RD SHARMA

Solutions

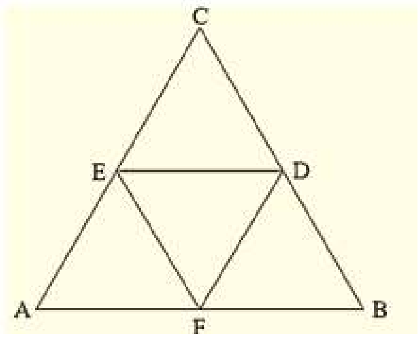
Class 9 Maths

Chapter 14

Ex *14.4*

Q1) In ΔABC, D, E and F are, respectively the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7cm, 8cm and 9cm, respectively, find the perimeter of ΔDEF.

Solution:



Given that,

AB = 7cm, BC = 8cm, AC = 9cm

In ΔABC,

F and E are the mid points of AB and AC.

$\therefore EF = \frac{1}{2}BC$

Similarly

$DF = \frac{1}{2}AC \text{ and } DE = \frac{1}{2}AB$

Perimeter of ΔDEF = DE + EF + DF

$= \frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$

$= \frac{1}{2} * 7 + \frac{1}{2} * 8 + \frac{1}{2} * 9$

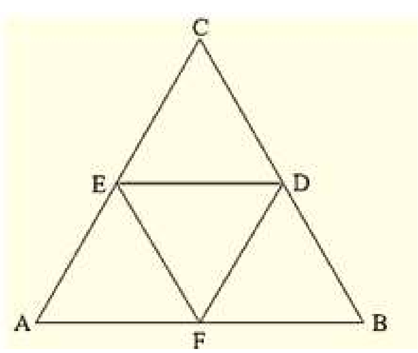
=3.5 + 4 + 4.5

=12cm

$\therefore \text{Perimeter of } \Delta DEF = 12\text{cm}$

Q2) In a ΔABC, $\angle A = 50^0$, $\angle B = 60^0$ and $\angle C = 70^0$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution:



In ΔABC,

D and E are mid points of AB and BC.

By Mid point theorem,

$$DE \parallel AC,\ DE = \frac{1}{2}AC$$

F is the midpoint of AC

Then, $DE = \frac{1}{2}AC = CF$

In a Quadrilateral DECF

$$DE \parallel AC,\ DE = CF$$

Hence DECF is a parallelogram.

$\therefore \angle C = \angle D = 70^0$ [Opposite sides of a parallelogram]

Similarly

BEFD is a parallelogram, $\angle B = \angle F = 60^0$

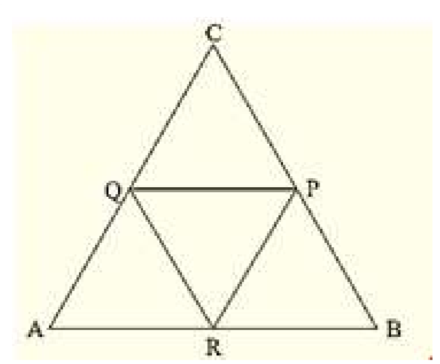
ADEF is a parallelogram, $\angle A = \angle E = 50^0$

$\therefore$ Angles of ΔDEF are

$$\angle D = 70^0,\ \angle E = 50^0,\ \angle F = 60^0$$

Q3) In a triangle, P, Q and R are the mid points of sides BC, CA and AB respectively. If AC = 21cm, BC = 29cm and AB = 30cm, find the perimeter of the quadrilateral ARPQ.

Solution:



In ΔABC,

R and P are mid points of AB and BC

$$RP \parallel AC,\ RP = \frac{1}{2}AC \quad \text{[By Midpoint Theorem]}$$

In a quadrilateral,

[A pair of side is parallel and equal]

$$RP \parallel AQ,\ RP = AQ$$

Therefore, RPQA is a parallelogram

$\Rightarrow AR = \frac{1}{2}AB = \frac{1}{2} * 30 = 15cm$

AR = QP = 15cm [Opposite sides are equal]

$\Rightarrow RP = \frac{1}{2}AC = \frac{1}{2} * 21 = 10.5cm$

RP = AQ = 10.5cm [Opposite sides are equal]

Now,

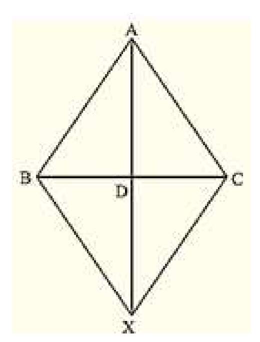
Perimeter of ARPQ = AR + QP + RP +AQ

= 15 +15 +10.5 +10.5

= 51cm

Q4) In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Solution:



In a quadrilateral ABXC, we have

AD = DX [Given]

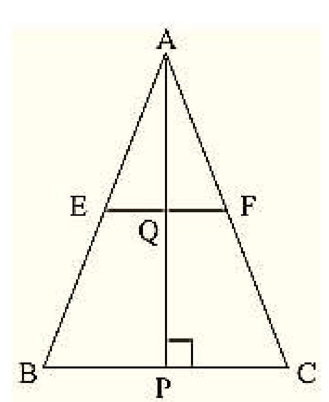
BD = DC [Given]

So, diagonals AX and BC bisect each other.

Therefore, ABXC is a parallelogram.

Q5) In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Solution:



In a ΔABC

E and F are mid points of AB and AC

$\therefore$ EF $\parallel$ FE, $\frac{1}{2}$BC = FE [By mid point theorem]

In ΔABP

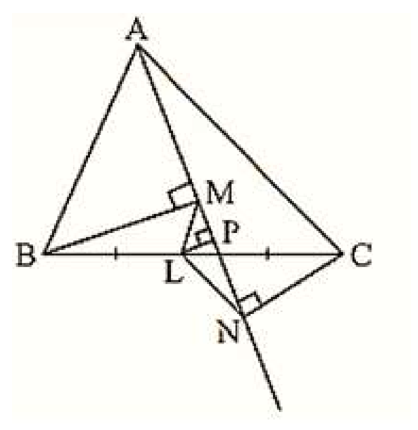
F is the mid-point of AB and $\therefore$ FQ $\parallel$ BP [$\because$ EF $\parallel$ BP]

Therefore, Q is the mid-point of AP [By mid-point theorem]

Hence, AQ = QP.

Q6) In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Solution:



Given that,

In ΔBLM and ΔCLN

$\angle BML = \angle CNL = 90^0$

BL = CL [L is the mid-point of BC]

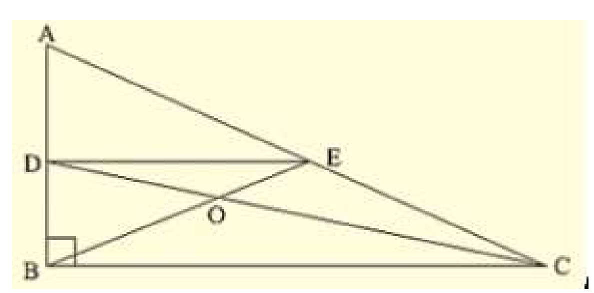
$\angle MLB = \angle NLC$ [Vertically opposite angle]

$\therefore \Delta BLM = \Delta CLN$

$\therefore$ LM = LN [corresponding parts of congruent triangles]

Q7)In figure 14.95, Triangle ABC is a right angled triangle at B. Given that AB = 9cm, AC = 15cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC ***(ii)*** The area of ΔADE.



Solution:

In ΔABC, $\angle B = 90^0$,

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$=>15^2 = 9^2 + BC^2$

$=>BC = \sqrt{15^2 - 9^2}$

$=>BC = \sqrt{225 - 81}$

$=> BC = \sqrt{144}$ = 12cm

In ΔABC,

D and E are mid-points of AB and AC

$\therefore DE \parallel BC, \ DE = \frac{1}{2}BC \quad [\text{By mid} - \text{point theorem}]$

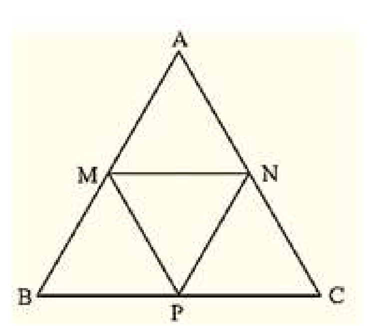
$AD = DB = \frac{AB}{2} = \frac{9}{2} = 4.5\text{cm} \quad [\because D \text{ is the mid} - \text{point of AB}]]$

$\text{Area of } \Delta ADE = \frac{1}{2} * AD * DE$

$= \frac{1}{2} * 4.5 * 6$

$=13.5cm^2$

Q8) In figure 14.96, M, N and P are mid-points of AB, AC and BC respectively. If MN = 3cm, NP = 3.5cm and MP = 2.5cm, calculate BC, AB and AC.



Solution:

Given MN = 3cm, NP = 3.5cm and MP = 2.5cm.

To find BC, AB and AC

In ΔABC

M and N are mid-points of AB and AC

$\therefore MN = \frac{1}{2}BC, \ MN \parallel BC \quad [\text{By mid} - \text{point theorem}]$

$\Rightarrow 3 = \frac{1}{2}BC$

$\Rightarrow 3 * 2 = BC$

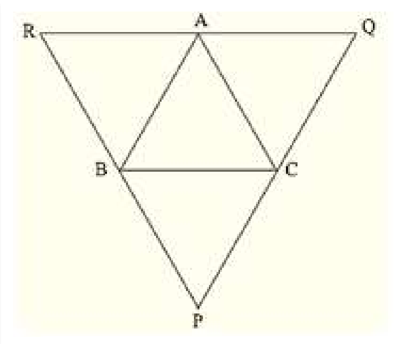
$\Rightarrow BC = 6cm$

Similarly

AC = 2MP = 2 (2.5) = 5cm

AB = 2 NP = 2 (3.5) = 7cm

Q9) ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.



Solution:

Clearly ABCQ and ARBC are parallelograms.

Therefore, BC = AQ and BC = AR

=>AQ = AR

=>A is the mid-point of QR

Similarly B and C are the mid points of PR and PQ respectively.

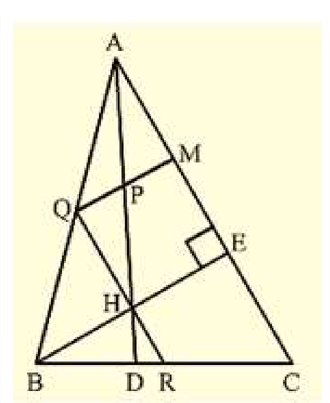
$\therefore AB = \frac{1}{2}PQ, \ BC = \frac{1}{2}QR, \ CA = \frac{1}{2}PR$

=>PQ = 2AB, QR = 2BC and PR = 2CA

=>PQ + QR + RP = 2 (AB + BC + CA)

=>Perimeter of ΔPQR = 2 (perimeter of ΔABC)

Q10) In figure 14.97, $BE \perp AC$, AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that $\angle PQR = 90^0$



Solution:

Given,

$BE \perp AC$ and P, Q and R are respectively mid-point of AH, AB and BC.

To prove: $\angle PQR = 90^0$

Proof: In ΔABC, Q and R are mid-points of AB and BC respectively.

$\therefore$ QR $\parallel$ AC ……(i)

In ΔABH, Q and P are the mid-points of AB and AH respectively

$\therefore$ QP $\parallel$ BH ….(ii)

But, $BE \perp AC$

Therefore, from equation (i) and equation (ii) we have,

$QP \perp QR$

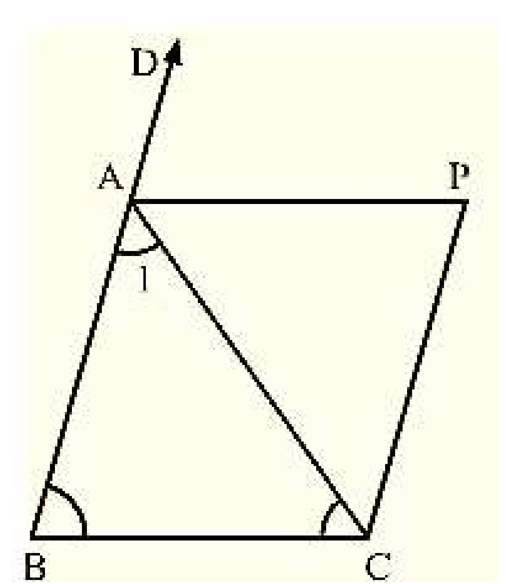
=>$\angle PQR = 90^0$

Hence Proved.

Q11) In figure 14.98, AB=AC and CP||BA and AP is the bisector of exterior $\angle CAD$ of ΔABC . Prove that

(i) $\angle PAC = \angle BCA$**.**

(ii) ABCP is a parallelogram.



Solution:

Given,

AB = AC and $CD \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of ΔABC

To prove:

(i) $\angle PAC = \angle BCA$

(ii) ABCP is a parallelogram.

Proof:

(i) We have,

AB=AC

=>$\angle ACB = \angle ABC$ [Opposite angles of equal sides of triangle are equal]

Now, $\angle CAD = \angle ABC + \angle ACB$

=>$\angle PAC + \angle PAD = 2\angle ACB$ $[\therefore \angle PAC = \angle PAD]$

=>$2\angle PAC = 2\angle ACB$

=>$\angle PAC = \angle ACB$

(ii) Now,

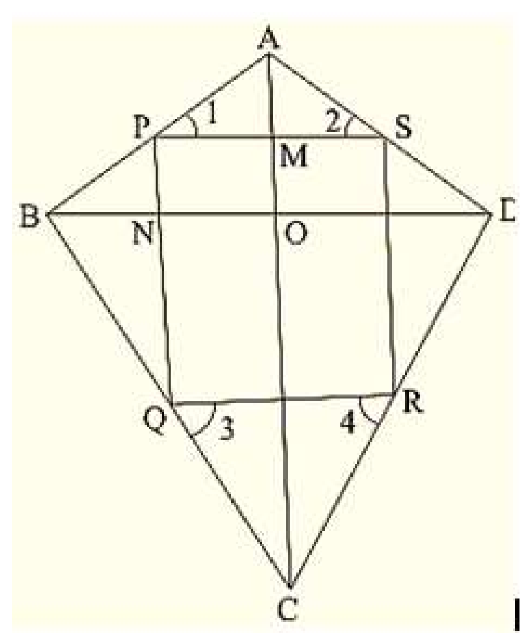
$\angle PAC = \angle BCA$

=>$AP \parallel BC$ and $CP \parallel BA$ [Given]

Therefore, ABCP is a parallelogram.

Q12) ABCD is a kite having AB=AD and BC=CD. Prove that the figure found by joining the mid points of the sides, in order, is a rectangle.

Solution:



Given,

A kite ABCD having AB=AD and BC=CD. P, Q, R, S are the mid-points of sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove:

PQRS is a rectangle.

Proof:

In ΔABC, P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \ldots..(i)$

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2}AC \ldots..(ii)$

From (i) and (ii) we have

$PQ \parallel RS$ and $PQ = RS$

Thus, in quadrilateral PQRS, a pair of opposite sides is equal and parallel. So, PQRS is a parallelogram. Now, we shall prove that one angle of parallelogram PQRS is a right angle.

Since AB=AD

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AD$

$\Rightarrow AP = AS \ldots.(iii) \quad [\because P \text{ and } S \text{ are mid points of } AB \text{ and } AD]$

$\Rightarrow \angle 1 = \angle 2 \ldots.(iv)$

Now, in ΔPBQ and ΔSDR, we have

PB = SD $\quad [\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB]$

BQ = DR [Since PB = SD]

And PQ = SR [Since, PQRS is a parallelogram]

So, by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SDR$

$\Rightarrow \angle 3 = \angle 4 \quad [CPCT]$

Now, $\Rightarrow \angle 3 + \angle SPQ + \angle 2 = 180^0$

And $\angle 1 + \angle PSR + \angle 4 = 180^0$

$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow \angle SPQ = \angle PSR \quad [\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively.

$\therefore \angle SPQ + \angle PSR = 180^0$

=>$2\angle SPQ = 180^0$

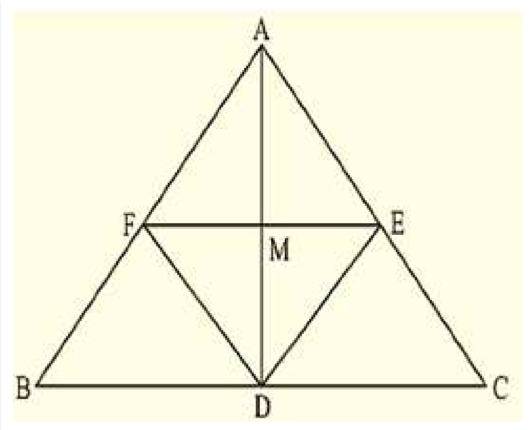
=>$\angle SPQ = 90^0 \quad [\because \angle PSR = \angle SPQ]$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90^0$.

Hence, PQRS is a parallelogram.

Q13) Let ABC be an isosceles triangle in which AB=AC. If D, E, F be the mid points of the, sides BC,CA and AB respectively, show that the segment AD and EF bisect each other at right angles.

Solution:



Since D, E and F are mid-points of sides BC, CA and AB respectively.

$\therefore$ AB $\parallel$ DE and AC $\parallel$ DF

$\therefore$ AF $\parallel$ DE and AE $\parallel$ DF

ABDE is a parallelogram.

AF = DE and AE = DF

$\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

DE = DF [Since, AB = AC]

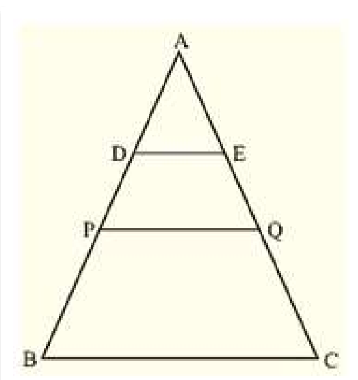
AE = AF = DE = DF

ABDF is a rhombus.

=>AD and FE bisect each other at right angle.

Q14) ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ ***and E is a point on AC such that*** $AE = \frac{1}{4}AC$. ***Prove that*** $DE = \frac{1}{4}BC$.

Solution:



Let P and Q be the mid-points of AB and AC respectively.

Then $PQ \parallel BC$

$PQ = \frac{1}{2}BC$.....(i)

In ΔAPQ, D and E are the mid-points of AP and AQ respectively.

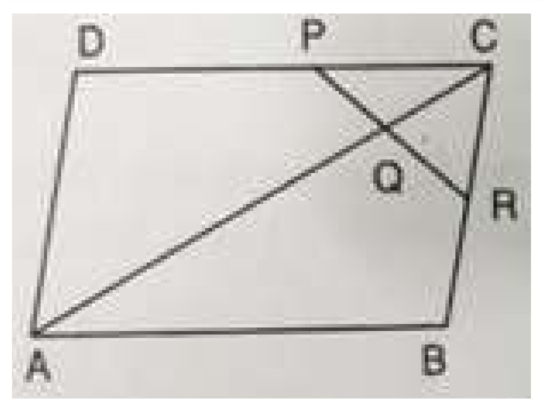
$\therefore$ DE $\parallel$ PQ, and DE $= \frac{1}{2}$PQ.....(ii)

From (i) and (ii): DE $= \frac{1}{2}$ PQ $= \frac{1}{2}$ ($\frac{1}{2}$BC)

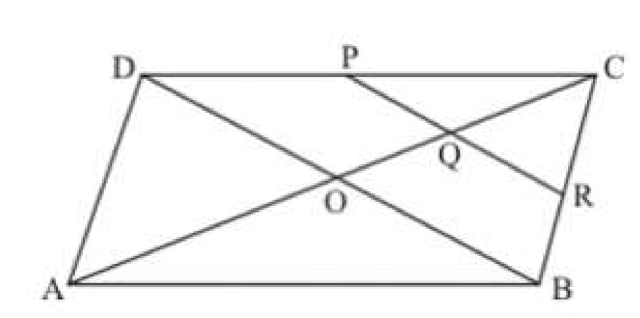
$\therefore$ DE $= \frac{1}{4}$BC

Hence proved.

Q15) In Figure 14.99, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$***. If PQ produced meets BC at R, prove that R is a mid-point of BC.***



Solution:



Join B and D.

Suppose AC and BD intersect at O.

Then $OC = \frac{1}{2}AC$

Now,

$CQ = \frac{1}{4}AC$

$\Rightarrow CQ = \frac{1}{2}(\frac{1}{2}AC)$

= $\frac{1}{2}OC$

In ΔDCO, P and Q are mid points of DC and OC respectively.

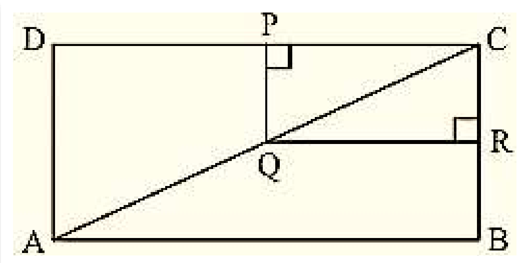
$\therefore PQ \parallel DO$

Also in ΔCOB, Q is the mid-point of OC and $QR \parallel OB$

Therefore, R is the mid-point of BC.

Q16) In figure 14.100, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

(i) DP = PC ***(ii)*** $PR = \frac{1}{2}AC$



Solution:

(i) In ΔADC, Q is the mid-point of AC such that $PQ \parallel AD$

Therefore, P is the mid-point of DC.

=>DP = DC [Using mid-point theorem]

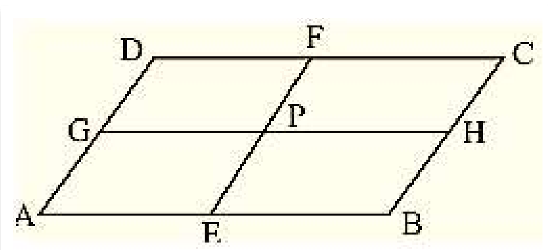
(ii) Similarly, R is the mid-point of BC

$\therefore PR = \frac{1}{2}BD$

$PR = \frac{1}{2}AC$ [Diagonal of rectangle are equal, BD = AC]

Q17) ABCD is a parallelogram; E and f are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Solution:



Since E and F are mid-points of AB and CD respectively

$AE = BE = \frac{1}{2}AB$

And $CF = DF = \frac{1}{2}CD$

But, AB = CD

$\frac{1}{2}AB = \frac{1}{2}CD$

=>BE = CF

Also, $BE \parallel CF$ $[\because AB \parallel CD]$

Therefore, BEFC is a parallelogram

$BC \parallel EF$ and BE = PH .....(i)

Now, $BC \parallel EF$

=>AD ∥ EF [∵ BC ∥ AD as ABCD is a parallelogram]

Therefore, AEFD is a parallelogram.

=>AE = GP

But E is the mid-point of AB.

So, AE = BF

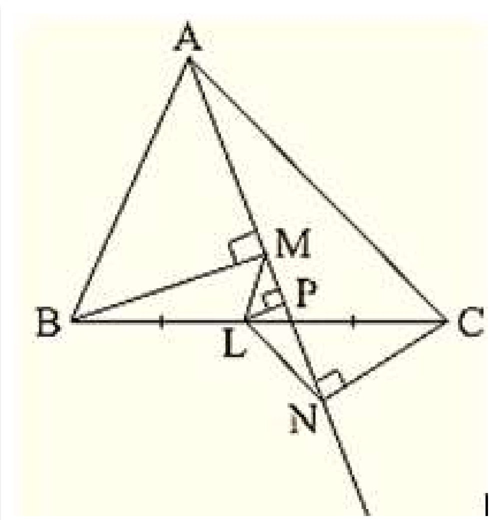
Therefore, GP = PH.

Q18) BM and CN are perpendiculars to a line passing through the vertex A of triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Solution:

To prove LM = LN

Draw LS as perpendicular to line MN.



Therefore, the lines BM, LS and CN being the same perpendiculars on line MN are parallel to each other.

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

In the figure, MB, LS and NC are three parallel lines and the two transversal lines are MN and BC.

We have, BL = LC [As L is the given mid-point of BC]

Using the intercept theorem, we get

MS = SN …. (i)

Now in ΔMLS and ΔLSN

MS = SN using equation (i).

$\angle LSM = \angle LSN = 90^0 \quad [LS \perp MN]$

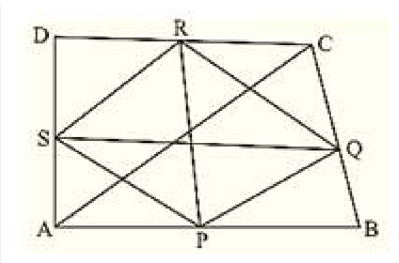
And SL = LS is common.

$\therefore \Delta MLS \cong \Delta LSN \quad$ [SAS Congruency Theorem]

$\therefore$ LM = LN [CPCT]

Q19) Show that, the line segments joining the mid-points of opposite sides of a quadrilateral bisects each other.

Solution:



Let ABCD is a quadrilateral in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.

So, by using mid-point theorem we can say that

$$SP \parallel BD \text{ and } SP = \frac{1}{2}BD \ldots\ldots(i)$$

Similarly in ΔBCD

$$QR \parallel BD \text{ and } QR = \frac{1}{2}BD \ldots\ldots(ii)$$

From equations (i) and (ii), we have

$$SP \parallel QR \text{ and } SP = QR$$

As in quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other.

So, SPQR is a parallelogram since the diagonals of a parallelogram bisect each other.

Hence PR and QS bisect each other.

Q20) Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is ___________.

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is ___________.

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is ___________.

Solution:

(i) Isosceles

(ii) Right triangle

(iii) Parallelogram