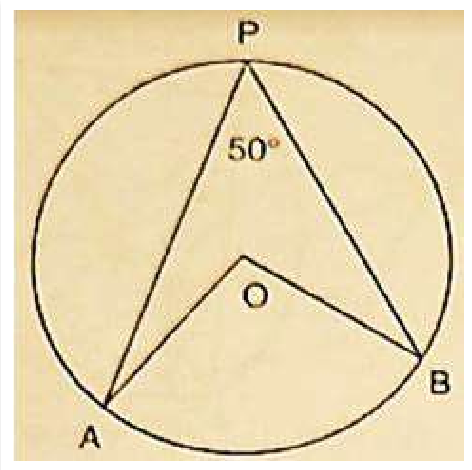
RD SHARMA
Solutions
Class 9 Maths
Chapter 16
Ex *16.4*

Q1) In figure 16.120, O is the centre of the circle. If $\angle APB = 50^0$, find $\angle AOB$ and $\angle OAB$.



Solution:

$\angle APB = 50^0$

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \angle APB = 2 \times 50^0 = 100^0$

since $OA = OB$ [Radius of circle]

Then $\angle OAB = \angle OBA$ [Angles opposite to equal sides]

Let $\angle OAB = x$

In ΔOAB, by angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180^0$

=>x + x + 100^0 = 180^0

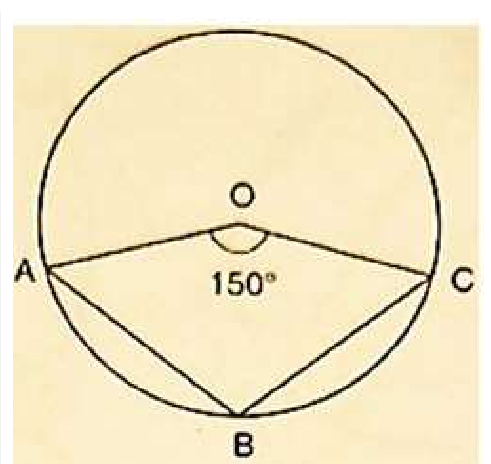
=>2x = 180^0 – 100^0

=>2x = 80^0

=>x = 40^0

$\angle OAB = \angle OBA = 40^0$

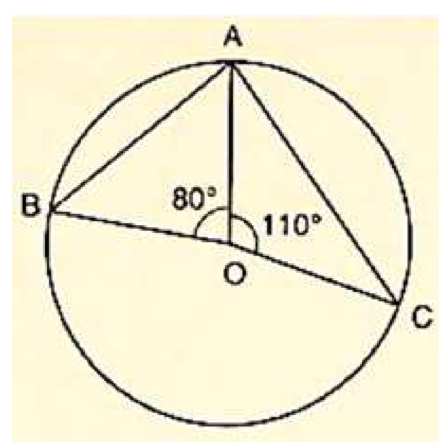
Q2) In figure 16.121, it is given that O is the centre of the circle and $\angle AOC = 150^0$. Find $\angle ABC$.



Solution:

$$\angle AOC = 150^0$$
$$\therefore \angle AOC + \text{reflex } \angle AOC = 360^0 \qquad \text{[Complex angle]}$$
$$\Rightarrow 150^0 + \text{reflex } \angle AOC = 360^0$$
$$\Rightarrow \text{reflex } \angle AOC = 360^0 - 150^0$$
$$\Rightarrow \text{reflex } \angle AOC = 210^0$$
$$\Rightarrow 2\angle ABC = 210^0 \qquad \text{[By degree measure theorem]}$$
$$\Rightarrow \angle ABC = \frac{210^0}{2} = 105^0$$

Q3) In figure 16.22, O is the centre of the circle. Find $\angle BAC$.



Solution:

We have $\angle AOB = 80^0$

And $\angle AOC = 110^0$

Therefore, $\angle AOB + \angle AOC + \angle BOC = 360^0 \qquad \text{[Complete angle]}$

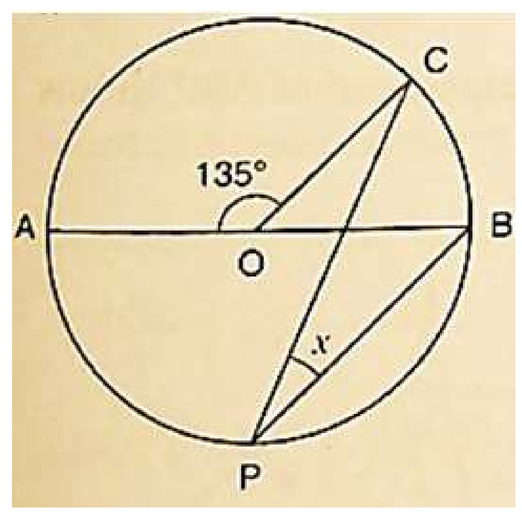
$$\Rightarrow 80^0 + 100^0 + \angle BOC = 360^0$$
$$\Rightarrow \angle BOC = 360^0 - 80^0 - 110^0$$
$$\Rightarrow \angle BOC = 170^0$$

By degree measure theorem

$$\angle BOC = 2\angle BAC$$
$$\Rightarrow 170^0 = 2\angle BAC$$
$$\Rightarrow \angle BAC = \frac{170^0}{2} = 85^0$$

Q4) If O is the centre of the circle, find the value of x in each of the following figures.

(i)



Solution:

$\angle AOC = 135^0$

$\therefore \angle AOC + \angle BOC = 180^0$ [Linear pair of angles]

$\Rightarrow 135^0 + \angle BOC = 180^0$

$\Rightarrow \angle BOC = 180^0 - 135^0$

$\Rightarrow \angle BOC = 45^0$

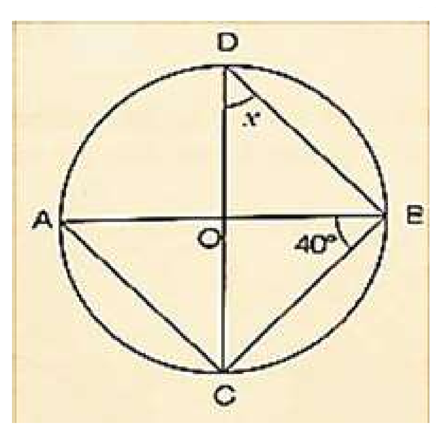
By degree measure theorem

$\angle BOC = 2\angle CPB$

$\Rightarrow 45^0 = 2x$

$\Rightarrow x = \frac{45^0}{2} = 22\frac{1}{2}^0$

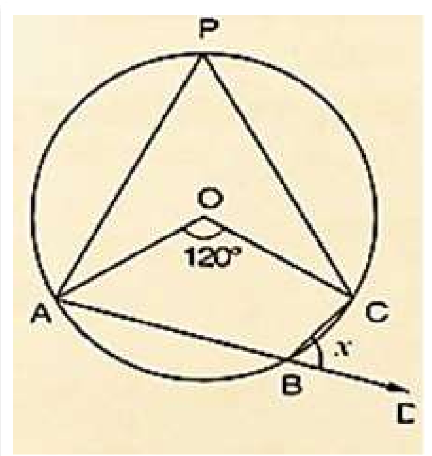
(ii)



Solution:

We have
$\angle ABC = 40^0$
$\angle ACB = 90^0$ [Angle in semicircle]
In ΔABC, by angle sum property
$\angle CAB + \angle ACB + \angle ABC = 180^0$
$\Rightarrow \angle CAB + 90^0 + 40^0 = 180^0$
$\Rightarrow \angle CAB = 180^0 - 90^0 - 40^0$
$\Rightarrow \angle CAB = 50^0$
Now,
$\angle CDB = \angle CAB$ [Angle is same in segment]
$\Rightarrow x = 50^0$

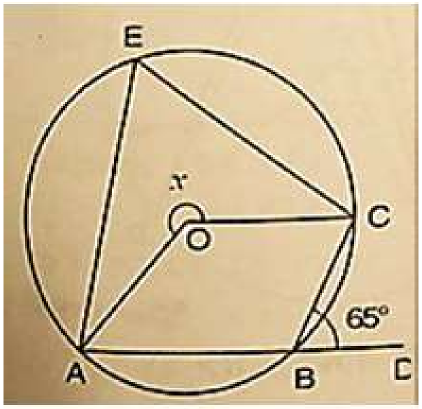
(iii)



Solution:

We have
$\angle AOC = 120^0$
By degree measure theorem.
$\angle AOC = 2\angle APC$
$\Rightarrow 120^0 = 2\angle APC$
$\Rightarrow \angle APC = \frac{120^0}{2} = 60^0$
$\angle APC + \angle ABC = 180^0$ [Opposite angles of cyclic quadrilaterals]
$\Rightarrow 60^0 + \angle ABC = 180^0$
$\Rightarrow \angle ABC = 180^0 - 60^0$
$\Rightarrow \angle ABC = 120^0$
$\therefore \angle ABC + \angle DBC = 180^0$ [Linear pair of angles]
$\Rightarrow 120 + x = 180^0$
$\Rightarrow x = 180^0 - 120^0 = 60^0$

(iv)



Solution:

We have

$\angle CBD = 65^0$

$\therefore \angle ABC + \angle CBD = 180^0$ [Linear pair of angles]

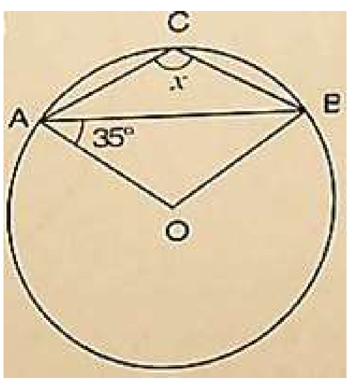
$\Rightarrow \angle ABC = 65^0 = 180^0 \Rightarrow \angle ABC = 180^0 - 65^0 = 115^0$

$\therefore$ reflex $\angle AOC = 2\angle ABC$ [By degree measure theorem]

$\Rightarrow x = 2 \times 115^0$

$\Rightarrow x = 230^0$

(v)



Solution:

We have

$\angle OAB = 35^0$

Then, $\angle OBA = \angle OAB = 35^0$ [Angles opposite to equal radii]

In ΔAOB, by angle sum property

$\Rightarrow \angle AOB + \angle OAB + \angle OBA = 180^0$

$\Rightarrow \angle AOB + 35^0 + 35^0 = 180^0$

$\Rightarrow \angle AOB = 180^0 - 35^0 - 35^0 = 110^0$

$\therefore \angle AOB + \text{reflex}\angle AOB = 360^0$ [Complex angle]

$\Rightarrow 110^0 + \text{reflex}\angle AOB = 360^0$

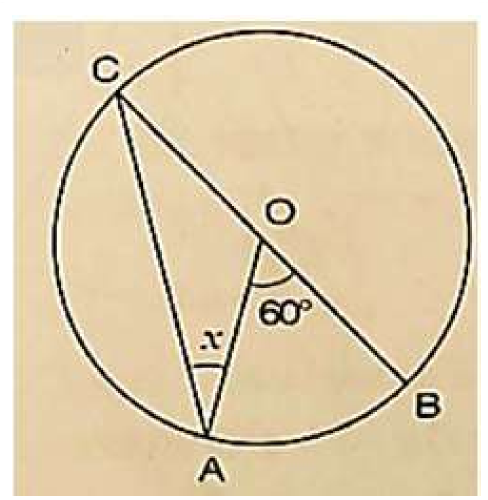
$\Rightarrow \text{reflex}\angle AOB = 360^0 - 110^0 = 250^0$

By degree measure theorem reflex$\angle AOB = 2\angle ACB$

$\Rightarrow 250^0 = 2x$

$\Rightarrow x = \frac{250^0}{2} = 125^0$

(vi)



Solution:

We have

$\angle AOB = 60^0$

By degree measure theorem reflex

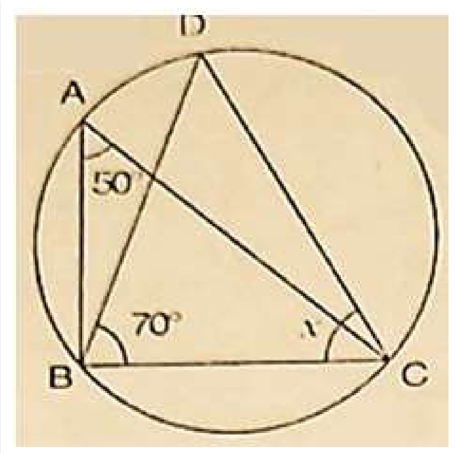
$\angle AOB = 2\angle ACB$

$\Rightarrow 60^0 = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{60^0}{2} = 30^0$ [Angles opposite to equal radii]

$\Rightarrow x = 30^0.$

(vii)



Solution:

We have

$\angle BAC = 50^0$ and $\angle DBC = 70^0$

$\therefore \angle BDC = \angle BAC = 50^0$ [Angle in same segment]

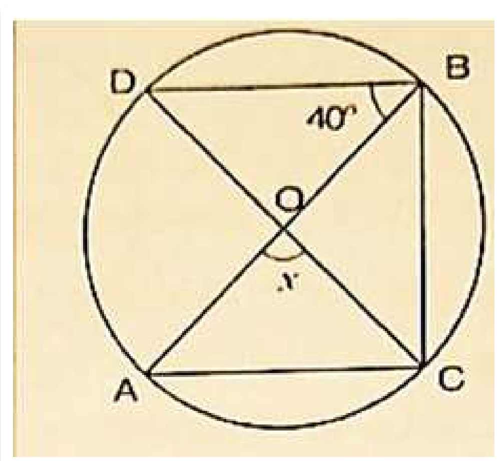
In ΔBDC, by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180^0$

$\Rightarrow 50^0 + x + 70^0 = 180^0$

$\Rightarrow x = 180^0 - 50^0 - 70^0 = 60^0$

(viii)



Solution:

We have,

$\angle DBO = 40^0$ and $\angle DBC = 90^0$ [Angle in a semi circle]

$\Rightarrow \angle DBO + \angle OBC = 90^0$

$\Rightarrow 40^0 + \angle OBC = 90^0$

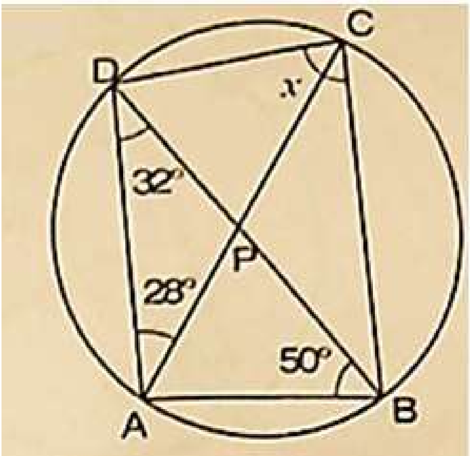
$\Rightarrow \angle OBC = 90^0 - 40^0 = 50^0$

By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow x = 2 \times 50^0 = 100^0$

(ix)



Solution:

In ΔDAB, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180^0$

$\Rightarrow 32^0 + \angle DAB + 50^0 = 180^0$

$\Rightarrow \angle DAB = 180^0 - 32^0 - 50^0$

$\Rightarrow \angle DAB = 98^0$

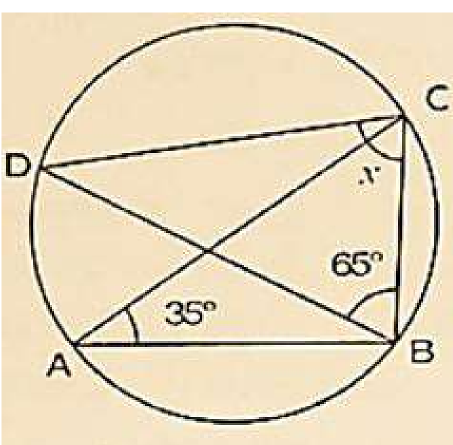
Now,

$\angle OAB + \angle DCB = 180^0$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 98^0 + x = 180^0$

$\Rightarrow x = 180^0 - 98^0 = 82^0$

(x)



Solution:

We have,

$\angle BAC = 35^0$

$\angle BDC = \angle BAC = 35^0$ [Angle in same segment]

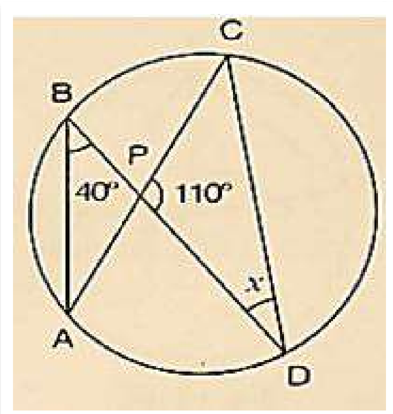
In ΔBCD, by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180^0$

$\Rightarrow 35^0 + x + 65^0 = 180^0$

$\Rightarrow x = 180^0 - 35^0 - 65^0 = 80^0$

(xi)



Solution:

We have,

$\angle ABD = 40^0$

$\angle ACD = \angle ABD = 40^0$ [Angle in same segment]

In ΔPCD, by angle sum property

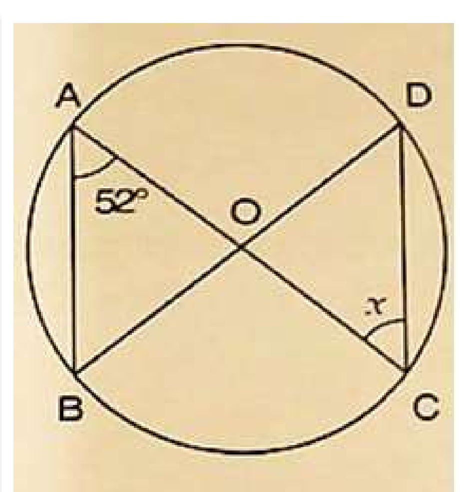
$\angle PCD + \angle CPO + \angle PDC = 180^0$

$\Rightarrow 40^0 + 110^0 + x = 180^0$

$\Rightarrow x = 180^0 - 150^0$

$\Rightarrow x = 30^0$

(xii)



Solution:

Given that,

$\angle BAC = 52^0$

Then $\angle BDC = \angle BAC = 52^0$ [Angle in same segment]

Since OD = OC

Then $\angle ODC = \angle OCD$ [Opposite angle to equal radii]

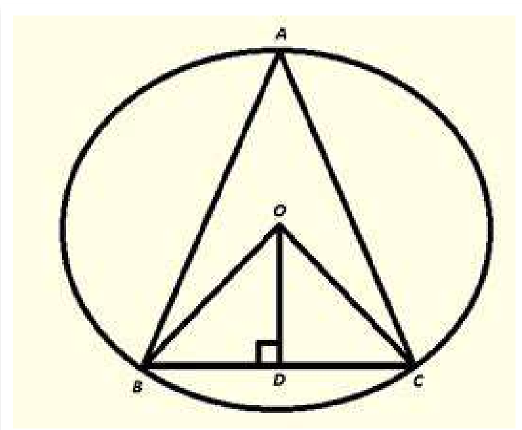
$\Rightarrow x = 52^0$

Q5) O is the circumference of the triangle ABC and Odis perpendicular on BC. Prove that $\angle BOD = \angle A$.

Solution:

Given O is the circum centre of triangle ABC and $OD \perp BC$

To prove $\angle BOD = 2\angle A$



Proof:

In

ΔOBD and ΔOCD

$\angle ODB = \angle ODC$ [Each 90^0]

OB = OC [Radius of circle]

OD = OD [Common]

Then $\Delta OBD \cong \Delta OCD$ *[By RHS Condition].*

$\therefore \angle BOD = \angle COD \ldots\ldots(i)$ [PCT].

By degree measure theorem

$\angle BOC = 2\angle BAC$

$\Rightarrow 2\angle BOD = 2\angle BAC$ [By using (i)]

$\Rightarrow \angle BOD = \angle BAC$

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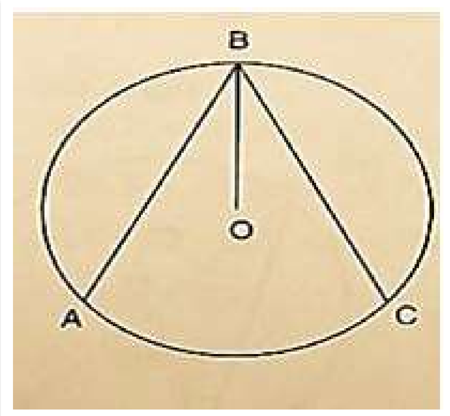
Q6) In figure 16.135, O is the centre of the circle, BO is the bisector of $\angle ABC$. Show that AB = AC.



Solution:

Given, BO is the bisector of $\angle ABC$

To prove AB = BC

Proof:

Since, BO is the bisector of $\angle ABC$.

Then, $\angle ABO = \angle CBO \ldots..(i)$

Since, OB = OA [Radius of circle]

Then, $\angle ABO = \angle DAB \ldots..(ii)$ [opposite angles to equal sides]

Since OB = OC [Radius of circle]

Then, $\angle OAB = \angle OCB \ldots..(iii)$ [opposite angles to equal sides]

Compare equations (i), (ii) and (iii)

$\angle OAB = \angle OCB \ldots..(iv)$

In ΔOAB and ΔOCB

$\angle OAB = \angle OCB$ [From (iv)]

$\angle OBA = \angle OBC$ [Given]

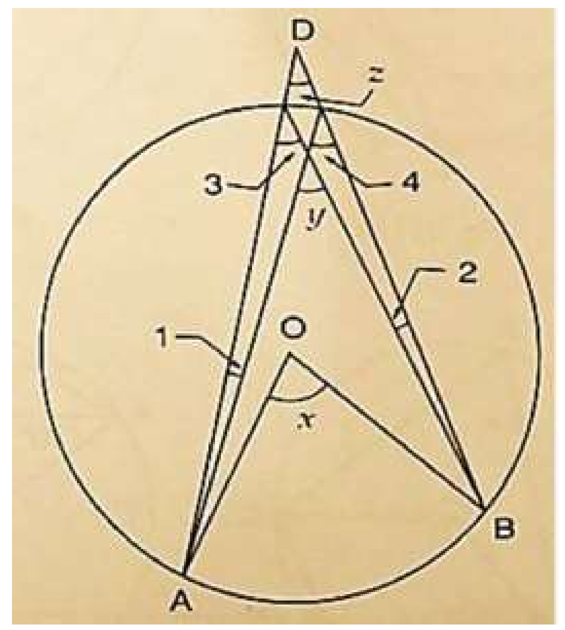
OB = OB [Common]

Then,

$$\Delta OAB \cong \Delta OCB \quad [\text{By AAS condition}]$$

$$\therefore AB = BC \quad [CPCT]$$

Q7) In figure 16.136, O is the centre of the circle, then prove that $\angle x = \angle y + \angle z$.



Solution:

We have,

$\angle 3 = \angle 4$ [Angles in same segment]

$\therefore \angle x = 2\angle 3$ [By degree measure theorem]

$\Rightarrow \angle x = \angle 3 + \angle 3 \Rightarrow \angle x = \angle 3 + \angle 4 \ldots\ldots(i)$ [$\angle 3 =$ angle4]

But $\angle y = \angle 3 + \angle 1$ [By exterior angle property]

$\Rightarrow \angle 3 = \angle y - \angle 1 \ldots\ldots(ii)$

from (i) and (ii)

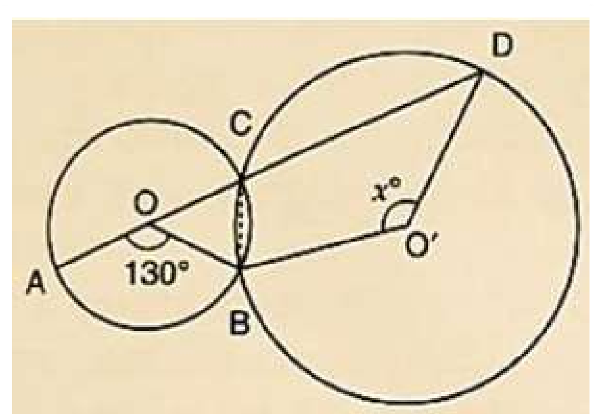
$\angle x = \angle y - \angle 1 + \angle 4$

$\Rightarrow \angle x = \angle y + \angle 4 - \angle 1$

$\Rightarrow \angle x = \angle y + \angle z + \angle 1 - \angle 1$ [By exterior angle property]

$\Rightarrow \angle x = \angle y + \angle z$

Q8) In figure 16.137, O and O' are centers of two circles intersecting at B and C. ACD is a straight line, find x.



Solution:

By degree measure theorem

$$\angle AOB = 2\angle ACB$$

$$\Rightarrow 130^0 = 2\angle ACB \Rightarrow \angle ACB = \frac{130^0}{2} = 65^0$$

$$\therefore \angle ACB + \angle BCD = 180^0 \quad \text{[Linera pair of angles]}$$

$$\Rightarrow 65^0 + \angle BCD = 180^0$$

$$\Rightarrow \angle BCD = 180^0 - 65^0 = 115^0$$

By degree measure theorem

$$\text{reflex}\angle BOD = 2\angle BCD$$

$$\Rightarrow \text{reflex}\angle BOD = 2 \times 115^0 = 230^0$$

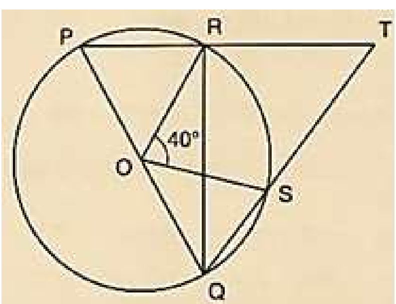
$$\text{Now, reflex}\angle BOD + \angle BO'D = 360^0 \quad \text{[Complex angle]}$$

$$\Rightarrow 230^0 + x = 360^0$$

$$\Rightarrow x = 360^0 - 230^0$$

$$\therefore x = 130^0$$

Q9) In figure 16.138, O is the centre of a circle and PQ is a diameter. If $\angle ROS = 40^0$, find $\angle RTS$..



Solution:

Since PQ is diameter

Then,

$$\angle PRQ = 90^0 \quad \text{[Angle in semi circle]}$$

$$\therefore \angle PRQ + \angle TRQ = 180^0 \quad \text{[Linear pair of angle]}$$

$$90^0 + \angle TRQ = 180^0$$

$$\angle TRQ = 180^0 - 90^0 = 90^0.$$

By degree measure theorem

$$\angle ROS = 2\angle RQS$$

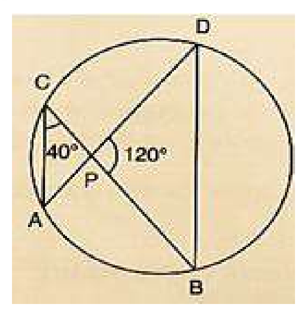
$$\Rightarrow 40^0 = 2\angle RQS$$

$$\Rightarrow \angle RQS = \frac{40^0}{2} = 20^0$$

In ΔRQT, by Angle sum property

$$\angle RQT + \angle QRT + \angle RTS = 180^0$$
$$\Rightarrow 20^0 + 90^0 + \angle RTS = 180^0$$
$$\Rightarrow \angle RTS = 180^0 - 20^0 - 90^0 = 70^0$$

Q10) In figure 16.139, if $\angle ACB = 40^0,\ \angle DPB = 120^0,$ find $\angle CBD.$



Solution:

We have,

$$\angle ACB = 40^0;\ \angle DPB = 120^0$$

$$\therefore \angle APB = \angle DCB = 40^0 \qquad \text{[Angle in same segment]}$$

In ΔPOB, by angle sum property

$$\angle PDB + \angle PBD + \angle BPD = 180^0$$
$$\Rightarrow 40^0 + \angle PBD + 120^0 = 180^0$$
$$\Rightarrow \angle PBD = 180^0 - 40^0 - 120^0$$
$$\Rightarrow \angle PBD = 20^0$$
$$\therefore \angle CBD = 20^0$$

Q11) A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

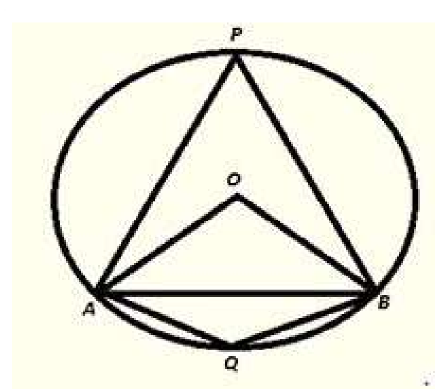
Solution:

We have,

Radius OA = Chord AB

=>OA = OB = AB

Then triangle OAB is an equilateral triangle.



$\therefore \angle AOB = 60^0$ [one angle of equilateral triangle]

By degree measure theorem

$$\angle AOB = 2\angle APB$$
$$\Rightarrow 60^0 = 2\angle APB$$
$$\Rightarrow \angle APB = \frac{60^0}{2} = 30^0$$

Now, $\angle APB + \angle AQB = 180^0$ [opposite angles of cyclic quadrilateral]

$$\Rightarrow 30^0 + \angle AQB = 180^0$$
$$\Rightarrow \angle AQB = 180^0 - 30^0 = 150^0.$$

Therefore, Angle by chord AB at minor arc = 150^0

Angle by chord AB at major arc = 30^0