RD SHARMA

Solutions

Class 9 Maths

Chapter 21

Ex *21.2*

Q1. Find the volume of a sphere whose radius is: (i) 2 cm (ii) 3.5 cm (iii) 10.5 cm.

Sol.

(i) Radius(r)= 2cm

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (2)^3$

=33.52cm^3

(ii) Radius(r)= 3.5cm

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (3.5)^3$ = 179.666cm^3

(iii) Radius(r)= 10.5cm

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (10.5)^3$ = 4851cm^3

Q2. Find the volume of a sphere whose diameter is: (i) 14 cm (ii) 3.5 dm (iii) 2.1 m

Sol.

(i) Diameter=14cm, Radius(r)= $\frac{14}{2}$= 7cm

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (7)^3$ = 1437.33cm^3

(ii) Diameter=3.5dm, Radius(r)= $\frac{3.5}{2}$= 1.75dm

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (1.75)^3$

=22.46dm^3

(iii) Diameter=2.1m, Radius(r)= $\frac{2.1}{2}$= 1.05m

Therefore volume=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \frac{22}{7} \times (1.05)^3$ = 4.851m^3

Q3. A hemispherical tank has the inner radius of 2.8 m. Find its capacity in liters.

Sol.

Radius of the tank= 2.8m

Therefore Capacity= $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (2.8)^3 = 45.994 m^3$

$1m^3 = 1000l$

Therefore capacity in litres = 45994 litres

Q4. A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Sol.

Inner radius = 5cm

Outer radius = 5 + 0.25 = 5.25

Volume of steel used= Outer volume-Inner volume

$= \frac{2}{3} \times \pi \times ((5.25)^3 - (5)^3)$

$= \frac{2}{3} \times \frac{22}{7} \times ((5.25)^3 - (5)^3)$

$= 41.282 cm^3$

Q5. How many bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Sol.

Cube edge= 22cm

Therefore volume of the cube= $(22)^3$ = $10648 cm^3$

And,

Volume of each bullet=

$$\frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (1)^3$$

$$= \frac{4}{3} \times \frac{22}{7}$$

$$= \frac{88}{21} cm^3$$

Number of bullets=

$$\frac{\text{Volume of cube}}{\text{Volume of bullet}}$$

$$= \frac{10648}{\frac{88}{21}}$$

$$= 2541$$

Q6. A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made?

Sol.

Volume of laddoo having radius= 5cm

i.e Volume$(V_1) = \frac{4}{3}\pi r^3$

$(V_1) = \frac{4}{3} \times \frac{22}{7} \times (5)^3$

$(V_1) = \frac{11000}{21}\ cm^3$

Also Volume of laddoo having radius 2.5cm

i.e Volume$(V_2) = \frac{4}{3}\pi r^3$

$(V_2) = \frac{4}{3} \times \frac{22}{7} \times (2.5)^3$

$(V_2) = \frac{1375}{21}\ cm^3$

Therefore number of laddoos$= \frac{V_1}{V_2} = \frac{11000}{1375} = 8$

Q7. A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be $\frac{3}{2}$ cm and 2 cm, find the diameter of the third ball.

Sol.

Volume of lead ball$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (\frac{3}{2})^3$

Diameter of first ball$d_1 = \frac{3}{2}$cm

Radius of first ball$r_1 = \frac{\frac{3}{2}}{2} = \frac{3}{4}$cm

Diameter of second balld_2 = 2cm

Radius of second ball$r_2 = \frac{2}{2}$cm = 1cm

Diameter of third balld_3 = d

Radius of third ball$r_3 = \frac{d}{2}$cm

Volume of lead ball$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$

Volume of lead ball$= \frac{4}{3} \times \pi \times (\frac{3}{4})^3 + \frac{4}{3} \times \pi \times (\frac{2}{2})^3 + \frac{4}{3} \times \pi \times (\frac{d}{2})^3$

$\frac{4}{3} \times \frac{22}{7} \times (\frac{3}{2})^3 = \frac{4}{3} \times \pi \times (\frac{3}{4})^3 + \frac{4}{3} \times \pi \times (\frac{2}{2})^3 + \frac{4}{3} \times \pi \times (\frac{d}{2})^3$

$\frac{4}{3}\pi [(\frac{3}{2})^3] = \frac{4}{3}\pi [(\frac{3}{4})^3 + (\frac{2}{2})^3 + (\frac{d}{2})^3]$

$\frac{27}{8} = \frac{27}{64} + 1 + \frac{d^3}{8}$

$d^3 = 8 [\frac{27}{8} - \frac{27}{64} - 1]$

$\frac{d^3}{8} = \frac{125}{64}$

$\frac{d}{2} = \frac{5}{4}$

$d = \frac{10}{4}$

d=2.5cm

Q8. A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$cm. Find the radius of the cylinder.

Sol.

Radius of cylinder=r

Radius of sphere= 5cm

Volume of sphere=$\frac{4}{3}\pi r^3$

=$\frac{4}{3} \times \pi \times (5)^3$

Height of water rised= $\frac{5}{3}$cm

Volume of water rised in cylinder=$\pi r^2 h$

Therefore, Volume of water rises in cylinder= Volume of sphere

Let r be the radius of the cylinder

$\pi r^2 h = \frac{4}{3}\pi r^3$

$r^2 \times \frac{5}{3} = \frac{4}{3} \times \pi \times (5)^3$

$r^2 \times \frac{5}{3} = \frac{4}{3} \times \frac{22}{7} \times 125$

$r^2 = 20 \times 5$

$r = \sqrt{100}$

r = 10cm

Q9. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Sol.

Let v_1 and v_2 be the volumes of the first and second sphere respectively

Radius of the first sphere = r

Radius of the second sphere = 2r

Therefore, $\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (2r)^3}$

= $\frac{1}{8}$

Q10. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol.

Given that

Volume of the cone= Volume of the hemisphere

$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$r^2 h = 2r^3$

h=2r

$\frac{h}{r} = \frac{1}{1} \times 2 = 2$

Therefore

Ratio of their heights= 2:1

Q11. A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Sol.

Given that

Volume of water in the hemispherical bowl= Volume of water in the cylinder

Let h be the height to which water rises in the cylinder

Inner radii of the bowl= r_1 =3.5cm

Inner radii of the bowl= r_2 =7cm

$\frac{2}{3}\pi r_1^3 = \pi r_2^2 h$

$h = \frac{2r_1^3}{3r_2^2}$

$h = \frac{2(3.5)^3}{3(7^2)}$

$h = \frac{7}{12}\text{cm}$

Q12. A cylinder whose height is two thirds of its diameter has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Sol.

Given that

Height of the cylinder= $\frac{2}{3}\text{diameter}$

We know that

Diameter= 2(radius)

$h = \frac{2}{3} \times 2r = \frac{4}{3}r$

Volume of the cylinder=Volume of the sphere

$\pi r^2 h = \frac{4}{3}\pi r^3$

$\pi \times r^2 \times (\frac{4}{3}r) = \frac{4}{3}\pi(4)^3$

$(r)^3 = (4)^3$

r = 4cm

Q13. A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Sol.

It is given that

Volume of water in hemispherical bowl= Volume of cylinder

$\frac{2}{3}\pi r_1^3 = \pi r_2^2 h$

$\frac{2}{3}\pi(6)^3 = \pi(4)^2 h$

$h = \frac{2}{3} \times \frac{6\times6\times6}{4\times4}$

h = 9cm

Q14. A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Sol.

Let r be the radius of the iron ball

Radius of the cylinder=16cm

Then,

Volume of iron ball= Volume of water raised in the hub

$\frac{4}{3}\pi r^3 = \pi r^2 h$

$\frac{4}{3}r^3 = (16)^2 \times 9$

$r^3 = \frac{27\times16\times16}{4}$

$r^3 = 1728$

r = 12cm

Therefore radius of the ball=12cm.

Q15. A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball. (Use = $\frac{22}{7}$).

Sol.

Given that:

Radius of the cylinder = 12cm = r_1

Raised in raised = 6.75 cm = r_2

Volume of water raised = Volume of the sphere

= $\pi r_1^2 h = \frac{4}{3}\pi r_2^3$

= $12 \times 12 \times 6.75 = \frac{4}{3} r_2^3$

= $r_2^3 = \frac{12 \times 12 \times 6.75 \times 3}{4}$

= $r_2^3 = 729$

= $r_2 = 9cm$

Radius of the sphere is 9cm

Q16. The diameter of a copper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.

Sol.

Given that diameter of a copper sphere=18cm

Radius of the sphere= 9cm

Length of the wire=108m=10800cm

Volume of cylinder= Volume of sphere

$$\pi r_1^2 h = \frac{4}{3}\pi r_2^3$$
$$r_1^2 \times 10800 = \frac{4}{3} \times 9 \times 9 \times 9$$
$$r_1^2 = 0.009$$
$$r_1 = 0.3cm$$

Therefore $\text{Diameter} = 2 \times 0.3 = 0.6cm$

Q17. A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimeters?

Sol.

Given that,

Radius of the cylinder jar=6cm=r_1

Level to be rised=2cm

Radius of each iron sphere=1.5cm=r_2

$$\text{Number of sphere} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3}$$

$$= \frac{r_1^2 h}{\frac{4}{3} r_2^3}$$

$$= \frac{6\times6\times2}{\frac{4}{3}\times1.5\times1.5\times1.5}$$

$$\text{Number of sphere} = 16$$

Q18. A measuring jar of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely. What will be the change in the level of water in the jar?

Sol.

Given that,

Diameter of jar=10cm

Radius of jar=5cm

Let the level of water be raised by h

Diameter of the spherical bowl=2cm

Radius of the ball=1cm

Volume of jar=4(Volume of spherical ball)

$$\pi r_1^2 h = 4\left(\frac{4}{3}\pi r_2^3\right)$$

$$r_1^2 h = 4\left(\frac{4}{3} r_2^3\right)$$

$$5 \times 5 \times h = 4 \times \frac{4}{3} r_2^3$$

$$5 \times 5 \times h = 4 \times \frac{4}{3} \times 1 \times 1 \times 1$$

$$h = \frac{4\times4\times1}{3\times5\times5}$$

$$h = \frac{16}{75}\text{cm}$$

$$\text{Heightofwaterinjar} = \frac{16}{75}\text{cm}$$

Q19. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Sol.

Given that

Diameter of sphere=6cm

$$\text{Radius of sphere} = \frac{d}{2} = \frac{6}{2} = 3\text{cm} = r_1$$

$$\text{Diameter of the wire} = 0.2\text{cm}$$

$$\text{Radius of the wire} = 0.1\text{cm} = r_2$$

Volume of sphere=Volume of wire

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$= \frac{4}{3} \times 3 \times 3 \times 3 = 0.1 \times 0.1 \times h$$

$$h = \frac{4 \times 3 \times 3}{0.1 \times 0.1}$$

$$h = 3600\text{cm}$$

$$h = 36\text{m}$$

Therefore length of wire=36m

Q20. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height $22/3$cm. Find the diameter of the cylinder.

Sol.

Given that,

Internal radius of the sphere=3cm=r_1

External radius of the sphere=5cm=r_2

Height of the cylinder=$\frac{8}{3}\text{cm} = h$

Volume of the spherical shell = Volume of cylinder

$$\frac{4}{3}\pi(r_2^3 - r_1^3) = \pi r_3^2 h$$

$$\frac{4}{3}(5^3 - 3^3) = \frac{8}{3}r_3^2$$

$$r_3^2 = \frac{4 \times 98 \times 3}{3 \times 8}$$

$$r_3 = \sqrt{49}$$

$$r_3 = 7\text{cm}$$

Therefore diameter of the cylinder=2(radius)=14cm

Q21. A hemisphere of the lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Sol.

Given

Radius of the hemisphere=Volume of cone

$$\frac{2}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$
$$\frac{2}{3} \times 7^3 = \frac{1}{3} r_2^2 \times 49$$
$$r_2^2 = \frac{2\times7\times7\times7\times3}{3\times49}$$
$$r_2^2 = \frac{2058}{147}$$
$$r_2 = 3.47\text{cm}$$

Therefore radius of the base=3.74cm

Q22. A hollow sphere of internal and external radii 2cm and 4 cm respectively is melted into a cone of base radius 4cm. Find the height and slant height of the cone.

Sol.

Given that

Hollow sphere external radii=r_2=4cm

Internal radii=r_1=2cm

Cone base radius(R)=4cm

Height=h

Volume of cone=Volume of sphere

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi (r_2^2 - r_1^2)$$
$$4^2 h = 4 (4^3 - 2^3)$$
$$h = \frac{4\times56}{16}$$
$$h = 14\text{cm}$$
$$\text{Slant height}\,(l) = \sqrt{r^2 + h^2}$$
$$\text{Slant height}\,(l) = \sqrt{r_2^2 + h^2}$$
$$l = \sqrt{4^2 + (14^2)}$$
$$l = \sqrt{16 + 196}$$
$$l = \sqrt{212}$$
$$l = 14.56\text{cm}$$

Q23. A metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained.

Sol.

Given that

Metallic sphere of radius=10.5cm

Cone radius=3.5cm

Height of radius=3cm

Let the number of cones obtained be x

$$v_S = x \times v_{cone}$$

$$\frac{4}{3}\pi r^3 = x \times \frac{1}{3}\pi r^2 h$$

$$x = \frac{4\times10.5\times10.5\times10.5}{3.5\times3.5\times3}$$

$$x = 126$$

Therefore number of cones=126

Q24. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol.

Given that

A cone and a hemisphere have equal bases and volumes

$$v_{cone} = v_{hemisphere}$$

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$r^2 h = 2r^3$$

$$h = 2r$$

$$\frac{h}{r} = \frac{2}{1}$$

$$h : r = 2 : 1$$

Therefore the ratio is 2:1

Q25. A cone, a hemisphere, and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1 : 2 : 3.

Sol.

Given that

A cone, a hemisphere and a cylinder stand on one equal bases and have the same weight

We know that

$$v_{cone} : v_{hemisphere} : v_{cylinder}$$

$$\frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$$

multiplying by 3

$$\pi r^2 h : 2\pi r^3 : 3\pi r^2 h$$

$$\pi r^3 : 2\pi r^3 : 3\pi r^3 \ (\because r = h \text{ and } r^2 h = r^3)$$

$$1 : 2 : 3$$

Therefore the ratio is 1 : 2 : 3

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Q26. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Sol.

Radius of cylindrical tub=12cm

Depth=20cm

Let r be the radius of the ball

Then

Volume of the ball= Volume of water raised

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$r^3 = \frac{3.14\times(12)^2\times 6.75\times 3}{4}$$

$$r^3 = 729$$

$$r = \sqrt[3]{729}$$

$$r = 9cm$$

Therefore radius of the ball=9cm

Q27. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere.

Sol.

Side of cube=10.5cm

Volume of sphere=v

Diameter of the largest sphere=10.5cm

2r=10.5

r=5.25cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 5.25\times 5.25\times 5.25$$

$$v = \frac{11\times 441}{8} cm^3$$

$$v = 606.375 cm^3$$

Q28. A sphere, a cylinder, and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes.

Sol.

Let r be the common radius

Height of the cone=height of the cylinder=2r

Let

v_1 =Volume of sphere=$\frac{4}{3}\pi r^3$

v_1 =Volume of cylinder=$\pi r^2 h$= $\pi r^2 \times 2r$

v_1 =Volume of cone=$\frac{1}{3}\pi r^2 h$= $\frac{1}{3}\pi r^3$

Now

$$v_1 : v_2 : v_3 = \frac{4}{3}\pi r^3 : 2\pi r^3 : \frac{2}{3}\pi r^3$$
$$= 4 : 6 : 2$$
$$= 2 : 3 : 1$$

Q29. A cube of side 4 cm contains a sphere touching its side. Find the volume of the gap in between.

Sol.

It is given that

Cube side=4cm

Volume of cube=$(4\text{cm})^3$=64cm^3

Diameter of the sphere= Length of the side of the cube=4cm

Therefore radius of the sphere=2cm

Volume of the sphere=$\frac{4}{3}\pi r^3$=

$$\frac{4}{3} \times \frac{22}{7} \times (2)^3 = 33.52\text{cm}^3$$

Volume of gap=Volume of cube-Volume of sphere

=64cm^3-33.52cm^3=30.48cm^3

Q30. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol.

Given that,

Inner radius of the hemispherical tank=1m=r_1

Thickness of the hemispherical tank=1cm=0.01m

Outer radius of hemispherical tank=(1+0.01)=1.01m==r_2

Volume of iron used to make the tank=

$$\frac{2}{3}\pi(r_2^3 - r_1^3)$$
$$= \frac{2}{3} \times \frac{22}{7}[(1.01)^3 - 1^3]$$
$$= \frac{44}{21}[(1.0303) - 1]\,\text{m}^3$$
$$= 0.06348\text{m}^3$$

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Q31. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (mm^3) is needed to fill this capsule?

Sol.

Given that

Diameter of capsule=3.5mm

Radius=$\frac{3.5}{2}$=1.75mm

Volume of spherical sphere=

$$\frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (1.75)^3$$

$$= 22.458 mm^3$$

Therefore $22.46 mm^3$ of medicine is required

Q32. The diameter of the moon is approximately one-fourth of the diameter of the earth. What is the earth the volume of the moon?

Sol.

Diameter of moon= $\frac{1}{4}$thdiameter of earth

Let the diameter of earth be d, so radius=$\frac{d}{2}$

Then diameter of moon=$\frac{d}{4}$

Radius=$\frac{\frac{d}{2}}{4}$= $\frac{d}{8}$

Volume of moon=$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{d}{8})^3 = \frac{4}{3} \times \frac{1}{512}\pi d^3$

Volume of earth=$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{d}{2})^3 = \frac{4}{3} \times \frac{1}{8}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{4}{3}\times\frac{1}{512}\pi d^3}{\frac{4}{3}\times\frac{1}{8}\pi d^3}$$

= $\frac{1}{64}$

Thus the volume of the moon is $\frac{1}{64}$ of volume of earth