RD SHARMA

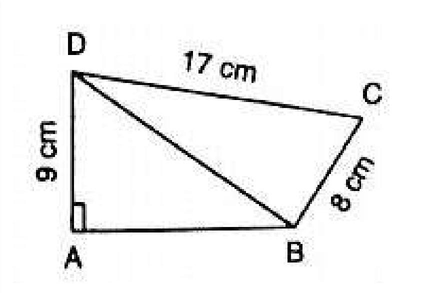
Solutions

Class 9 Maths

Chapter 15

Ex *15.3*

Q 1. In figure, compute the area of quadrilateral ABCD.



Solution:

Given:

DC = 17 cm, AD = 9 cm and BC = 8 cm

In ΔBCD we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow 17^2 = BD^2 + 8^2$

$\Rightarrow BD^2 = 289 - 64$

$\Rightarrow BD = 15$

In ΔABD we have

$AB^2 + AD^2 = BD^2$

$\Rightarrow 15^2 = AB^2 + 9^2$

$\Rightarrow AB^2 = 225 - 81 = 144$

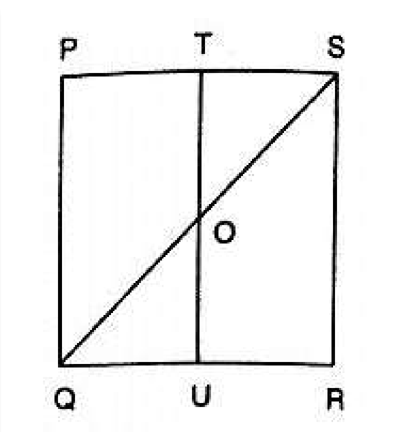
$\Rightarrow AB = 12$

$ar\,(quad\ ABCD) = ar\,(\Delta ABD) + ar\,(\Delta BCD)$

$ar\,(quad\ ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68 = 122\ cm^2$

$ar\,(quad\ ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = 54 + 60 = 114\ cm^2$

Q2. In figure, PQRS is a square and T and U are, respectively, the midpoints of PS and QR . Find the area of ΔOTS ***if PQ = 8 cm.***



Solution:

From the figure,

T and U are mid points of PS and QR respectively

$\therefore TU || PQ$

$\Rightarrow TO || PQ$

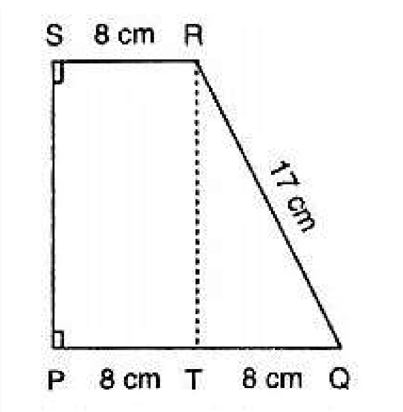
Thus , in ΔPQS , T is the mid point of PS and TO||PQ

$\therefore TO = \frac{1}{2}PQ = 4 \text{ cm}$

Also, TS = $\frac{1}{2}PS = 4 \text{ cm}$

$\therefore ar(\Delta OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) \text{ cm}^2 = 8 \text{ cm}^2$

Q3. Compute the area of trapezium PQRS in figure



Solution:

We have,

$ar(\text{trap. } PQRS) = ar(\text{rect. } PSRT) + ar(\Delta QRT)$

$\Rightarrow ar(\text{trap. } PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$

= $= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In ΔQRT , we have

$QR^2 = QT^2 + RT^2$

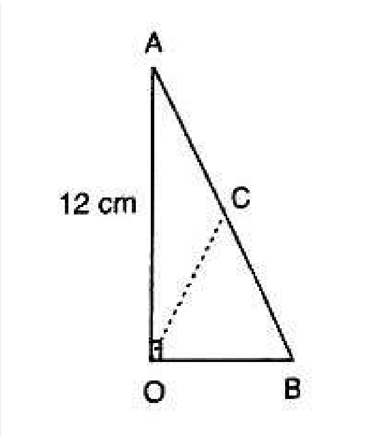
$\Rightarrow RT^2 = QR^2 - QT^2$

$\Rightarrow RT^2 = 17^2 - 8^2 = 225$

$\Rightarrow RT = 15$

Hence , area of trapezium = $12 \times 15 \text{ cm}^2$ = 180 cm^2

Q4. In figure, $\angle AOB = 90^\circ$, AC = BC , OA = 12 cm and OC = 6.5 cm .Find the area of ΔAOB.



Solution:

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$\therefore CA = CB = OC$

$\Rightarrow CA = CB = 6.5 \text{ cm}$

$\Rightarrow AB = 13 \text{ cm}$

In right angled triangle OAB , we have

$AB^2 = OB^2 + OA^2$

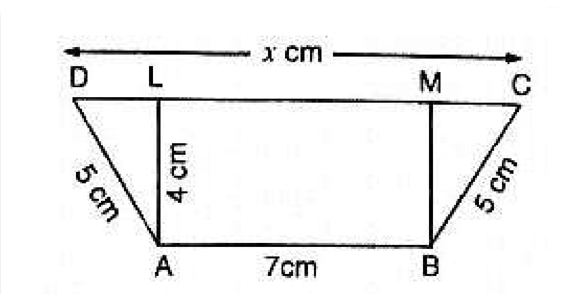
$\Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow OB = 5$

$\therefore ar(\Delta AOB) = \frac{1}{2}(12 \times 5) = 30 \text{ cm}^2$

Q5. In figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm , and distance between AB and DC is 4 cm . Find the value of x and area of trapezium ABCD.



Solution:

Draw AL ⊥ DC, BM ⊥ DC then ,

AL = BM = 4 cm and LM = 7 cm.

In Δ ADL , we have

$AD^2 = AL^2 + DL^2$

$\Rightarrow 25 = 16 + DL^2$

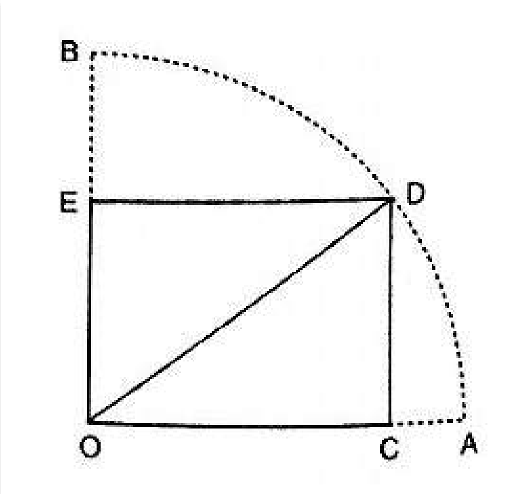
$\Rightarrow DL = 3 \text{ cm}$

Similarly, MC = $\sqrt{BC^2 - BM^2} = \sqrt{25 - 16} = 3 \text{ cm}$

$\therefore x = CD = CM + ML + LD = (3 + 7 + 3)$ cm = 13 cm

$ar\,(\text{trap. } ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4 \text{ cm}^2 = 40 \text{ cm}^2$

Q 6. In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$ cm , find the area of the rectangle .



Solution:

Given OD = 10 cm and OE = $2\sqrt{5}$ cm

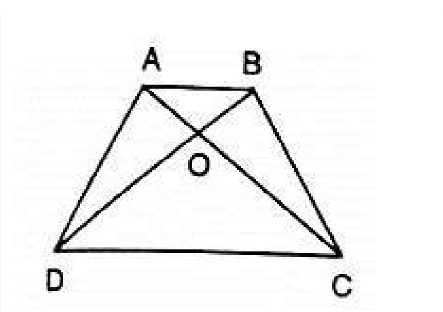
By using Pythagoras theorem

$\therefore OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{10^2 - (2\sqrt{5})^2} = 4\sqrt{5} \text{ cm}$

$\therefore$ Area of rectangle OCDE = $OE \times DE = 2\sqrt{5} \times 4\sqrt{5} \text{ cm}^2 = 40 \text{ cm}^2$

Q 7. In figure, ABCD is a trapezium in which AB || DC. Prove that $ar\,(\Delta AOD) = ar\,(\Delta BOC)$



Solution:

Given: ABCD is a trapezium in which AB || DC

To prove: $ar\,(\Delta AOD) = ar\,(\Delta BOC)$

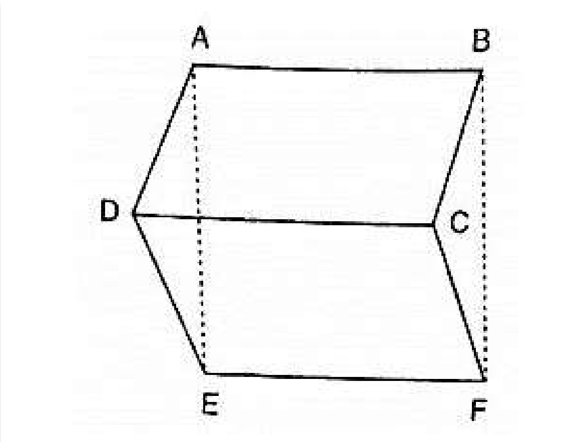
Proof:- Since , ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

Then, ar(ΔADC) = ar(ΔBDC)

$\Rightarrow \text{ar}(\Delta AOD) + \text{ar}(\Delta DOC) = \text{ar}(\Delta BOC) + \text{ar}(\Delta DOC)$

$\Rightarrow \text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$

Q 8. In figure, ABCD, ABFE and CDEF are parallelograms. Prove that $\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$.



Solution:

Given that

ABCD is parallelogram $\Rightarrow AD = BC$

CDEF is parallelogram $\Rightarrow DE = CF$

ABFE is parallelogram $\Rightarrow AE = BF$

Thus, in Δs ADF and BCF , we have

AD = BC, DE = CF and AE = BF

So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta BCF$

$\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$

Q 9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that :
$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

Solution:

Construction: – Draw BQ ⊥AC and DR ⊥ AC

Proof:-

L.H.S

= $\text{ar}(\Delta APB) \times \text{ar}(\Delta CDP)$

= $\frac{1}{2}[(AP \times BQ)] \times (\frac{1}{2} \times PC \times DR)$

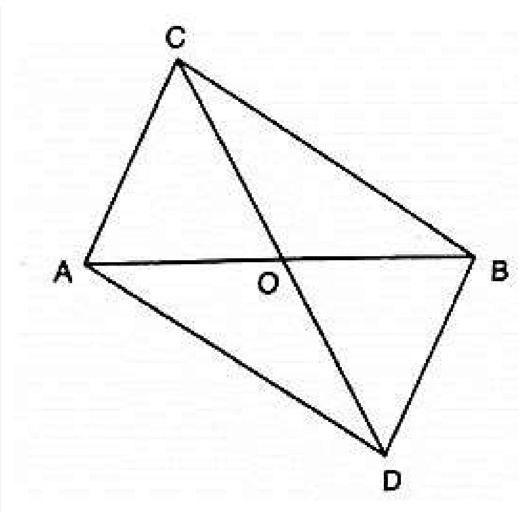
= $(\frac{1}{2} \times PC \times BQ) \times (\frac{1}{2} \times AP \times DR)$

= $\text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

= R.H.S

Hence proved.

Q 10. In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that $ar(\Delta ABC) = ar(\Delta ABD)$.



Solution:

Given that CD is bisected by AB at O

To prove: $ar(\Delta ABC) = ar(\Delta ABD)$.

Construction: Draw CP ⊥ AB and DQ ⊥ AB .

Proof:

$ar(\Delta ABC) = \frac{1}{2} \times AB \times CP \cdots\cdots\cdot (1)$

$ar(\Delta ABD) = \frac{1}{2} \times AB \times DQ \cdots\cdots\cdot (2)$

In ΔCPO and ΔDQO

$\angle CPO = \angle DQO \qquad [\text{each } 90^\circ]$

Given that, CO = OD

$\angle COP = \angle DOQ \qquad [\text{Vertically opposite angles are equal}]$

Then , $\Delta CP0 \cong \Delta DQO \qquad [\text{By AAS condition}]$

$\therefore CP = DQ \qquad (3)\,[\text{c. p. c. t}]$

Compare equation (1), (2) and (3)

$\therefore ar(\Delta ABC) = ar(\Delta ABD)$.

Q 11. If P is any point in the interior of a parallelogram ABCD , then prove that area of the triangle APB is less than half the area of parallelogram.

Solution:

Draw DN ⊥ AB and PM ⊥ AB

Now ,

$ar(||^{gm} ABCD) = AB \times DN, ar(\Delta APB) = \frac{1}{2}(AB \times PM)$

Now , PM < DN

$\Rightarrow AB \times PM < AB \times DN$

$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$

$\Rightarrow ar(\Delta APB) < \frac{1}{2} ar(||^{gm} ABCD)$

Q 12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of the median AD, prove that $ar(\Delta BGC) = 2ar(\Delta AGC)$.

Solution:

Draw AM ⊥ BC

Since, AD is the median of ΔABC

∴ BD = DC

$\Rightarrow BD = AM = DC \times AM$

$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow ar(\Delta ABD) = ar(\Delta ACD) \cdots\cdots\cdots (1)$

In ΔBGC , GD is the median

$\therefore ar(\Delta BGD) = ar(\Delta CGD) \cdots\cdots\cdots (2)$

In ΔACD , CG is the median

$\therefore ar(\Delta AGC) = ar(\Delta CGD) \cdots\cdots\cdots (3)$

From (2) and (3) we have,

$ar(\Delta BGD) = ar(\Delta AGC)$

But, $ar(\Delta BGC) = 2ar(\Delta BGD)$

$\therefore ar(\Delta BGC) = 2ar(\Delta AGC)$

Q 13. A point D is taken on the side BC of a ΔABC ***, such that BD = 2DC . Prove that*** $ar(\Delta ABD) = 2ar(\Delta ADC)$.

Solution:

Given that,

In ΔABC, BD = 2DC

To prove: $ar(\Delta ABD) = 2ar(\Delta ADC)$.

Construction:

Take a point E on BD such that BE = ED

Proof: Since, BE = ED and BD = 2 DC

Then, BE = ED = DC

We know that median of triangle divides it into two equal triangles.

$\therefore$ In ΔABD , AE is the median .

Then , $ar(\Delta ABD) = 2ar(\Delta AED) \cdots\cdots (1)$

In ΔAEC , AD is the median .

Then, $ar(\Delta AED) = 2ar(\Delta ADC) \cdots\cdots (2)$

Compare equation 1 and 2

$ar(\Delta ABD) = 2ar(\Delta ADC).$

Q 14. ABCD is a parallelogram whose diagonals intersect at O .If P is any point on BO, prove that :

(i) . $ar(\Delta ADO) = ar(\Delta CDO).$

(ii) . $ar(\Delta ABP) = 2ar(\Delta CBP).$

Solution:

Given that ABCD is the parallelogram

To Prove: (i) $ar(\Delta ADO) = ar(\Delta CDO).$

(ii) $ar(\Delta ABP) = 2ar(\Delta CBP).$

Proof: we know that diagonals of parallelogram bisect each other

$\therefore$ AO = OC and BO = OD

(i) . In Δ DAC , since DO is a median .

Then $ar(\Delta ADO) = ar(\Delta CDO).$

(ii) . In Δ BAC , since BO is a median .

Then $ar(\Delta BAO) = ar(\Delta BCO) \cdots\cdots\cdot (1)$

In Δ PAC , since PO is a median .

Then $ar(\Delta PAO) = ar(\Delta PCO) \cdots\cdots\cdot (2)$

Subtract equation 2 from 1.

$\Rightarrow ar(\Delta BAO) - ar(\Delta PAO) = ar(\Delta BCO) - ar(\Delta PCO)$

$\Rightarrow ar(\Delta ABP) = 2ar(\Delta CBP).$

Q 15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) . Prove that $ar(\Delta ADF) = ar(\Delta ECF).$

(ii) . If the area of ΔDFB = $3\ cm^2$ ***, find the area of*** $||^{gm}$ ***ABCD .***

Solution:

In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternate interior angles, Since $AD||BE$]

AD = EC [since $AD = BC = CE$]

And $\angle DFA = \angle CFA$ [Vertically opposite angles]

So, by AAS congruence criterion, we have

$\Delta ADF \cong \Delta ECF$

$\Rightarrow ar(\Delta ADF) = ar(\Delta ECF)$ and DF = CF .

Now, DF = CF

$\Rightarrow$ BF is a median in Δ BCD.

$\Rightarrow ar(\Delta BCD) = 2ar(\Delta BDF)$

$\Rightarrow ar(\Delta BCD) = 2 \times 3\ cm^2 = 6\ cm^2$

Hence, area of a parallelogram = $2ar(\Delta BCD) = 2 \times 6\ cm^2 = 12\ cm^2$

Q 16. ABCD is a parallelogram whose diagonals AC and BD intersect at O . A line through O intersects AB at P and DC at Q. Prove that $ar(\Delta POA) = ar(\Delta QOC)$.

Solution:

In triangles POA and QOC, we have

$\angle AOP = \angle COQ$

AO = OC

$\angle PAC = \angle QCA$

So, by ASA congruence criterion , we have

$\Delta POA \cong \Delta QOC$

$\Rightarrow ar(\Delta POA) = ar(\Delta QOC)$.

Q 17. ABCD is a parallelogram. E is a point on BA such that BE = 2EA and F is point on DC such that DF = 2FC. Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Solution:

Draw FG $\perp$ AB

We have,

BE = 2 EA and DF = 2FC

$\Rightarrow$ AB − AE = 2 AE and DC − FC = 2 FC

$\Rightarrow$ AB = 3 AE and DC = 3 FC

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC \cdots\cdots\cdots(1)$

But AB = DC

Then, AE = FC [opposite sides of $||^{gm}$]

Thus, AE = FC and AE || FC

Then, AECF is a parallelogram

Now, area of parallelogram AECF = $AE \times FG$

$\Rightarrow ar(||^{gm}\ AECF) = \frac{1}{3}AB \times FG$ from(1)

$\Rightarrow 3\ ar(||^{gm}\ AECF) = AB \times FG \cdots\cdots (2)$

And $ar(||^{gm}\ ABCD) = AB \times FG \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow 3ar(||^{gm}\ AECF) = ar(||^{gm}\ ABCD)$

$\Rightarrow ar(||^{gm}\ AECF) = \frac{1}{3} ar(||^{gm}\ ABCD)$

Q 18. In a triangle ABC, P and Q are respectively the mid points of AB and BC and R is the mid point of AP. Prove that :

(i) . $ar(\Delta PBQ) = ar(\Delta ARC)$.

(ii) . $ar(\Delta PRQ) = \frac{1}{2} ar(\Delta ARC)$.

(iii) . $ar(\Delta RQC) = \frac{3}{8} ar(\Delta ABC)$.

Solution:

We know that each median of a triangle divides it into two triangles of equal area.

(i) . Since CR is the median of Δ CAP

$\therefore ar(\Delta CRA) = \frac{1}{2} ar(\Delta CAP) \cdots\cdots\cdots (1)$

Also , CP is the median of a Δ CAB

$\therefore ar(\Delta CAP) = ar(\Delta CPB) \cdots\cdots\cdots (2)$

From 1 and 2 , we get

$\therefore ar(\Delta ARC) = \frac{1}{2} ar(\Delta CPB) \cdots\cdots\cdots (3)$

PQ is the median of a Δ PBC

$\therefore ar(\Delta CPB) = 2ar(\Delta PBQ) \cdots\cdots\cdots (4)$

From 3 and 4, we get

$\therefore ar(\Delta ARC) = ar(\Delta PBQ) \cdots\cdots\cdots (5)$

(ii) . Since QP and QR medians of triangles QAB and QAP respectively

$\therefore ar(\Delta QAP) = ar(\Delta QBP) \cdots\cdots\cdots (6)$

And $ar(\Delta QAP) = 2ar(\Delta QRP) \cdots\cdots\cdots (7)$

From 6 and 7, we get

$ar(\Delta PRQ) = \frac{1}{2} ar(\Delta PBQ) \cdots\cdots\cdots (8)$

From 5 and 8, we get

$ar(\Delta PRQ) = \frac{1}{2} ar(\Delta ARC)$

(iii) . Since, LR is a median of Δ CAP

$\therefore$ ar $(\Delta ARC) = \frac{1}{2}$ar (ΔCAD)

$= \frac{1}{2} \times \frac{1}{2}$ar (ΔABC)

$= \frac{1}{4}$ar (ΔABC)

Since RQ is the median of Δ RBC.

$\therefore$ ar $(\Delta RQC) = \frac{1}{2}$ar (ΔRBC)

$= \frac{1}{2}\{$ar $(\Delta ABC) -$ ar $(\Delta ARC)\}$

$= \frac{1}{2}\{$ar $(\Delta ABC) - \frac{1}{4}$ar $(\Delta ABC)\}$

$= \frac{3}{8}$ar (ΔABC)

Q 19. ABCD is a parallelogram. G is a point on AB such that AG = 2GB and E is point on DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) . ar (ADEG) = ar (GBCE).

(ii) . ar $(\Delta EGB) = \frac{1}{6}$ar (ABCD).

(iii) . ar $(\Delta EFC) = \frac{1}{2}$ar (ΔEBF).

(iv) . ar $(\Delta EGB) = \frac{3}{2} \times$ ar (ΔEFC)

(v) . Find what portion of the area of parallelogram is the area of Δ EFG.

Solution:

Given: ABCD is a parallelogram

AG = 2 GB, CE = 2 DE and BF = 2 FC

To prove:

(i) . ar (ADEG) = ar (GBCE).

(ii) . ar $(\Delta EGB) = \frac{1}{6}$ar (ABCD).

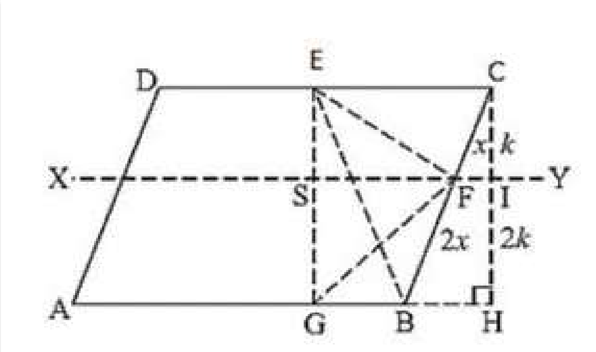
(iii) . ar $(\Delta EFC) = \frac{1}{2}$ar (ΔEBF).

(iv) . ar $(\Delta EGB) = \frac{3}{2} \times$ ar (ΔEFC)

(v) . Find what portion of the area of parallelogram is the area of Δ EFG.

Construction: Draw a parallel line to AB through point F and a perpendicular line to AB from C

Proof:



(i) .Since ABCD is a parallelogram

So, AB = CD and AD = BC

Consider the two trapezium s ADEG and GBCE

Since AB = DC, EC = 2DE, AG = 2GB

$\Rightarrow \mathrm{ED} = \frac{1}{3}\mathrm{CD} = \frac{1}{3}\mathrm{AB}$ and $\mathrm{EC} = \frac{2}{3}\mathrm{CD} = \frac{2}{3}\mathrm{AB}$

$\Rightarrow \mathrm{AG} = \frac{2}{3}\mathrm{AB}$ and $\mathrm{BG} = \frac{1}{3}\mathrm{AB}$

So , $\mathrm{DE} + \mathrm{AG} = \frac{1}{3}\mathrm{AB} + \frac{2}{3}\mathrm{AB} = \mathrm{AB}$ and $\mathrm{EC} + \mathrm{BG} = \frac{2}{3}\mathrm{AB} + \frac{1}{3}\mathrm{AB} = \mathrm{AB}$

Since the two trapezium ADEG and GBCE have same height and their sum of two parallel sides are equal

Since Area of trapezium = $\frac{\text{sum of parallel sides}}{2} \times \text{height}$

So, $\mathrm{ar}(\mathrm{ADEG}) = \mathrm{ar}(\mathrm{GBCE})$.

(ii) . Since we know from above that

$\mathrm{BG} = \frac{1}{2}\mathrm{AB}$. So

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \mathrm{GB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \frac{1}{3} \times \mathrm{AB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{6} \times \mathrm{AB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB})= \frac{1}{6}\mathrm{ar}(\mathrm{ABCD})$.

(iii) . Since height if triangle EFC and EBF are equal.So

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \mathrm{FC} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \frac{1}{2} \times \mathrm{FB} \times \mathrm{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2}\mathrm{ar}(\mathrm{EBF})$

Hence , $\mathrm{ar}(\Delta\mathrm{EFC})= \frac{1}{2}\mathrm{ar}(\Delta\mathrm{EBF})$.

(iv) . Consider the trapezium in which

$\mathrm{ar}(\mathrm{EGBC}) = \mathrm{ar}(\Delta\mathrm{EGB}) + \mathrm{ar}(\Delta\mathrm{EBF}) + \mathrm{ar}(\Delta\mathrm{EFC})$

$\Rightarrow \frac{1}{2}\text{ar}(ABCD) = \frac{1}{6}\text{ar}(ABCD) + 2\text{ar}(\Delta EFC) + \text{ar}(\Delta EFC)$

$\Rightarrow \frac{1}{3}\text{ar}(ABCD) = 3\text{ar}(\Delta EFC)$

$\Rightarrow \text{ar}(\Delta EFC) = \frac{1}{9}\text{ar}(ABCD)$

Now from (ii)part we have

$\text{ar}(\Delta EGB) = \frac{1}{6}\text{ar}(\Delta EFC)$

$\text{ar}(\Delta EGB) = \frac{3}{2} \times \frac{1}{9}\text{ar}(ABCD)$

$\text{ar}(\Delta EGB) = \frac{3}{2}\text{ar}(\Delta EFC)$

$\therefore \text{ar}(\Delta EGB) = \frac{3}{2}\text{ar}(\Delta EFC)$

(v) . In the figure it is given that FB = 2CF .Let CF = x and FB = 2x.

Now consider the two triangles CFI and CBH which are similar triangle.

So by the property of similar triangle CI = k and IH = 2k

Now consider the triangle EGF in which

$\text{ar}(\Delta EFG) = \text{ar}(\Delta ESF) + \text{ar}(\Delta SGF)$

$\text{ar}(\Delta EFG) = \frac{1}{2}SF \times k + \frac{1}{2}SF \times 2k$

$\text{ar}(\Delta EFG) = \frac{3}{2}SF \times k \cdots\cdots (i)$

Now ,

$\text{ar}(\Delta EGBC) = \text{ar}(SGBF) + \text{ar}(ESFC)$

$\text{ar}(\Delta EGBC) = \frac{1}{2}(SF + GB) \times 2k + \frac{1}{2}(SF + EC) \times k$

$\text{ar}(\Delta EGBC) = \frac{3}{2}k \times SF + (GB + \frac{1}{2}EC) \times k$

$\text{ar}(\Delta EGBC) = \frac{3}{2}k \times SF + (\frac{1}{3}AB + \frac{1}{2} \times \frac{2}{3}AB) \times k$

$\frac{1}{2}\text{ar}(\Delta ABCD) = \frac{3}{2}k \times SF + \frac{2}{3}AB \times k$

$\Rightarrow \text{ar}(\Delta ABCD) = 3k \times SF + \frac{4}{3}AB \times k$ [Multiply both sides by 2]

$\Rightarrow \text{ar}(\Delta ABCD) = 3k \times SF + \frac{4}{9}\text{ar}(ABCD)$

$\Rightarrow k \times SF = \frac{5}{27}\text{ar}(ABCD) \cdots\cdots\cdots (2)$

From 1 and 2 we have ,

$\text{ar}(\Delta EFG) = \frac{3}{2} \times \frac{5}{27}\text{ar}(ABCD)$

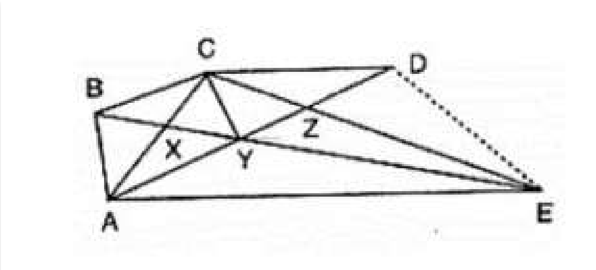
$\text{ar}(\Delta EFG) = \frac{5}{18}\text{ar}(ABCD)$

Q 20. In figure, CD || AE and CY || BA.

(i) . Name a triangle equal in area of ΔCBX

(ii) . Prove that $ar(\Delta ZDE) = ar(\Delta CZA)$.

(iii) . Prove that $ar(BCZY) = ar(\Delta EDZ)$.



Solution:

Since, triangle BCA and triangle BYA are on the same base BA and between same parallel s BA and CY.

Then $ar(\Delta BCA) = ar(\Delta BYA)$

$\Rightarrow ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$

$\Rightarrow ar(\Delta CBX) = ar(\Delta AXY) \cdots\cdots (1)$

Since, triangles ACE and ADE are on the same base AE and between same parallels CD and AE

Then, $ar(\Delta ACE) = ar(\Delta ADE)$

$ar(\Delta CZA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$

$ar(\Delta CZA) = ar(\Delta DZE) \cdots\cdots (2)$

Adding $ar(\Delta CYG)$ on both sides , we get

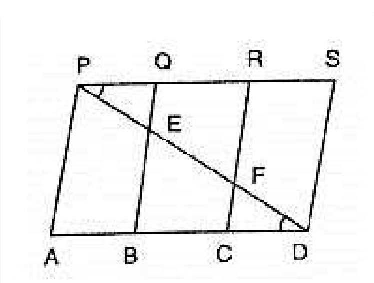
$\Rightarrow ar(\Delta CBX) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$

$\Rightarrow ar(BCZY) = ar(\Delta CZA) \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow ar(BCZY) = ar(\Delta DZE)$

Q 21. In figure, PSDA is a parallelogram in which PQ = QR =RS and AP || BQ ||CR. Prove that $ar(\Delta PQE) = ar(\Delta CFD)$.



Solution:

Given that PSDA is a parallelogram

Since, AP || BQ || CR ||DS and AD ||PS

Therefore, PQ = CD (equ. 1)

In triangle BED, C is the midpoint of BD and CF || BE

Therefore, F is the midpoint of ED

⇒ EF = PE

Smiliarly,

EF = PE

Therefore, PE = FD (equ . 2)

In triangles PQE and CFD, we have

PE = FD

Therefore, ∠ EPQ = ∠FDC [Alternate angles]

So, by SAS criterion , we have

$\Delta PQE \cong \Delta DCF$

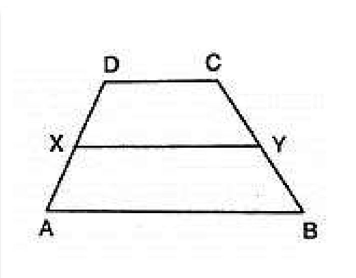
$\Rightarrow ar(\Delta PQE) = ar(\Delta DCF)$

Q 22. In figure, ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm .If X and Y are , respectively , the mid points of AD and BC , prove that :

(i) . XY = 50 cm

(ii) . DCYX is a trapezium

(iii) . $ar(\text{trap. } DCYX) = \frac{9}{11} ar(XYBA)$.



Solution:

(i) Join DY and produce it to meet AB produced at P.

In triangles BYP and CYD we have,

∠BYP = ∠ CYD [Vertically opposite angles]

∠DCY = ∠ PBY [Since , DC ||AP]

And BY = CY

So, by ASA congruence criterion, we have

$(\Delta BYP) \cong (\Delta CYD)$

⇒ DY = Yp and DC = BP

⇒ Y is the midpoint of DP

Also , x is the mid point of AD

Therefore, XY ||AP and XY ||$\frac{1}{2}$AP

$\Rightarrow$ XY $= \frac{1}{2}(AB + BP)$

$\Rightarrow$ XY $= \frac{1}{2}(AB + DC)$

$\Rightarrow$ XY $= \frac{1}{2}(60 + 40)$

= 50 cm

(ii) We have, XY ||AP

$\Rightarrow$ XY || AB and AB ||DC

$\Rightarrow$ XY || DC

$\Rightarrow$ DCYX is a trapezium

(iii) Since x and y are the mid points of Ad and BC respectively.

Therefore, trapezium DCYX and ABYX are of the same height say h cm

Now,

$ar(trap.\ DCXY) = \frac{1}{2}(DC + XY) \times h$

$\Rightarrow ar(trap.\ DCXY) = \frac{1}{2}(50 + 40) \times h\ cm^2 = 45h\ cm^2$

$\Rightarrow ar(trap.\ ABYX) = \frac{1}{2}(AB + XY) \times h$

$\Rightarrow ar(trap.\ ABYX) = \frac{1}{2}(60 + 50) \times h\ cm^2 = 55h\ cm^2$

$\frac{ar(trap.DCYX)}{ar(trap.ABYX)} = \frac{45h}{55h} = \frac{9}{11}$

$\Rightarrow ar(trap.\ DCYX) = \frac{9}{11} ar(trap.\ ABYX)$

Q 23. In figure ABC and BDE are two equilateral triangles such that D is the midpoint of BC. AE intersects BC in F. Prove that:

(i) . $ar(\Delta BDE)$= $\frac{1}{4} ar(\Delta ABC)$.

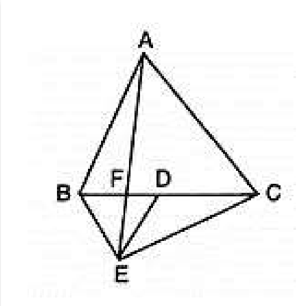
(ii) . $ar(\Delta BDE)$= $\frac{1}{2} ar(\Delta BAE)$.

(iii) . $ar(\Delta BFE)$= $ar(\Delta AFD)$.

(iv) . $ar(\Delta ABC)$= **2** $ar(\Delta BEC)$.

(v) . $ar(\Delta FED)$= $\frac{1}{8} ar(\Delta AFC)$.

(vi) . $ar(\Delta BFE)$= **2** $ar(\Delta EFD)$.



Solution:

Given that ABC and BDE are two equilateral triangles.

Let AB = BC = CA = x. Then, BD = $\frac{x}{2}$ = DE = BE

(i) We have,

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}x^2 \text{ and } ar(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{1}{4} \times \frac{\sqrt{3}}{4}x^2$$

Therefore , $ar(\Delta BDE) = \frac{1}{4}ar(\Delta ABC)$.

(ii) . It is given that triangles, ABC and BED are equilateral triangles

$\angle ACB = \angle DBE = 60°$

⇒ BE || AC (Since , alternative angles are equal)

Trinagles BAF and BEC are on the same base BE and between same parallels BF and AC .

Therefore, $ar(\Delta BAE) = ar(\Delta BEC)$

$\Rightarrow ar(\Delta BAE) = 2ar(\Delta BDE)$ [Since , ED is a median of triangle EBC ; $ar(\Delta BEC) = 2ar(\Delta BDE)$]

$\therefore ar(\Delta BDE) = \frac{1}{2}ar(\Delta BAE)$

(iii) Since, triangles ABC and BDE are equilateral triangles

$\therefore \angle ABC = 60° \text{ and } \angle BDE = 60°$

$\angle ABC = \angle BDE$

⇒ AB ||DE (since, alternate angles are equal)

Triangles BED and AED are on the same base ED and between same parallels AB and DE.

Therefore, $ar(\Delta BED) = ar(\Delta AED)$

$\Rightarrow ar(\Delta BED) - ar(\Delta EFD) = ar(\Delta AED) - ar(\Delta EFD)$

$\Rightarrow ar(\Delta BEF) = ar(\Delta AFD)$

(iv) Since ED is the median of triangle BEC

Therefore, $ar(\Delta BEC) = 2ar(\Delta BDE)$

$\Rightarrow ar(\Delta BEC) = 2 \times \frac{1}{4}ar(\Delta ABC)$ [From 1 , $ar(\Delta BDE) = \frac{1}{4}ar(\Delta ABC)$]

$\Rightarrow ar(\Delta BEC) = \frac{1}{2}ar(\Delta ABC)$

$\Rightarrow ar(\Delta ABC) = 2ar(\Delta BEC)$

(v) $ar(\Delta AFC) = ar(\Delta AFD) + ar(\Delta ADC)$

$\Rightarrow ar(\Delta BFE) + \frac{1}{2}ar(\Delta ABC)$ [using part (iii) , and AD is the median of triangle ABC]

$= ar(\Delta BFE) + \frac{1}{2} \times 4ar(\Delta BDE)$ (using part (i))

$= ar(\Delta BFE) = 2ar(\Delta FED) \cdots\cdot (3)$

$ar(\Delta BDE) = ar(\Delta BFE) + ar(\Delta FED)$

$\Rightarrow 2ar(\Delta FED) + ar(\Delta FED)$

$\Rightarrow 3ar(\Delta FED) \cdots\cdots (4)$

From 2, 3 and 4 we get ,

$ar(\Delta AFC) = 2ar(\Delta FED) + 2 \times 3ar(\Delta FED) = 8ar(\Delta FED)$

$ar(\Delta FED) = \frac{1}{8}ar(\Delta AFC)$

(vi) Let h be the height of vertex E, corresponding to the side BD in triangle BDE.

Let H be the height of vertex A, corresponding to the side BC in triangle ABC

From part (i)

$ar(\Delta BDE) = \frac{1}{4}ar(\Delta ABC)$

$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4}(\frac{1}{2} \times BC \times h)$

$\Rightarrow BD \times h = \frac{1}{4}(2BD \times H)$

$\Rightarrow h = \frac{1}{2}H \cdots\cdots\cdot (1)$

From part (iii)

$ar(\Delta BFE) = ar(\Delta AFD)$

$ar(\Delta BFE) = \frac{1}{2} \times FD \times H$

$ar(\Delta BFE) = \frac{1}{2} \times FD \times 2h$

$ar(\Delta BFE) = 2(\frac{1}{2} \times FD \times h)$

$ar(\Delta BFE) = 2ar(\Delta EFD)$

Q 24. D is the midpoint of side BC of ΔABC and E is the midpoint of BD. If O is the midpoint of AE, Prove that $ar(\Delta BOE) = \frac{1}{8}ar(\Delta ABC)$.

Solution:

Given that

D is the midpoint of sides BC of triangle ABC

E is the midpoint of BD and O is the midpoint of AE

Since AD and AE are the medians of triangles, ABC and ABD respectively

$\therefore \operatorname{ar}(\Delta ABD) = \frac{1}{2}\operatorname{ar}(\Delta ABC) \cdots\cdots\cdot (1)$

$\therefore \operatorname{ar}(\Delta ABE) = \frac{1}{2}\operatorname{ar}(\Delta ABD) \cdots\cdots\cdot (2)$

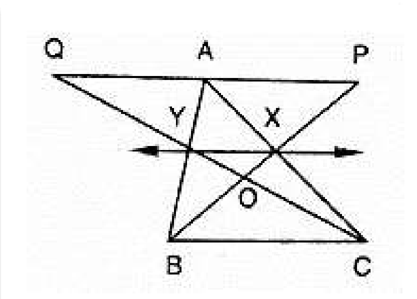
OB is the median of triangle ABE

Therefore, $\therefore \operatorname{ar}(\Delta BOE) = \frac{1}{2}\operatorname{ar}(\Delta ABE)$

From 1, 2 and 3 , we have

$\therefore \operatorname{ar}(\Delta BOE) = \frac{1}{8}\operatorname{ar}(\Delta ABC)$

Q 25. In figure, X and Y are the mid points of AC and AB respectively, QP ||BC and CYQ and BXP are straight lines . Prove that $\operatorname{ar}(\Delta ABP) = \operatorname{ar}(\Delta ACQ)$**.**



Solution:

Since X and Y are the mid points of AC and AB respectively.

Therefore, XY || BC

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels Xy and BC

$\therefore \operatorname{ar}(\Delta BYC) = \operatorname{ar}(\Delta BXC)$

$\Rightarrow \operatorname{ar}(\Delta BYC) - \operatorname{ar}(\Delta BOC) = \operatorname{ar}(\Delta BXC) - \operatorname{ar}(\Delta BOC)$

$\Rightarrow \operatorname{ar}(\Delta BOY) = \operatorname{ar}(\Delta COX)$

$\Rightarrow \operatorname{ar}(\Delta BOY) + \operatorname{ar}(\Delta XOY) = \operatorname{ar}(\Delta COX) + \operatorname{ar}(\Delta XOY)$

$\Rightarrow \operatorname{ar}(\Delta BXY) = \operatorname{ar}(\Delta CXY)$ (2)

We observed that the quadrilaterals XYAP and XYAQ are on the same base XY and between same parallels XY and PQ.

$\therefore \operatorname{ar}(\text{quad. } XYAP) = \operatorname{ar}(\text{quad}XYQA) \cdots\cdots (2)$

Adding 1 and 2 , we get

$\therefore \operatorname{ar}(\Delta BXY) + \operatorname{ar}(\text{quad. } XYAP) = \operatorname{ar}(\Delta CXY) + \operatorname{ar}(\text{quad}XYQA)$

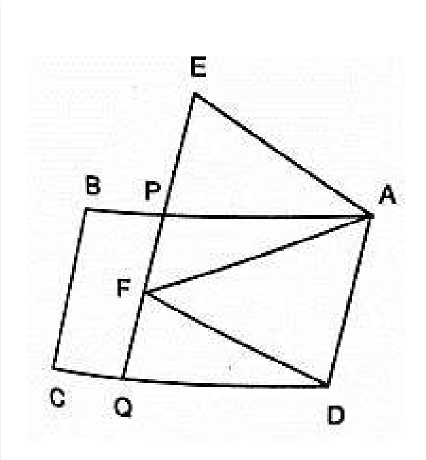
$\Rightarrow \operatorname{ar}(\Delta ABP) = \operatorname{ar}(\Delta ACQ)$

Q 26. In figure, ABCD and AEFD are two parallelograms. Prove that

(i) . PE = FQ

(ii) . $\operatorname{ar}(\Delta APE) : \operatorname{ar}(\Delta PFA) = \operatorname{ar}(\Delta QFD) : \operatorname{ar}(\Delta PFD)$

***(iii)* . ar (ΔPEA)= ar (ΔQFD).**



Solution:

Given that, ABCD and AEFD are two parallelograms

(i) . In triangles, EPA and FQD

$\angle PEA = \angle QFD$ [corresponding angles]

$\angle EPA = \angle FQD$ [corresponding angles]

PA = QD [opposite sides of parallelogram]

Then, $\Delta EPA \cong \Delta FQD$ [By AAS condition]

Therefore, EP = FQ [C.P.C.T]

(ii) . Since triangles, PEA and QFD stand on equal bases PE and FQ lies between the same parallels EQ and AD

Therefore, ar $(\Delta PEA) = \text{ar}(\Delta QFD)$ (1)

Since, triangles PEA and PFD stand on the same base PF and between same parallels PF and AD

Therefore, ar $(\Delta PFA) = \text{ar}(\Delta PFD)$ (2)

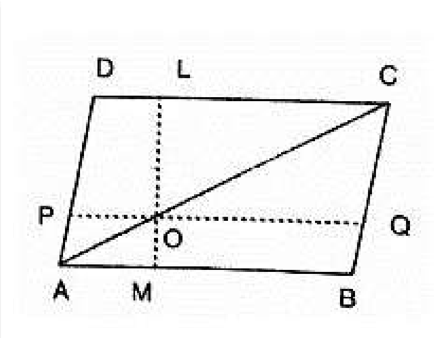
Divide the equation 1 by equation 2

$$\frac{\text{ar}(\Delta PEA)}{\text{ar}(\Delta PFA)} = \frac{\text{ar}(\Delta QFD)}{\text{ar}(\Delta PFD)}$$

(iii) . From part (i), $\Delta EPA \cong \Delta FQD$

Then, ar (ΔPEA)= ar (ΔQFD).

Q 27. In figure, ABCD is a parallelogram . O is any point on AC. PQ ||AB and LM || AD. Prove that :
ar $(||^{gm} DLOP) = \text{ar}(||^{gm} BMOQ)$.



Solution:

Since a diagonal of a parallelogram divides it into two triangles of equal area

Therefore, $\text{ar}(\Delta\text{ADC}) = \text{ar}(\Delta\text{ABC})$

$\Rightarrow \text{ar}(\Delta\text{APO}) + \text{ar}(||^{\text{gm}}\text{DLOP}) + \text{ar}(\Delta\text{OLC})$

$\Rightarrow \text{ar}(\Delta\text{AOM}) + \text{ar}(||^{\text{gm}}\text{BMOQ}) + \text{ar}(\Delta\text{OQC})$ (1)

Since AO and Oc are diagonals of parallelograms AMOP and OQCL respectively.

$\therefore \text{ar}(\Delta\text{APO}) = \text{ar}(\Delta\text{AMO})$ (2)

And $\text{ar}(\Delta\text{OLC}) = \text{ar}(\Delta\text{OQC})$ (3)

Subtracting 2 and 3 from 1, we get

$\text{ar}(||^{\text{gm}}\text{DLOP}) = \text{ar}(||^{\text{gm}}\text{BMOQ})$.

Q 28. In a triangle ABC, if L and M are points on AB and AC respectively such that LM || BC. Prove that:

(i) . $\text{ar}(\Delta\text{LCM}) = \text{ar}(\Delta\text{LBM})$.

(ii) . $\text{ar}(\Delta\text{LBC}) = \text{ar}(\Delta\text{MBC})$.

(iii) . $\text{ar}(\Delta\text{ABM}) = \text{ar}(\Delta\text{ACL})$.

(iv) . $\text{ar}(\Delta\text{LOB}) = \text{ar}(\Delta\text{MOC})$.

Solution:

(i) . Clearly triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

$\therefore \text{ar}(\Delta\text{LMB}) = \text{ar}(\Delta\text{LMC})$ (1)

(ii) . We observe that triangles LBC and MBC are on the same base BC and between same parallels LM and BC.

$\therefore \text{ar}(\Delta\text{LBC}) = \text{ar}(\Delta\text{MBC})$ (2)

(iii) . We have,

$\text{ar}(\Delta\text{LMB}) = \text{ar}(\Delta\text{LMC})$ [From 1]

$\Rightarrow \text{ar}(\Delta\text{ALM}) + \text{ar}(\Delta\text{LMB}) = \text{ar}(\Delta\text{ALM}) + \text{ar}(\Delta\text{LMC})$

$\Rightarrow \text{ar}(\Delta\text{ABM}) = \text{ar}(\Delta\text{ACL})$

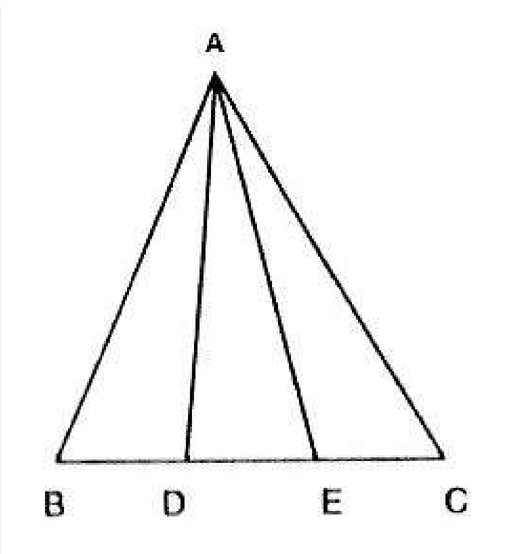
(iv) . We have,

$ar(\Delta LBC) = ar(\Delta MBC)$ [From 1]

$\Rightarrow ar(\Delta LBC) - ar(\Delta BOC) = ar(\Delta MBC) - ar(\Delta BOC)$

$\Rightarrow ar(\Delta LOB) = ar(\Delta MOC)$.

Q 29. In figure, D and E are two points on BC such that BD = DE = EC. Show that $ar(\Delta ABD) = ar(\Delta ADE) = ar(\Delta AEC)$.



Solution:

Draw a line l through A parallel to BC.

Given that, BD = DE = EC

We observed that the triangles ABD and AEC are on the equal bases and between the same parallels l and BC. Therefore, their areas are equal.

Hence, $ar(\Delta ABD) = ar(\Delta ADE) = ar(\Delta AEC)$.

Q 30. In figure, ABC is a right angled triangle at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that

(i) . $\Delta MBC \cong \Delta ABD$

(ii) . $ar(BYXD) = 2ar(\Delta MBC)$

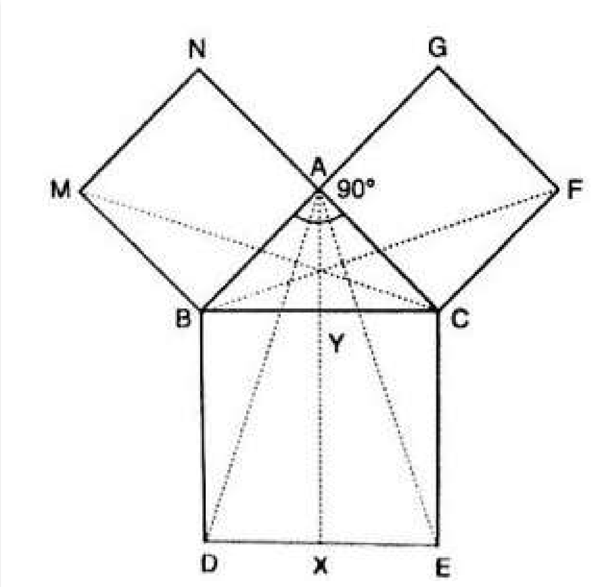
(iii) . $ar(BYXD) = ar(ABMN)$

(iv) . $\Delta FCB \cong \Delta ACE$

(v) . $ar(CYXE) = 2ar(\Delta FCB)$

(vi) . $ar(CYXE) = ar(ACFG)$

(vii) . $ar(BCED) = ar(ABMN) + ar(ACFG)$



Solution:

(i) . In ΔMBC and Δ ABD , we have

MB = AB

BC = BD

And ∠ MBC = ∠ ABD [since ,∠MBC and ∠ABC are obtained by adding ∠ABC to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta MBC \cong \Delta ABD$$

$$\Rightarrow ar(\Delta MBC) = ar(\Delta ABD) \cdots\cdots (1)$$

(ii) . Clearly, triangle ABC and rectangle BYXD are on the same base BD and between the same parallels AX and BD .

$$\therefore ar(\Delta ABD) = \frac{1}{2} ar(\text{rect } BYXD)$$

$$\Rightarrow ar(\text{rect } BYXD) = 2ar(\Delta ABD)$$

$$\Rightarrow ar(\text{rect } BYXD) = 2ar(\Delta MBC) \cdots\cdots\cdots (2)$$ [From equ .1]

(iii) . Since triangles MBC and square MBAN are on the same base Mb and between the same parallels MB and NC.

$$\therefore 2ar(\Delta MBC) = ar(MBAN) \cdots\cdots\cdots (3)$$

From equ. 2 and 3, we have

$$ar(\text{sq. } MBAN) = ar(\text{rect} BYXD)$$

(iv) . In triangles FCB and ACE, we have

FC = AC

CB = CE

And , ∠FCB = ∠ ACE [since ,∠FCB and ∠ACE are obtained by adding ∠ACB to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta FCB \cong \Delta ACE$$

(v) . We have,

$$\Delta FCB \cong \Delta ACE$$

$$\Rightarrow ar(\Delta FCB) = ar(\Delta ACE)$$

Clearly, triangle ACE and rectangle CYXE are on the same base CE and between same parallels CE and AX.

$$\therefore 2ar(\Delta ACE) = ar(CYXE)$$

$$\Rightarrow 2ar(\Delta FCB) = ar(\Delta CYXE) \cdots\cdots\cdot (4)$$

(vi) . Clearly , triangle FCb and rectangle FCAG are on the same base FC and between the same parallels FC and BG.

$$\therefore 2ar(\Delta FCB) = ar(FCAG) \cdots\cdots (5)$$

From 4 and 5, we get

$$ar(CYXE) = ar(ACFG)$$

(vii) . Applying Pythagoras theorem in triangle ACB, we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$

$$\Rightarrow ar(BCED) = ar(ABMN) + ar(ACFG)$$