RD SHARMA

Solutions

Class 8 Maths

Chapter 3

Ex 3.1

1.) Which of the following numbers are perfect squares?

(i) 484

$484 = 22^2$

(ii) 625

$625 = 25^2$

(iii)576

$576 = 24^2$

(iv) 941

Perfect squares closest to 941 are 900 (30^2) and 961 (31^2). Since 30 and 31 are consecutive numbers, there are no perfect squares between 900 and 961. Hence, 941 is not a perfect square.

(v) 961

$961 = 31^2$

(vi)2500

$2500 = 50^2$

Hence, all numbers except that in (iv), i.e. 941, are perfect squares.

2.) Show that each of the following numbers is a perfect square. Also, find the number whose square is the given number in each case:

Answer:

In each problem, factorize the number into its prime factors.

(i) 1156 = 2 x 2 x 17 x 17

Grouping the factors into pairs of equal factors, we obtain:

1156 = (2 x 2) x (17 x 17) No factors are left over. Hence, 1156 is a perfect square. Moreover, by grouping 1156 into equal factors:

$1156 = (2 \times 17) \times (2 \times 17) = (2 \times 17)^2$

Hence, 1156 is the square of 34, which is equal to 2 x 17.

(ii) 2025 =3x3x3x3x5x5

Grouping the factors into pairs of equal factors, we obtain:

2025 = (3 x 3) x (3 x 3) x (5 x 5)

No factors are left over. Hence, 2025 is a perfect square. Moreover, by grouping 2025 into equal factors:

$2025 = (3 \times 3 \times 5) \times (3 \times 3 \times 5) = (3 \times 3 \times 5)^2$

Hence, 2025 is the square of 45, which is equal to 3 x 3 x 5.

(iii) 14641 = 11 x 11 x 11 x 11

Grouping the factors into pairs of equal factors, we obtain:

14641 = (11 x 11) x (11 x 11)

No factors are left over. Hence, 14641 is a perfect square. The above expression is already grouped into equal factors:

14641 = (11 x 11) x (11 x 11) = (11 x 11)2 Hence, 14641 is the square of 121, which is equal to 11 x 11.

(iv) 4761 = 3 x 3 x 23 x 23

Grouping the factors into pairs of equal factors, we obtain:

4761 = (3 x 3) x (23 x 23)

No factors are left over. Hence, 4761 is a perfect square. The above expression is already grouped into equal factors:

4761 = (3 x 23) x (3 x 23) = $(3 \times 23)^2$

Hence, 4761 is the square of 69, which is equal to 3 x 23.

3.) Find the smallest number by which of the following number must be multiplied so that the product is a perfect square:

Answer:

Factorize each number into its factors

(i) 23805 = 3 x 3 x 5 x 23 x 23

3	**23805**
3	**7935**
5	**2645**
23	**529**
23	**23**
	1

Grouping 23805 into pairs of equal factors:

23805 = (3 x 3) x (23 x 23) x 5

Here, the factor 5 does not occur in pairs. To be a perfect square, every prime factor has to be in pairs. Hence, the smallest number by which 23805 must be multiplied is 5.

(ii) 12150 = 2 x 3 x 3 x 3 x 3 x 3 x 5 x 5

2	**12150**
3	**6075**
3	**2025**
3	**675**
3	**225**
3	**75**
5	**25**
5	**5**
	1

Grouping 12150 into pairs of equal factors:

12150 = (3 x 3 x 3 x 3) x (5 x 5) x 2 x 3

Here, 2 and 3 do not occur in pairs. To be a perfect square, every prime factor has to be in pairs.

Hence, the smallest number by which 12150 must be multiplied is 2 x 3, i.e. by 6.

(iii) 7688 = 2 x 2 x 2 x 31 x 31

2	**7688**
2	**3844**
2	**1922**
31	**961**
31	**31**
	1

Grouping 7688 into pairs of equal factors:

7688 = (2 x 2) x (31 x 31) x 2

Here, 2 do not occur in pairs. To be a perfect square, every prime factor has to be in pairs. Hence the smallest number by which 7688 must be multiplied is 2.

4.) Find the smallest number by which the given number must be divided so that the resulting number is a perfect square:

Answer:

For each question, factorize the number into its prime factors.

(i) 14283 = 3 x 3 x 3 x 23 x x23

3	**14283**
3	**4761**
3	**1587**
23	**529**
23	**23**
	1

Grouping the factors into pairs:

14283 = (3 x 3) x (23 x 23) x 3

Here, the factor 3 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 14283 must be divided for it to be a perfect square is 3.

(ii) 1800 = 2 x 2 x 2 x 3 x 3 x 5 x 5

2	**1800**
2	**900**
2	**450**
3	**225**
3	**75**
5	**25**

5	**5**
	1

Grouping the factors into pairs:

1800 = (2 x2) x (3 x 3) x (5x 5) x2

Here the factor 2 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 1800 must be divided for it to be a perfect square is 2.

(iii) 2904 = 2 x 2 x 2 x3 x 11 x 11

2	**2904**
2	**1452**
2	**726**
3	**363**
11	**121**
11	**11**
	1

Grouping the factors into pairs:

2904= (2 x2) x (11 x 11) x 2 x 3

Here the factor 2 and 3 does not occur in pairs. To be a perfect square, all the factors have to be in pairs. Hence, the smallest number by which 2304 must be divided for it to be a perfect square is 2 x 3, i.e. 6.

5.) Which of the following numbers are perfect squares?

Answer:

11: The perfect squares closest to 11 are 9 ($9 = 3^2$) and 16 ($16 = 4^2$). Since 3 and 4 are consecutive numbers, there are no perfect squares between 9 and 16, which mean that 11 is not a perfect square.

12: The perfect squares closest to 12 are 9 ($9 =3^2$) and 16 ($16 = 4^2$). Since 3 and 4 are consecutive numbers, there are no perfect squares between 9 and 16, which mean that 12 is not a perfect square.

$16 = 4^2$

32: The perfect squares closest to 32 are 25 (25 = 52) and 36 (36 = 62). Since 5 and 6 are consecutive numbers, there are no perfect squares between 25 and 36, which means that 32 is not a perfect square.

$36 = 6^2$

50: The perfect squares closest to 50 are 49 (49 = 72) and 64 (64 = 82). Since 7 and 8 are consecutive numbers, there are no perfect squares between 49 and 64, which means that 50 is not a perfect square. $64 = 8^2$

79: The perfect squares closest to 79 are 64 (64 = 82) and 81 (81 = 92). Since 8 and 9 are consecutive numbers, there are no perfect squares between 64 and 81, which mean that 79 is not a perfect square.

81 = 92

111: The perfect squares closest to 111 are 100 (100 = 102) and 121 (121 = 112). Since 10 and 11 are consecutive numbers, there are no perfect squares between 100 and 121, which means that 111Is not a perfect square.

$121 = 11^2$

Hence, the perfect squares are 16, 36, 64, 81 and 121.

6.) Using prime factorization method, find which of the following numbers are perfect squares?

(i) 189 = 3 x 3 x 3 x 7

3	189
3	63
3	21
7	7
	1

Grouping them into pairs of equal factors:

189 = (3 x 3) x 3 x 7

The factors 3 and 7 cannot be paired. Hence, 189 is not a perfect square.

(ii) 225 = 3 x 3 x 5 x 5

3	**225**
3	**75**
5	**25**
5	**5**
	1

Grouping them into pairs of equal factors:

225 = (3 x 3) x (5 x 5)

There are no left out of pairs. Hence, 225 is a perfect square.

(iii) 2048 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

2	**2048**
2	**1024**
2	**512**
2	**256**
2	**128**
2	**64**
2	**32**
2	**16**
2	**8**
2	**4**
2	**2**
	1

Grouping them into pairs of equal factors:

2048 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x 2

The last factor, 2 cannot be paired.. Hence, 2048 is a perfect square.

(iv) 343 = 7 x 7 x 7

7	**343**
7	**49**
7	**7**
	1

Grouping them into pairs of equal factors:

343 = (7 x 7) x 7

The last factor, 7 cannot be paired. Hence, 343 is not a perfect square.

(v) 441 = 3 x 3 x 7 x 7

3	**441**
3	**147**
7	**49**
7	**7**
	1

Grouping them into pairs of equal factors:

441 = (3 x 3) x (7 x 7)

There are no left out of pairs. Hence, 441 is a perfect square.

(vi) 2916 = 2 x 2 x 3 x 3 x 3 x 3 x 3 x 3

2	**2916**
2	**1458**
3	**729**
3	**243**
3	**81**
3	**27**
3	**9**
3	**3**
	1

Grouping them into pairs of equal factors:

2916 = (2 x 2) x (3 x 3) x (3 x 3) x (3 x 3)

There are no left out of pairs. Hence, 2916 is a perfect square.

(vii) 11025= 3 x 3 x5 x7 x 7

3	**11025**
3	**3675**
5	**1225**
5	**245**
7	**49**
7	**7**
	1

Grouping them into pairs of equal factors:

11025= (3 x 3) x (5 x 5) x (7 x 7)

There are no left out of pairs. Hence, 11025 is a perfect square.

(viii) 3549 = 3 x 7 x 13 x 13

3	**3549**
7	**1183**
13	**169**
13	**13**
	1

Grouping them into pairs of equal factors:

3549 = (13 x 13) x 3 x 7

The last factors, 3 and 7 cannot be paired. Hence, 3549 is not a perfect square.

Hence, the perfect squares are 225, 441, 2916 and 11025.

7.) By what number should each of the following numbers be multiplied to get a perfect square in each case? Also, find the number whose square is the new number.

Factorizing each number

(i) 8820 = 2 x 2 x 2 x 3 x 5 x 7 x 7

2	**8820**

2	**4410**
3	**2205**
3	**735**
5	**245**
7	**49**
7	**7**
	1

Grouping them into pairs of equal of equal factors:

8820= (2 x 2) x (3 x 3) x (7 x 7) x 5

The factor, 5 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 8820 must be multiplied by 5 for it to be a perfect square.

The new number would be (2×2) x (3 x 3) x (7 x7) x (5 x 5).

Furthermore, we have:

(2 x 2) x (3 x 3) x (7 x 7) x (5 x 5)= (2 x 3 x 5 x 7) x (2 x 3 x 5 x 7)

Hence, the number whose square is the new number is:

2 x 3 x5 x 7 = 210

(ii) 3675 = 3 x 5 x 5 x 7 x 7

3	**3675**
5	**1225**
5	**245**
7	**49**
7	**7**
	1

Grouping them into pairs of equal factors:

3675 = (5 x 5) x (7 x 7) x 3

The factor 3 is not paired. For a number to be the perfect square, each prime factor has to be paired.

Hence, 3675 must be multiplied by 3 for it to be a perfect square.

The new number would be (5 x 5) x (7 x 7) x (3 x 3).

Furthermore, we have:

(5 x 5) x (7 x 7) x (3 x 3) = (3 x 5 x 7) x (3 x 5 x 7)

Hence, the number whose square is the new number is:

3 x 5 x 7 = 105

(iii) 605 = 5 x 11 x 11

5	**605**
11	**121**

11	**11**
	1

Grouping them into pairs of equal factors:

605 =5 x (11 x 11)

The factor 5 is not paired. For a number to be perfect square, each prime factor has to be paired.

Hence, 605 must be multiplied by 5 for it to be a perfect square.

The new number would be (5 x 5) x (11 x 11)

Furthermore, we have:

(5 x 5) x (11 x 11) = (5 x 11) x (5x 11)

Hence, the number whose square is the new number is:

5 x 11 = 55

(iv) 2880 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 5

2	**2880**
2	**1440**
2	**720**
2	**360**
2	**180**
2	**90**
3	**45**
3	**15**
5	**5**
	1

Grouping them into pairs of equal factors:

2880 = (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x 5

There is a 5 as the leftover. For a number to be a perfect square, each prime factor has to be paired.

Hence, 2880 must be multiplied by 5 to be a perfect square.

The new number would be (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x (5 x 5).

Furthermore, we have:

(2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) x (5 x 5) = (2 x2x2x3x 5) x (2 x2x2x3x 5) Hence, the number whose square is the new number is:

2x2x2x3x5= 120

(v) 4056 =2 x 2 x 2 x 3 x 13 x 13

2	**4056**
2	**2028**
2	**1014**

3	**507**
13	**169**
13	**13**

Grouping them into pairs of equal factors:

4056 = (2 x 2) x(13 x 13) x 2 x 3

The factors at the end, 2 and 3 are not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 4056 must me multiplied by 6 (2 x 3) for it to be a perfect square.

The new number would be (2 x 2) x (2 x2) x (3 x 3) x (13 x 13).

Furthermore, we have

(2 x 2) x (2 x2) x (3 x 3) x (13 x 13) = (2 x 2 x 3 x 13) x (2 x 2 x 3 x13)

Hence, the number whose square is the new number is:

2 x 2 x 3 x13 = 156

(vi) 3468 = 2 x 2 x 3 x 17 x 17

2	**3468**
2	**1734**
3	**864**
17	**289**
17	**17**
	1

Grouping them into pairs of equal factors:

3468 = (2 x 2) x (17 x 17) x 3

The factor at the end, 3 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 3468 must me multiplied by 3 for it to be a perfect square.

The new number would be (2 x 2) x (17 x 17) x (3 x 3).

Furthermore, we have

(2 x 2) x (17 x17) x (3 x3) = (2 x 3 x 17) x (2 x 3 x17)

Hence, the number whose square is the new number is:

2 x 3 x17 = 102

(viii) 7776 = 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 3

2	**7776**
2	**3888**
2	**1944**
2	**972**
2	**486**
3	**243**

3	**81**
3	**27**
3	**9**
3	**3**
	1

Grouping them into pairs of equal factors:

7776= (2 x 2) x (2 x 2) x (3 x 3) x (3 x 3) x 2 x 3

The factor at the end, 2 and 3 are not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 7776 must me multiplied by 6 (2 x 3) for it to be a perfect square.

The new number would be (2 x 2) x (2 x 2) x (2 x 2)(3 x 3) x (3 x 3) x (3 x 3) .

Furthermore, we have

(2 x 2) x (2 x 2) x (2 x 2) (3 x 3) x (3 x 3) x (3 x 3)= (2 x 2 x 2 x 3 x 3 x 3) x (2 x 2 x 2 x 3 x 3 x 3)

Hence, the number whose square is the new number is:

2 x 2 x 2 x 3 x 3 x 3 = 216

8.) By what numbers should each of the following be divided to get a perfect square in each case? Also, find the number whose square is the new number.

Answer:

Factorizing each number

(i) 16562 = 2 x 7 x 7 x 13 x13

2	**16562**
7	**8281**
7	**1183**
13	**169**
13	**13**
	1

Grouping them into pairs of equal factors:

16562= 2 x(7 x 7) x (13 x 13)

The factor at the end, 2 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 16652 must me multiplied by 2 for it to be a perfect square.

The new number would be (7x 7) x (13 x13).

Furthermore, we have

(7x 7) x (13 x13) = (7 x 13) x (7 x 13)

Hence, the number whose square is the new number is:

7 x 13= 91

(ii) 3698 = 2 x 43 x 43

2	**3698**

43	**1849**
43	**43**
	1

Grouping them into pairs of equal factors:

3698 = 2 x (43 x 43)

The factor at the end, 2 is not paired. For a number to be a perfect square, each prime factor has to be paired. Hence, 3698 must me multiplied by 2 for it to be a perfect square.

The new number would be (43 x43)

Hence, the number whose square is the new number is 43.

(iii) 5103 = 3 x 3 x 3 x 3 x 3 x 3 x 7

3	**5103**
3	**1701**
3	**567**
3	**189**
3	**63**
3	**21**
7	**7**
	1

Grouping them into pairs of equal factors: 5103 = (3 x 3) x (3 x 3) x (3 x 3) x 7

The factor, 7 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 5103 must be divided by 7 for it to be a perfect square. The new number would be (3 x 3) x (3 x 3) x (3 x 3).

Furthermore, we have: (3 x 3) x (3 x 3) x (3 x 3) = (3 x 3 x 3) x (3 x 3 x 3) Hence, the number whose square is the new number is:

3 x 3 x 3=27

(iv) 3174= 2 x 3 x 23 x 23

2	**3174**
	1587
	529
	23
	1

Grouping them into pairs of equal factors:

3174 = 2 x 3 x (23 x 23)

The factors, 2 and 3 are not paired.

For a number to be a perfect square, each prime factor has to be paired.

Hence, 3174 must be divided by 6 (2 x 3) for it to be a perfect square.

The new number would be (23 x 23).

Hence, the number whose square is the new number is 23.

(v) 1575= 3 x3 x 5 x 7

3	**1575**
3	**525**
5	**175**
5	**35**
7	**7**
	1

Grouping them into pairs of equal factors:

1575 = (3 x 3) x (5 x 5) x 7

The factor, 7 is not paired. For a number to be a perfect square, each prime factor has to be paired.

Hence, 1575 must be divided by 7 for it to be a perfect square.

The new number would be (3 x 3) x (5 x 5).

Furthermore, we have:

(3 x 3) x (5 x 5) = (3 x 5) x (3 x 5)

Hence, the number whose square is the new number is: 3 x 5 = 15

9.) Find the greatest number of two digits which is a perfect square.

Answer:

We know that 102 is equal to 100 and 9^2 is equal to 81.

Since 10 and 9 are consecutive numbers, there is no perfect square between 100 and 81.

Since 100 is the first perfect square that has more than two digits, 81 is the greatest two-digit perfect square.

10.) Find the least number of three digits which is a perfect square.

Answer:

Let us make a list of the squares starting from 1.

$1^2=1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2=25$

$6^2 = 36$

$7^2= 49$

$8^2 = 64$

$9^2 = 81$

$10^2=100$

The square of 10 has three digits. Hence, the least three-digit perfect square is 100.

11.) Find the smallest number by which 4851 must be multiplied so that the product becomes a perfect square.

Answer:

4581 = 3 x 3 x 7 x 7 x11

3	**4851**
3	**1617**
7	**539**
7	**77**
11	**11**
	1

Grouping them into pairs of equal factors:

4851 = (3 x 3) x (7 x 7) x 11

The factor, 11 is not paired. The smallest number by which 4851 must be multiplied such that the resulting number is a perfect square is 11.

12.) Find the smallest number by which 28812 must be divided so that the quotient becomes a perfect square.

Prime factorization of 28812:

28812 = 2 x 2 x 3 x 7 x 7 x 7 x7

2	**22812**
2	**14406**
3	**7203**
7	**2401**
7	**343**
7	**49**
7	**7**
	1

Grouping them into pairs of equal factors:

28812 = (2 x2) x (7 x 7) x (7x 7) x 3

The factor, 3 is not paired. Hence, the smallest number by which 28812 must be divided such that the resulting number is a perfect square is 3.

13.) Find the smallest number by which 1152 must be divided so that it becomes a perfect square. Also, find the number whose square is the resulting number.

Answer:

Prime factorization of 1152:

1152 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

2	**1152**
2	**576**
2	**288**
2	**144**
2	**72**
2	**36**
2	**18**
3	**9**
3	**3**
	1

Grouping them into pairs of equal factors:

1152=(2 x 2)x(2 x 2)x(2 x 2)x(3 x 3)x 2

The factor, 2 at the end is not paired.

For a number to be a perfect square, each prime factor has to be paired.

Hence, 1152 must be divided by 2 for it to be a perfect square.

The resulting number would be (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3).

Furthermore, we have:

(2 x 2) x (2 x 2) x (2 x 2) x (2 x 2) x (3 x 3) = (2 x 2 x 2 x 3) x (2 x 2 x 2 x 3)

Hence, the number whose square is the resulting number is: 2 x 2 x 2 x 3 = 24