RD SHARMA

Solutions

Class 8 Maths

Chapter 3

Ex 3.2

1.) The following number are not perfect squares. Give reason:

A number ending with 2, 3, 7 or 8 cannot be a perfect square.

(i) 1547

Its last digit is 7. Hence, 1547 cannot be a perfect square.

(ii) 45743

Its last digit is 3. Hence, 45743 cannot be a perfect square.

(iii) 22453

Its last digit is 8. Hence, 8948 cannot be a perfect square.

(iv) 333333

Its last digit is 3. Hence, 333333 cannot be a perfect square.

2.) Show that the following numbers are not perfect squares:

A number ending with 2, 3, 7 or 8 cannot be a perfect square.

(i) 9327

Its last digit is 7. Hence, 9327 is not a perfect square.

(ii) 4058

Its last digit is 8. Hence, 4058 is not a perfect square.

(iii) 22453

Its last digit is 3. Hence, 22453 is not a perfect square.

(iv)743522

Its last digit is 2. Hence, 743522 is not a perfect square.

3.) The square of which of the following numbers would be an odd number?

The square of an odd number is always odd.

(i) 731

731 is an odd number. Hence, its square will be an odd number.

(ii)3456

3456 is an even number. Hence, its square will not be an odd number.

(iii) 5559

5559 is an odd number. Hence, its square will not be an odd number.

(iv) 42008

42008 is an even number. Hence, its square will not be an odd number.

Hence, only the squares of 731 and 5559 will be odd numbers.

4.) What will be the unit digit of the squares of the following numbers?

The unit's digit is affected only by the last digit of the number.

Hence, for each question, we only need to examine the square of its last digit.

(i) 52

Its last digit is 2. Hence, the unit's digit is 2^2, which is equal to 4.

(ii) 977

Its last digit is 7. Hence, the unit's digit is the last digit of 49 (49 = 72), which is 9.

(iii) 4583

Its last digit is 3. Hence, the unit's digit is 3^2, which is equal to 9.

(iv) 78367

Its last digit is 7. Hence, the unit's digit is the last digit of 49 (49 = 72), which is 9.

(v) 52698

Its last digit is 8. Hence, the unit's digit is the last digit of 64 (64 = 82), which is 4.

(vi) 99880

Its last digit is 0. Hence, the unit's digit is 02, which is equal to 0.

(vii 12796

Its last digit is 6. Hence, the unit's digit is the last digit of 36 (36 = 62), which is 6.

(viii) 55555

Its last digit is 5. Hence, the unit's digit is the last digit of 25 (25 = 52), which is 5.

(ix) 53924

Its last digit is 4. Hence, the unit's digit is the last digit of 16 (16 = 42), which is 6.

5.) Observe the following pattern:

$1 + 3 = 2^2$

$1+ 3 + 5 = 3^2$

$1+ 3 +5 + 7= 4^2$ and write the value of $1+ 3 + 5 + 7 + 9 + \ldots$ up to n terms.

From the pattern, we can say that the sum of the first n positive odd numbers is equal to the square of the n-th positive number. Putting that into formula:

$1 + 3 + 5 + 7 + n = n^2$, where the left hand side consists of n terms.

6.) Observe the following pattern:

$2^2 – 1^2 = 2 +1$

$3^2 – 2^2 = 3 +2$

$4^2 – 3^2 = 4 + 3$

$5^2 – 4^2 = 5 + 4$

From the pattern, we can say that the difference between the squares of two consecutive numbers is the sum of the numbers itself. In a formula:

$(n+1)^2 — (n)^2 = (n+1) + n$

Using this formula, we get:

(i) $100^2 – 99^2 = (99 + 1) + 99 = 199$

(ii) $111^2 – 109^2 = 111^2 – 110^2 + 110^2 – 109^2 = (111 + 110) + (110 + 109) = 440$

(iii) $99^2 – 96^2 = 99^2 – 98^2 + 98^2 – 97^2 + 97^2 – 96^2$

$= 99 + 98 + 98 + 97 + 97 + 96 = 585$

7.) Which of the following triplets is Pythagorean?

Only (i), (ii), (iv) and (v) are Pythagorean triplets.

A triplet (a, b, c) is called Pythagorean if the sum of the squares of the two smallest numbers is equal to the square of the biggest number.

(i) (8, 15, 17)

The two smallest numbers are 8 and 15. The sum of their squares is:

$8^2 + 15^2 = 289 = 17^2$

Hence, (8, 15, 17) is a Pythagorean triplet.

(ii) (18, 80, 82)

The two smallest numbers are 18 and 80. The sum of their squares is: $18^2 + 80^2 = 6724 = 82^2$

Hence, (18, 80, 82) is a Pythagorean triplet.

(iii) (14, 48, 51)

The two smallest numbers are 14 and 48. The sum of their squares is:

$14^2 + 48^2 = 2500$, this is not equal to 512 = 2601

Hence, (14, 48, 51) is not a Pythagorean triplet.

(iv) (10, 24, 51)

The two smallest numbers are 10 and 24. The sum of their squares is:

$10^2 + 24^2 = 676 = 26^2$

Hence, (10, 24, 26) is a Pythagorean triplet.

(v) (16, 63, 65)

The two smallest numbers are 16 and 63. The sum of their squares is:

$16^2 + 63^2 = 4225 = 65^2$ Hence, (16, 63, 65) is a Pythagorean triplet.

(vi) (12, 35, 38)

The two smallest numbers are 12 and 35. The sum of their squares is:

$12^2 + 35^2 = 1369$, which is not equal to $38^2 = 1444$ Hence, (12, 35, 38) is not a Pythagorean triplet.

8.) Observe the following pattern:

$(1 \times 2) + (2 \times 3) = \frac{2\times3\times4}{3}$

$(1 \times 2) + (2 \times 3) + (3 \times 4) = \frac{3\times4\times5}{3}$

(1 x 2) + (2 x3) + (3 x 4) + (4x 5) = $\frac{4\times5\times6}{3}$ and find the value of (1 x 2) + (2 x 3) + (3 x 4) + (4 x5) + (5 x 6)

The RHS of the three equalities is a fraction whose numerator is the multiplication of three consecutive numbers and whose denominator is 3.

If the biggest number (factor) on the LHS is 3, the multiplication of the three numbers on the RHS begins with 2.

If the biggest number (factor) on the LHS is 4, the multiplication of the three numbers on the RHS begins with 3.

If the biggest number (factor) on the LHS is 5, the multiplication of the three numbers on the RHS begins with 4.

Using this pattern, (1 x 2) + (2 x 3) + (3 x 4) + (4 x 5) + (5 x 6) has 6 as the biggest number. Hence, the multiplication of the three numbers on the RHS will begin with 5. Finally, we have:

1 x 2+ 2 x3 + 3 x 4 + 4 x 5 + 5 x 6 = $\frac{5\times6\times7}{3}$ = 70

9.) Observe the following pattern:

1 = $\frac{1}{2}$ {1 x (1+ 1)}

1 +2 = $\frac{1}{2}$ {2 x {2 +2)}

1 + 2 +3 = $\frac{1}{2}$ {3 x (3 + 1)}

1 + 2 + 3 +4 =$\frac{1}{2}$ {4 x (4+ 1)} and find the values of each of the following :

(i) 1+ 2 + 3 + 4 + 5 … + 50

(ii) 31 + 32+ … + 50

Observing the three numbers for right hand side of the equalities:

The first equality, whose biggest number on the LHS is 1, has 1, 1 and 1 as the three numbers.

The second equality, whose biggest number on the LHS is 2, has 2, 2 and 1 as the three numbers

The third equality, whose biggest number on the LHS is 3, has 3, 3 and 1 as the three numbers.

The fourth equality, whose biggest number on the LHS is 4, has 4, 4 and 1 as the three numbers.

Hence, if the biggest number on the LHS is n, the three numbers on the RHS will be n, n and 1.

Using this property, we can calculate the sums for (i) and (ii) as follows:

(i) 1 + 2 + 3 +….. + 50 = $\frac{1}{2}$ x 50 x (50 + 1) = 1275

(ii) The sum can be expressed as the difference of the two sums as follows:

31 + 32 +…. + 50 = (1 + 2 + 3 +…. + 50) – (1 + 2 + 3 + +30) The result of the first bracket is exactly the same as in part (i).

1 + 2 + ….+ 50 = 1275

Then, the second bracket:

1 + 2+…. +30 = $\frac{1}{2}$ (30 x (30 + 1)) = 465

Finally, we have: 31 + 32 + + 50 = 1275 — 465 =810

11.) Which of the following numbers are squares of even numbers: 121, 225, 256, 324, 1296, 6561, 5476, 4489, 373758

The numbers whose last digit is odd can never be the square of even numbers. So, we have to leave out 121, 225, 6561 and 4489, leaving only 256, 324, 1296, 5476 and 373758.

For each number, use prime factorization method and make pairs of equal factors.

(i) 256 =2x2x2x2x2x2x2x2 = (2 x 2) x (2 x 2) x (2 x 2) x (2 x 2)

There are no factors that are not paired. Hence, 256 is a perfect square. The square of an even number is always even. Hence, 256 is the square of an even number.

(ii) 324 =2x2x3x3x3x3 = (2 x 2) x (3 x 3) x (3 x 3) There are no factors that are not paired. Hence, 324 is a perfect square. The square of an even number is always even. Hence, 324 is the square of an even number.

(iii) 1296 =2x2x2x2x3x3x3x3 = (2 x 2) x (2 x 2) x (3 x 3) x (3 x 3) There are no factors that are not paired. Hence, 1296 is a perfect square. The square of an even number is always even. Hence, 1296 is the square of an even number.

(iv) 5476 = 2 x 2 x 37 x 37 = (2 x 2) x (37 x 37) There are no factors that are not paired. Hence, 5476 is a perfect square. The square of an even number is always even. Hence, 5476 is the square of an even number.

(v) 373758 =2 x 3 x 7 x 11 x 809 Here, each factor appears only once, so grouping them into pairs of equal factors is not possible. It means that 373758 is not the square of an even number.

Hence, the numbers that are the squares of even numbers are 256, 324, 1296 and 5476.

12.) By just examining the units digit, can you tell which of the following cannot be whole squares?

If the unit's digit of a number is 2, 3, 7 or 8, the number cannot be a whole square.

(i) 1026

1026 has 6 as the unit's digit, so it is possibly a perfect square.

(ii) 1028

1028 has 8 as the unit's digit, so it cannot be a perfect square.

(iii) 1024

1024 has 4 as the unit's digit, so it is possibly a perfect square.

(iv) 1022

1022 has 2 as the unit's digit, so it cannot be a perfect square.

(v) 1023

1023 has 3 as the unit's digit, so it cannot be a perfect square.

(vi) 1027

1027 has 7 as the unit digit, so it cannot be a perfect square.

Hence, by examining the unit's digits, we can be certain that 1028, 1022, 1023 and 1027 cannot be whole squares.

13.) Write five numbers which you cannot decide whether they are squares.

Answer:

A number whose unit digit is 2, 3, 7 or 8 cannot be a perfect square.

On the other hand, a number whose unit digit is 1, 4, 5, 6, 9 or 0 might be a perfect square (although we will have to verify whether it is a perfect square or not).

Applying the above two conditions, we cannot quickly decide whether the following numbers are squares of any numbers:

1111, 1444, 1555, 1666, 1999

14.) Write five numbers which you cannot decide whether they are square just be looking at the units digit.

Answer:

A number whose unit digit is 2, 3, 7 or 8 cannot be a perfect square.

On the other hand, a number whose unit digit is 1, 4, 5, 6, 9 or 0 might be a perfect square although we have to verify that.

Applying these two conditions, we cannot determine whether the following numbers are squares just by looking at their unit digits:

1111, 1001, 1555, 1666 and 1999

15.) Write True (T) and false (F) for the following statements.

(i) The number of digits in a square number is even.

False

Example: 100 is the square of a number but its number of digits is three, which is not an even number.

(ii) The square of a prime number is prime.

False

If p is a prime number, its square is p2, which has at least three factors: 1, p and p2. Since it has more than two factors, it is not a prime number.

(iii) The sun of two square numbers is a square number.

False

1 is the square of a number (1 = 12). But 1 + 1 = 2, which is not the square of any number.

(iv) The difference of two square numbers is a square number.

False

4 and 1 are squares (4 = 22, 1 = 12). But 4 — 1 = 3, which is not the square of any number.

(v) The product of two square numbers is a square number.

True

If a2 and b2 are two squares, their product is a2 x b2 = (a x b)2, which is a square.

(vi) No square number is negative.

True

The square of a negative number will be positive because negative times negative is positive.

(vii) There is no square number between 50 and 60

True 72 = 49 and 82 = 64. 7 and 8 are consecutive numbers and hence there are no square numbers between 50 and 60.

(viii) There are fourteen square number up to 200.

True 142 is equal to 196, which is below 200. There are 14 square numbers below 200.