+b}{2}$

$$\frac{a+b}{2} = 25$$

$\Rightarrow a + b = 50$

$\Rightarrow b = 50 - a$ … (i)

Geometric mean of two numbers $= \sqrt{ab}$

$\Rightarrow \sqrt{ab} = 20$

$\Rightarrow ab = 400$

Substituting value of b from eqn. (i)

a(50 – a) = 400

⇒ $50a - a^2 = 400$

On rearranging

⇒ $a^2 - 50a + 400 = 0$

⇒ $a^2 - 40a - 10a + 400$

⇒ a(a – 40) – 10(a – 40) = 0

⇒ (a – 10) (a – 40) = 0

⇒ a = 10, 40

Substituting, a = 10 Or a = 40 in eqn. (i)

b = 40 Or b = 10

Therefore two numbers are 10 and 40

(ii) AM = 10 and GM = 8

To find: Two positive numbers a and b

Given: AM = 10 and GM = 8

Formula used: (i) Arithmetic mean between $a \text{ and } b = \frac{a+b}{2}$

(ii) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Arithmetic mean of two numbers $= \frac{a+b}{2}$

$$\frac{a+b}{2} = 10$$

⇒ a + b = 20

⇒ a = 20 – b … (i)

Geometric mean of two numbers $=\sqrt{ab}$

$\Rightarrow \sqrt{ab}=8$

$\Rightarrow ab=64$

Substituting value of a from eqn. (i)

b(20 – b) = 64

$\Rightarrow$ 20b – b^2 = 64

On rearranging

$\Rightarrow$ b^2 – 20b + 64 = 0

$\Rightarrow$ b^2 – 16b – 4b + 64

$\Rightarrow$ b(b – 16) – 4(b – 16) = 0

$\Rightarrow$ (b – 16) (b – 4) = 0

$\Rightarrow$ b = 16, 4

Substituting, b = 16 Or b = 4 in eqn. (i)

a = 4 Or b = 16

Therefore two numbers are 16 and 4

Q. 2. Find the GM between the numbers

(i) 5 and 125
(ii) 1 and $\frac{9}{16}$
(iii) 0.15 and 0.0015
(iv) -8 and -2
(v) -6.3 and -2.8
(vi) ad ab^3

Answer : (i) 5 and 125

To find: Geometric Mean

Given: The numbers are 5 and 125

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$$= \sqrt{5 \times 25}$$

$$= \sqrt{625}$$

= 25

The geometric mean between 5 and 125 is 25

(ii) 1 and $\frac{9}{16}$

To find: Geometric Mean

Given: The numbers are 1 and $\frac{9}{16}$

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$$= \sqrt{1 \times \frac{9}{16}}$$

$$= \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

The geometric mean between 1 and $\frac{9}{16}$ is $\frac{3}{4}$.

(iii) 0.15 and 0.0015

To find: Geometric Mean

Given: The numbers are 0.15 and 0.0015

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{0.15 \times 0.0015}$

$= \sqrt{0.000225}$

= 0.015

The geometric mean between 0.15 and 0.0015 is 0.015.

(iv) -8 and -2

To find: Geometric Mean

Given: The numbers are -8 and -2

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{-8 \times -2}$

$= \sqrt{16}$

= ± 4

Mean is a number which has to fall between two numbers.

Therefore we will take -4 as our answer as +4 doesn’t lie between -8 and -2

The geometric mean between -8 and -2 is -4.

(v) -6.3 and -2.8

To find: Geometric Mean

Given: The numbers are -6.3 and -2.8

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{-6.3 \times -2.8}$

$= \sqrt{17.64}$

$= \pm 4.2$

Mean is a number which has to fall between two numbers.

Therefore we will take -4.2 as our answer as +4.2 doesn't lie between -6.3 and -2.8

The geometric mean between -6.3 and -2.8 is -4.2.

(vi) a^3b and ab^3

To find: Geometric Mean

Given: The numbers are a^3b and ab^3

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{a^3b \times ab^3}$

$= \sqrt{a^4b^4}$

$= a^2b^2$

The geometric mean between a^3b and ab^3 is a^2b^2.

Q. 13. Insert two geometric means between 9 and 243.

Answer : To find: Two geometric Mean

Given: The numbers are 9 and 243

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of

geometric mean

Let G_1 and G_2 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{243}{9}\right)^{\frac{1}{2+1}}$

⇒ r $= 27^{\frac{1}{3}}$

⇒ r = 3

G_1 = ar = 9×3 = 27

G_2 = ar^2= $9×3^2$ = 9×9 = 81

Two geometric mean between 9 and 243 are 27 and 81.

Q. 4. Insert three geometric means between $\frac{1}{3}$ and 432.

Answer : To find: Three geometric Mean

Given: The numbers $\frac{1}{3}$ and 432

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of

geometric mean

Let G_1, G_2 and G_3 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{3+1}}$

⇒ r $= \left(\frac{432}{\left(\frac{1}{3}\right)}\right)^{\frac{1}{2+1}}$

⇒ r $= \left(\frac{432\times3}{1}\right)^{\frac{1}{3+1}}$

⇒ r $= (1296)^{\frac{1}{4}}$

⇒ r = 6

$G_1 = ar = \left(\frac{1}{3}\right)\times6 = 2$

$G_2 = ar^2 = \left(\frac{1}{3}\right)\times6^2 = \left(\frac{1}{3}\right)\times36 = 12$

$G_3 = ar^3 = \left(\frac{1}{3}\right)\times6^3 = \left(\frac{1}{3}\right)\times216 = 72$

Three geometric mean between $\frac{1}{3}$ and 432 are 2, 12 and 72.

Q. 5. Insert four geometric means between 6 and 192.

Answer : To find: Four geometric Mean

Given: The numbers 6 and 192

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of geometric mean

Let G_1, G_2, G_3 and G_4 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{4+1}}$

⇒ r $= \left(\frac{192}{6}\right)^{\frac{1}{4+1}}$

⇒ r $= (32)^{\frac{1}{5}}$

⇒ r = 2

G_1 = ar = 6×2 = 12

$G_2 = ar^2 = 6\times2^2 = 24$

$G_3 = ar^3 = 6\times2^3 = 48$

$G_4 = ar^4 = 6\times2^4 = 96$

Four geometric mean between 6 and 192 are 12, 24, 48 and 96.

Q. 6. The AM between two positive numbers a and b(a>b) is twice their GM. Prove that a:b $= \left(2+\sqrt{3}\right):\left(2-\sqrt{3}\right)$**.**

Answer : To prove: Prove that a:b $= \left(2+\sqrt{3}\right):\left(2-\sqrt{3}\right)$

Given: Arithmetic mean is twice of Geometric mean.

Formula used: (i) Arithmetic mean between a and b $= \frac{a+b}{2}$

(ii) Geometric mean between a and b $= \sqrt{ab}$

AM = 2(GM)

$$\frac{a+b}{2} = 2\left(\sqrt{ab}\right)$$

$\Rightarrow$ a + b = $4\left(\sqrt{ab}\right)$

Squaring both side

$\Rightarrow (a + b)^2 = 16ab$ … (i)

We know that $(a - b)^2 = (a + b)^2 - 4ab$

From eqn. (i)

$\Rightarrow (a - b)^2 = 16ab - 4ab$

$\Rightarrow (a - b)^2 = 12ab$ … (ii)

Dividing eqn. (i) and (ii)

$$\frac{(a+b)^2}{(a-b)^2} = \frac{16ab}{12ab}$$

$$\Rightarrow \left(\frac{a+b}{a-b}\right)^2 = \frac{16}{12}$$

Taking square root both side

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{2\sqrt{3}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$$

Applying componendo and dividend

$$\Rightarrow \frac{a+b+a-b}{a+b-a+b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{2a}{2b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Hence Proved

Q. 7. If a, b, c are in AP, x is the GM between a and b; y is the GM between b and c; then show that b^2 is the AM between x^2 and y^2.

Answer : To prove: b^2 is the AM between x^2 and y^2.

Given: (i) a, b, c are in AP

(ii) x is the GM between a and b

(iii) y is the GM between b and c

Formula used: (i) Arithmetic mean between $\text{a and b} = \frac{a+b}{2}$

(ii) Geometric mean between $\text{a and b} = \sqrt{ab}$

As a, b, c are in A.P.

⇒ 2b = a + c … (i)

As x is the GM between a and b

⇒ x = $\left(\sqrt{ab}\right)$

⇒ x^2 = ab … (ii)

As y is the GM between b and c

⇒ y = $\left(\sqrt{bc}\right)$

⇒ y^2 = bc … (iii)

Arithmetic mean of x^2 and y^2 is $\left(\frac{x^2+y^2}{2}\right)$

Substituting the value from (ii) and (iii)

$$\left(\frac{x^2+y^2}{2}\right) = \left(\frac{ab+bc}{2}\right)$$

$= \frac{b(a+c)}{2}$

Substituting the value from eqn. (i)

$= \frac{b(2b)}{2}$

$= b^2$

Hence Proved

Q. 8. Show that the product of n geometric means between a and b is equal to the nth power of the single GM between a and b.

Answer : To prove: Product of n geometric means between a and b is equal to the nth power of the single GM between a and b.

Formula used:(i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

(ii) Sum of n terms of A.P. $= \frac{(n)(n+1)}{2}$

Let the n geometric means between and b be $G_1, G_2, G_3, \ldots G_n$

Hence a, $G_1, G_2, G_3, \ldots G_n$, b are in GP

$\Rightarrow G_1 = ar$, $G_2 = ar^2$ and so on …

Now, we have n+2 term

$\Rightarrow b = ar^{n+2-1}$

$\Rightarrow b = ar^{n+1}$

$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ … (i)

The product of n geometric means is $G_1 \times G_2 \times G_3 \times \ldots G_n$

$= ar \times ar^2 \times ar^3 \times \ldots ar^n$

$= a^n \times r^{(1+2+3\ldots+n)}$

$= a^n \times r^{(n)\left(\frac{n+1}{2}\right)}$ $\left[\text{Sum of n terms of A.P.} = \frac{(n)(n+1)}{2}\right]$

Substituting the value of r from eqn. (i)

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{1}{n+1}\right)(n)\left(\frac{n+1}{2}\right)}$

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{n}{2}\right)}$

$= a^n \times \frac{b^{\frac{n}{2}}}{a^{\frac{n}{2}}}$

$= a^{\frac{n}{2}} \times b^{\frac{n}{2}}$

$= (ab)^{\frac{n}{2}}$

$= \left(\sqrt{ab}\right)^n$ … (ii)

Single geometric mean between a and b $= \sqrt{ab}$

n^{th} power of single geometric mean between a and b $= \left(\sqrt{ab}\right)^n$

Hence Proved

Q. 9. If AM and GM of the roots of a quadratic equation are 10 and 8 respectively then obtain the quadratic equation.

Answer : To find: The quadratic equation.

Given: (i) AM of roots of quadratic equation is 10

(ii) GM of roots of quadratic equation is 8

Formula used: (i) Arithmetic mean between $\text{a and b} = \frac{a+b}{2}$

(ii) Geometric mean between $\text{a and b} = \sqrt{ab}$

Let the roots be p and q

Arithmetic mean of roots p and q $= \frac{p+q}{2}$ = 10

⇒ $\frac{p+q}{2}$ = 10

⇒ p + q = 20 = sum of roots … (i)

Geometric mean of roots p and q = $\sqrt{pq}$ = 8

⇒ pq = 64 = product of roots … (ii)

Quadratic equation = x^2 – (sum of roots)x + (product of roots)

From equation (i) and (ii)

Quadratic equation = x^2 – (20)x + (64)

= x^2 –20x + 64

x^2 –20x + 64

Exercise 12G

Q. 1. Find the sum of each of the following infinite series :

$8 + 4\sqrt{2} + 4 + 2\sqrt{2} + \ldots\ldots \infty$

Answer : It is Infinite Geometric Series.

Here, a=8,

$$r = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$$\therefore \text{Sum} = \frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1}$$

$$\text{Sum} = \frac{8\sqrt{2}}{\sqrt{2}-1}$$

Q. 2. Find the sum of each of the following infinite series :

6 + 1.2 + 0.24 + ….. ∞

Answer : It is Infinite Geometric Series.

Here, a=6,

$$r = \frac{1.2}{6} = \frac{2}{10} = 0.2$$

The formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$$\therefore \text{Sum} = \frac{6}{1-0.2} = \frac{6}{0.8} = \frac{15}{2}$$

$$\text{Sum} = \frac{15}{2}$$

Q. 3. Find the sum of each of the following infinite series :

$$\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \ldots.. \infty$$

Answer : It is Infinite Geometric Series

Here, a=√2

$$r = \frac{\frac{-1}{\sqrt{2}}}{\sqrt{2}} = \frac{-1}{2}$$

$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\sqrt{2}}{1-\frac{-1}{2}} = \frac{\sqrt{2}}{1+\frac{1}{2}} = \frac{2\sqrt{2}}{3}$

$\text{Sum} = \frac{2\sqrt{2}}{3}$

Q. 4. Find the sum of each of the following infinite series :

10 – 9 + 8.1 - ……∞

Answer : It is Infinite Geometric Series

Here, a=10

$r = \frac{-9}{10} = -0.9$

$\therefore \text{Sum} = \frac{a}{1-r} = \frac{10}{1-(-0.9)} = \frac{10}{1+0.9} = \frac{10}{1.9} = \frac{100}{19}$

$\text{Sum} = \frac{100}{19}$

Q. 5. Find the sum of each of the following infinite series :

$\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \ldots\ldots \infty$

Answer : This geometric series is the sum of two geometric series:

$\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty \;\&\; \frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$

Sum of geometric series: $\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty$

Here, $a = \frac{2}{5}$

$$r = \frac{\frac{2}{5^3}}{\frac{2}{5}} = \frac{1}{5^2} = \frac{1}{25}$$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{\frac{2}{5}}{\frac{25-1}{25}} = \frac{2 \times 25}{24 \times 5} = \frac{5}{12}$$

Sum of geometric series: $\frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$

Here, a= $\frac{3}{5^2}$

$$r = \frac{\frac{3}{5^4}}{\frac{3}{5^2}} = \frac{1}{5^2} = \frac{1}{25}$$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{3}{5^2}}{1-\frac{1}{25}} = \frac{\frac{3}{5^2}}{\frac{25-1}{25}} = \frac{3 \times 25}{25 \times 24} = \frac{1}{8}$$

∴Sum of the given infinite series=sum of both the series $= \frac{5}{12} + \frac{1}{8} = \frac{(5\times2)+(1\times3)}{24}$

$$= \frac{10+3}{24} = \frac{13}{24}$$

Sum $= \frac{13}{24}$

Q. 6. Prove that $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty = 3$

Answer : L.H.S=$9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty$

$=9^{(1/3)+(1/9)+(1/27)+\ldots\infty}$

The series in the exponent is an infinite geometric series

Whose, a = $\frac{1}{3}$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1 \times 3}{1 \times 9} = \frac{1}{3}$$

∴Sum of the series in the exponent $= \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1\times3}{3\times2} = \frac{1}{2}$

∴L.H.S=$9^{1/2}$

=3=R.H.S

Hence, Proved that $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty = 3$

Q. 7. Find the rational number whose decimal expansion is given below :

(i) $0.\overline{3}$ **(ii)** $0.\overline{231}$

(iii) $3.\overline{52}$

Answer : (i) Let, x=0.3333…

⇒ x=0.3+0.03+0.003+…

⇒ x=3(0.1+0.01+0.001+0.0001+…∞)

⇒ x=3($\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots \infty$)

This is an infinite geometric series.

Here, a=1/10 and r=1/10

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

∴x $= 3 \times \frac{1}{9} = \frac{1}{3}$

$0.\overline{3} = \frac{1}{-}$

(ii) Let, x=0.231231231….

⇒ x=0.231+0.000231+0.000000231+…∞

⇒ x=231(0.001+0.000001+0.000000001+…∞)

⇒ x=231($\frac{1}{10^3}$+$\frac{1}{10^6}+\frac{1}{10^9}+\frac{1}{10^{12}}$+…∞)

This is an infinite geometric series.

Here, a$=\frac{1}{10^3}$ and r=$\frac{1}{10^3}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1 \times 1000}{999 \times 1000} = \frac{1}{999}$$

⇒ $x = 231 \times \frac{1}{999} = \frac{231}{999}$

$0.\overline{231}=\frac{231}{999}$

(iii) Let, x=3.525252552…

⇒ x=3+0.52+0.0052+0.000052+…∞

⇒ x=3+52(0.01+0.0001+…∞)

⇒ x=3+52($\frac{1}{10^2}$+$\frac{1}{10^4}+\frac{1}{10^6}+\frac{1}{10^8}$+…∞)

Here, a$=\frac{1}{10^2}$ and r=$\frac{1}{10^2}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1 \times 100}{99 \times 100} = \frac{1}{99}$$

⇒ $x = 3 + \left(52 \times \frac{1}{99}\right) = \frac{297+52}{999} = \frac{349}{999}$

$3.\overline{52} = \frac{349}{999}$

Q. 8. Express the recurring decimal 0.125125125 …. = $0.\overline{125}$ as a rational number.

Answer : Let, x=0.125125125… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=125.125125125… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=125.125125125-0.125125125=125

⇒ 999x=125

⇒ $x = \frac{125}{999}$

$0.\overline{125} = \frac{125}{999}$

Q. 9. Write the value of $0.\overline{423}$ in the form of a simple fraction.

Answer : Let, x=0.423423423… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=423.423423423… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=423.423423423-0.423423423=423

⇒ 999x=423

⇒ $x = \frac{423}{999} = \frac{47}{111}$

$0.\overline{423} = \frac{47}{111}$

Q. 10. Write the value of $2.\overline{134}$ in the form of a simple fraction.

Answer : Let, x=2.134134134… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=2134.134134134… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=2134.134134134-2.134134134=2132

⇒ 999x=2132

⇒ $x = \frac{2132}{999}$

$2.\overline{134} = \frac{2132}{999}$

Q. 11. The sum of an infinite geometric series is 6. If its first term is 2, find its common ratio.

Answer :

Given: $\frac{a}{1-r} = 6$, a=2

To find: r=?

$\therefore \frac{2}{1-r} = 6$

$\Rightarrow 1 - r = \frac{2}{6} = \frac{1}{3}$

$\Rightarrow$ 3(1-r)=1

$\Rightarrow$ 3-3r=1

$\Rightarrow$ 3r=3-1

$\Rightarrow$ r= $\frac{2}{3}$

Common ratio r= $\frac{2}{3}$

Q. 12. The sum of an infinite geometric series is 20, and the sum of the squares of these terms is 100. Find the series.

Answer :

Given: $\frac{a}{1-r} = 20$ & $\frac{a^2}{1-r^2} = 100$

(Because on squaring both first term a and common ratio r will be squared.)

To find: the series

a=20(1-r)…(i)

$\Rightarrow \frac{a^2}{1-r^2} = 100 = \frac{(20\times(1-r))^2}{(1-r)(1+r)}$ ...(from (i))

$\Rightarrow 100 = 400 \times \frac{1-r}{1+r}$

⇒ 100(1+r)=400(1-r)

⇒ 100+100r=400-400r

⇒ 100r+400r=400-100

⇒ 500r=300

⇒ 5r=3

$\Rightarrow r = \frac{3}{5}$

Put this value of r in equation (i) we get

$$a = 20\left(1 - \frac{3}{5}\right) = \frac{20 \times 2}{5} = 8$$

∴The infinite geometric series is:8, $\frac{24}{5}, \frac{72}{25}, \frac{216}{125}, \frac{648}{625}, \ldots \infty$

Q. 13. The sum of an infinite GP is 57, and the sum of their cubes is 9747. Find the GP.

Answer : Let the first term Of G.P. be a, and common ratio be r.

$\therefore \frac{a}{1-r} = 57 \ldots (1)$

On cubing each term will become,

$a^{3,}$ a^3r^3, ….

∴This sum $= \frac{a^3}{1-r^3} = 9747 \ldots (2)$

a=57(1-r) put this in equation 2 we get

$$\frac{(57 \times (1-r))^3}{1-r^3} = 9747$$

$\Rightarrow \frac{57^3 \times (1-r)^3}{1-r^3} = 9747$

$\Rightarrow \frac{(1-r)\times(1-r)^2}{(1-r)(1+r+r^2)} = \frac{9747}{57\times57\times57} = \frac{1}{19}$

$\Rightarrow 19(1-2r+r^2)=1+r+r^2$

$\Rightarrow 19r^2-r^2-38r-r+19-1=0$

$\Rightarrow 18r^2-39r+18=0$

$\Rightarrow 6r^2-13r+6=0$

$\Rightarrow (2r-3)(3r-2)=0$

$\Rightarrow r= 2/3, 3/2$

But $-1<r<1$

$\Rightarrow r=2/3$

Substitute this value of r in equation 1 we get

$$a = 57 \times \left(1 - \frac{2}{3}\right) = 19$$

Thus the first term of G.P. is 19, and the common ratio is 2/3

$\therefore G.P=19, \frac{38}{3}, \frac{76}{9}, ...$

$19, \frac{38}{3}, \frac{76}{9}, ...$

Exercise 12H

Q. 1. If the 5th term of a GP is 2, find the product of its first nine terms.

Answer : Given: 5^{th} term of a GP is 2.

To find: the product of its first nine terms.

First term is denoted by a, the common ratio is denote by r.

$\therefore ar^4 = 2$

We have to find the value of: $a \times ar^1 \times ar^2 \times ar^3 \times \ldots \times ar^8$

$= a^9r^{1 + 2 + 3 + 4 + \ldots + 8}$

$= a^9r^{36}$

$= (ar^4)^9$

$= (2)^9$

$= 512$

Ans: 512.

Q. 2. If the (p + q)th and (p – q)th terms of a GP are m and n respectively, find its pth term.

Answer : Let,

$t_{p + q} = m = Ar^{p + q - 1} = Ar^{p - 1}r^q$

And

$t_{p - q} = n = Ar^{p - q - 1} = Ar^{p - 1}r^{- q}$

We know that p^{th} term $= Ar^{p - 1}$

$\therefore m \times n = A^2r^{2p - 2}$

$\Rightarrow Ar^{p - 1} = (mn)^{1/2}$

$\Rightarrow p^{th}$ term $= (mn)^{1/2}$

Ans: p^{th} term $= (mn)^{1/2}$

Q. 3. If 2^{nd}, 3^{rd} and 6^{th} terms of an AP are the three consecutive terms of a GP then find the common ratio of the GP.

Answer : We have been given that 2nd, 3rd and 6th terms of an AP are the three consecutive terms of a GP.

Let the three consecutive terms of the G.P. be a, ar, ar^2.

Where a is the first consecutive term and r is the common ratio.

2nd, 3rd terms of the A.P. are a and ar respectively as per the question.

∴ The common difference of the A.P. = $ar - a$

And the sixth term of the A.P. = ar^2

Since the second term is a and the sixth term is ar^2(In A.P.)

We use the formula:$t = a + (n - 1)d$

∴ $ar^2 = a + 4(ar - a)$…(the difference between 2nd and 6th term is $4(ar - a)$)

⇒ $ar^2 = a + 4ar - 4a$

⇒ $ar^2 + 3a - 4ar = 0$

⇒ $a(r^2 - 4r + 3) = 0$

⇒ $a(r - 1)(r - 3) = 0$

Here, we have 3 possible options:

1)$a = 0$ which is not expected because all the terms of A.P. and G.P. will be 0.

2)$r = 1$,which is also not expected because all th terms would be equal to first term.

3)$r = 3$,which is the required answer.

Ans: Common ratio = 3

Q. 4. Write the quadratic equation, the arithmetic and geometric means of whose roots are A and G respectively.

Answer : Let the roots of the required quadratic equation be a and b.

The arithmetic and geometric means of roots are A and G respectively.

⇒ $A = (a + b)/2$…(i)

And G = $\sqrt{ab}$ …(ii)

We know that the equation whose roots are given is =

$x^2 - (a + b)x + ab = 0$

From (i) and (ii) we get:

$x^2 - 2A + G^2 = 0$

Thus, $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

Ans: $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

Q. 5. If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$ then prove that x, y, z are in AP.

Answer : It is given that:

$a^{1/x} = b^{1/y} = c^{1/z}$

Let $a^{1/x} = b^{1/y} = c^{1/z} = k$

⇒ $a^{1/x} = k$

⇒ $(a^{1/x})^x = k^x$…(Taking power of x on both sides.)

⇒ $a^{1/x \times x} = k^x$

⇒ $a = k^x$

Similarly $b = k^y$

And $c = k^z$

It is given that a,b,c are in G.P.

⇒ $b^2 = ac$

Substituting values of a,b,c calculated above, we get:

⇒ $(k^y)^2 = k^x k^z$

⇒ $k^{2y} = k^{x + z}$

Comparing the powers we get,

2y = x + z

Which is the required condition for x,y,z to be in A.P.

Hence, proved that x,y,z, are in A.P.

Q. 6. If a, b, c are in AP and x, y, z are in GP then prove that the value of x^{b-c}. y^{c-a}. z^{a-b} is 1.

Answer : To prove: x^{b-c}. y^{c-a}. z^{a-b} = 1….(i)

It is given that a,b,c are in A.P.

⇒ 2b = a + c…(ii)

And x,y,z, are in G.P.

⇒ $y^2 = xz$

⇒ $x = y^2/z$

Substitute this value of x in equation (i),we get

L.H.S =

⇒ $\left(\frac{y^2}{z}\right)^{b-c} \times y^{c-a} \times z^{a-b}$

⇒ $y^{2(b-c)+c-a} . z^{a-b-(b-c)}$

⇒ $y^{2b-2c+c-a} . z^{a+c-b-b}$

⇒ $y^{2b-c-a} . z^{a+c-2b}$

⇒ $y^0 . z^0$…(Using equation (i))

= 1 = R.H.S

Hence, proved that . If a, b, c are in AP and x, y, z are in GP then x^{b-c}. y^{c-a}. z^{a-b}= 1

Q. 7. Prove that $\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}....\infty\right)=\frac{3}{4}$

Answer : It is Infinite Geometric Series.

Here, a = 1,

$$r = \frac{\frac{-1}{3}}{1} = \frac{-1}{3}$$

Formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$$\therefore \text{Sum} = \frac{1}{1-\frac{-1}{3}} = \frac{1\times 3}{3+1} = \frac{3}{4} = \text{R.H.S.}$$

$$\text{Hence, Proved that } \left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}\ldots\infty\right)=\frac{3}{4}$$

Q. 8. Express $0.\overline{123}$ as a rational number.

Answer : Let, x = 0.123123123….

⇒ x = 0.123 + 0.000123 + 0.000000123 + …∞

⇒ x = 123(0.001 + 0.000001 + 0.000000001 + …∞)

⇒ $x = 123(\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}} + \ldots\infty)$

This is an infinite geometric series.

Here, $a = \frac{1}{10^3}$ and $r = \frac{1}{10^3}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1\times 1000}{999\times 1000} = \frac{1}{999}$$

$$\Rightarrow x = 123 \times \frac{1}{999} = \frac{123}{999}$$

Ans : $0.\overline{123} = \frac{123}{999}$

Q. 9. Express $0.\overline{6}$ as a rational number.

Answer : Let ,x = 0.6666…

⇒ x = 0.6 + 0.06 + 0.006 + …

⇒ x = 6(0.1 + 0.01 + 0.001 + 0.0001 + …∞)

⇒ x = $6\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \ldots\infty\right)$

This is an infinite geometric series.

Here,a = 1/10 and r = 1/10

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

$$\therefore x = 6 \times \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

Ans: $0.\overline{6} = \frac{2}{-}$

Q. 10. Express $0.\overline{68}$ as a rational number.

Answer : Let, x = 0.68686868…

⇒ x = 0.68 + 0.0068 + 0.000068 + …∞

⇒ x = 68(0.01 + 0.0001 + …∞)

⇒ x = $68\left(\frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + \frac{1}{10^8} + \ldots\infty\right)$

Here, a $= \frac{1}{10^2}$ and r = $\frac{1}{10^2}$

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1 \times 100}{99 \times 100} = \frac{1}{99}$$

$\Rightarrow x = \left(68 \times \frac{1}{99}\right) = \frac{68}{999} = \frac{68}{999}$

Ans: $0.\overline{68} = \frac{68}{999}$

Q. 11. The second term of a GP is 24 and its fifth term is 81. Find the sum of its first five terms.

Answer : Given: second term of a GP is 24 and its fifth term is 81.

To find: sum of first five terms of the G.P.

$ar = 24$ & $ar^4 = 81$

dividing these two terms we get:

$\Rightarrow \frac{ar^4}{ar} = \frac{81}{24}$

$\Rightarrow r^3 = \frac{27}{8}$

Taking cube root on both the sides we get,

$\Rightarrow r = \frac{3}{2}$

Substituting this value of r in ar = 24 we get

a = 24/(3/2) = (24 × 2)/3 = 16

∴ Sum of first Five terms of a G.P. = $a(r^n - 1)/(r - 1)$

$= 16 \times \frac{(\frac{3}{2})^5 - 1}{\frac{3}{2} - 1} = 16 \times \frac{\frac{243}{32} - 1}{\frac{3}{2} - 1}$

$= 16 \times \frac{242 \times 2}{32 \times 1} = 242$

Ans: 242

Q. 12. The ratio of the sum of first three terms is to that of first six terms of a GP is 125 : 152. Find the common ratio.

Answer : The first three terms of a G.P. are:a,ar,ar^2

The first six terms of a G.P. are:a,ar,ar^2,ar^3,ar^4,ar^5

It is given that the ratio of the sum of first three terms is to that of first six terms of a GP is 125 : 152.

⇒ $a + ar + ar^2 = 125x$ & $a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 152x$

⇒ $a + ar + ar^2 + r^3(a + ar + ar^2) = 152x$

⇒ $125x + r^3(125x) = 152x$

⇒ $r^3(125x) = 152x - 125x = 27x$

⇒ $r^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3$

⇒ r = 3/5

Ans: common ratio = $\frac{3}{5}$

Q. 13. The sum of first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio and these three terms.

Answer : Let the first three terms of G.P. be $\frac{a}{r}, a, ar$

It is given that $\frac{a}{r} \times a \times ar = 1$

⇒ $a^3 = 1$

⇒ a = 1

And

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

⇒ $a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$

$\Rightarrow \left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$ …(a = 1)

$\Rightarrow \left(\frac{1}{r} + r\right) = \frac{39}{10} - 1 = \frac{29}{10}$

$\Rightarrow 10(1 + r^2) = 29r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (2r - 5)(5r - 2) = 0$

$\Rightarrow r = \frac{5}{2}, \frac{2}{5}$

Therefore the first three terms are:

i) If $r = \frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii) If $r = \frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$

Ans: Common ratio $r = \frac{5}{2}, \frac{2}{5}$ and the first three terms are:

i) if $r = \frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii) If $r = \frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$