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Solutions

Class 8 Maths

Chapter 4

Ex *4.1*

Q1. Find the cubes of the following numbers:

(i) 7 (ii) 12 (iii) 16 (iv) 21 (v) 40 (vi) 55 (vii) 100 (viii) 302 (ix) 301

Answer:

Cube of a number is given by the number raised to the power three.

(i) Cube of 7 = 7^3 = 7 x 7 x 7 = 343

(ii) Cube of 12 = 12^3 = 12 x 12 x 12 = 1728

(iii) Cube of 16 = 16^3 = 16 x 16 x 16 = 4096

(iv) Cube of 21 = 2^3 = 21 x 21 x 21 = 9261

(v) Cube of 40 = 40^3 = 40 x 40 x 40 = 64000

(vi) Cube of 55 = 553 = 55 x 55 x 55 = 166375

(vii) Cube of 100 = 100^3 = 100 x 100 x 100 = 1000000

(viii) Cube of 302 = 302^3 = 302 x 302 x 302 = 27543608

(ix) Cube of 301 = 3013 = 301 x 301 x 301 = 27270901

Q2. Write the cubes of all natural numbers between 1 and 0 and verify the following statements:

(i) Cubes of all odd natural numbers are odd. 6.0

(ii) Cubes of all even natural numbers are even, 3.

Answer:

The cubes of natural numbers between 1 and 10 are listed and classified in the following table.

We can classify all natural numbers as even or odd number; therefore, to check whether the cube of a natural number is even or odd, it is sufficient to check its divisibility by 2.

If the number is divisible by 2, it is an even number, otherwise, it will an odd number.

(i) From the above table, it is evident that cubes of all odd natural numbers are odd.

(ii) From the above table, it is evident that cubes of all even natural numbers are even.

Number	Cube	Classification
1	1	Odd
2	8	Even (Last digit is even, i.e., 0, 2, 4, 6,

		8)
3	27	Odd (Not an even number)
4	64	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
5	125	Odd (Not an even number)
6	216	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
7	343	Odd (Not an even number)
8	512	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)
9	729	Odd (Not an even number)
10	1000	Even (Last digit is even, i.e., 0, 2, 4, 6, 8)

Q3. Observe the following pattern:

$$1^3 = 1$$

$$1^3 + 2^3 = (1+2)^2$$

$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3 + 10^3$ by the above pattern.

Answer:

Extend the pattern as follows:

$$1^3 = 1$$
$$1^3 + 2^3 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = (1+2+3+4+5+6)^2$$

Now, from the ab

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2 =$

$55^2 = 3025$

Thus, the required value is 3025

Q4. Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings: "The cube of a natural number which is a multiple of 3 is a multiple of 27".

Answer:

Five natural numbers, which are multiples of 3, are 3, 6, 9, 12 and 15.

Cubes of these five numbers are:

$3^3 = 3 \text{ x } 3 \text{ x } 3 = 27$

$6^3 = 6 \text{ x } 6 \text{ x } 6 = 216$

$9^3 = 9 \text{ x } 9 \text{ x } 9 = 729$

$12^3 = 12 \text{ x } 12 \text{ x } 12 = 1728$

$15^3 = 15 \text{ x } 15 \text{ x } 15 = 3375$

Now, let us write the cubes as a multiple of 27. We have:

27 = 27 x 1

216 = 27 x 8

729 = 27 x 27

1728 = 27 x 64

3375 = 27 x 125

It is evident that the cubes of the above multiples of 3 could be written as multiples of 27. Thus, it is verified that the cube of a natural number, which is a multiple of 3, is a multiple of 27.

Q5. Write the cubes of 5 natural numbers which are of the form 3n + 1 (e.g. 4, 7, 10, …) and verify the following:

'The cube of a natural number of the form 3n + I is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1'.

Answer :

Five natural numbers of the form (3n + 1) could be written by choosing n = 1, 2, 3, … etc.

Let five such numbers be 4, 7, 10, 13, and 16.

The cubes of these five numbers are: $4^3 = 64$, $7^3 = 343$, $10^3 = 1000$, $13^3 = 2197$ and $l6^3 = 4096$

The cubes of the numbers 4, 7, 10, 13, and 16 could be expressed as:

64 = 3 x 21 + 1, which is of the form (3n + 1) for n = 21

343 = 3 x 114 + 1, which is of the form (3n + 1) for n = 114

1000 = 3 x 333 + 1, which is of the form (3n + 1) for n = 333

2197 = 3 x 732 + 1, which is of the form (3n + 1) for n = 732

4096 = 3 x 1365 + 1, which is of the form (3n + 1) for n = 1365

The cubes of the numbers 4, 7, 10, 13, and 16 could be expressed as the natural numbers of the form (3n + 1) for some natural number n; therefore, the statement is verified.

Q6. Write the cubes of 5 natural numbers of the form 3n + 2 (i.e. 5, 8, 11,…) and verify the following:

"The cube of a natural number of the form 3n + 2 is a natural number of the same form i.e. when it is divided by 3 the remainder is 2".

Answer :

Five natural numbers of the form (3n + 2) could be written by choosing n = 1, 2, 3… etc.

Let five such numbers be 5, 8, 11, 14, and 17.

The cubes of these five numbers are: 5^3 = 125, 8^3 = 512, 11^3 = 1331, 14^3= 2744, and 17^3 = 4913.

The cubes of the numbers 5, 8, 11, 14 and 17 could be expressed as:

125 = 3 x 41 + 2, which is of the form (3n + 2) for n = 41

512 = 3 x 170 + 2, which is of the form (3n + 2) for n = 170

1331 = 3 x 443 + 2, which is of the form (3n + 2) for n = 443

2744 = 3 x 914 + 2, which is of the form (3n + 2) for n = 914

4913 = 3 x 1637 + 2, which is of the form (3n + 2) for n = 1637

The cubes of the numbers 5, 8, 11, 14, and 17 can be expressed as the natural numbers of the form (3n + 2) for some natural number n. Hence, the statement is verified.

Q7. Write the cubes of 5 natural numbers of which are multiples of 7 and verify the following:

"The cube of a multiple of 7 is a multiple of 7^3"

Answer :

Five multiples of 7 can be written by choosing different values of a natural number n in the expression 7n.

Let the five multiples be 7, 14, 21, 28 and 35.

The cubes of these numbers are: 7^3 = 343, 14^3 = 2744, 21^3 = 9261, 28^3 = 21952, and 35^3 = 42875

Now, write the above cubes as a multiple of 7^3. Proceed as follows:

343 = 7^3 x 1

2744 = 14^3 = 14 x 14 x 14 = (7 x 2) x (7 x 2) x (7 x 2) = (7 x7 x 7) x (2 x 2 x 2) = 7^3 x 2^3

9261 = 21^3 = 21 x 21 x 21 = (7 x 3) x (7 x 3) x (7 x 3) = 7^3 x 3^3

21952 = 28^3 = 28 x 28 x 28 = (7 x 4) x (7 x 4) x (7 x 4) = (7 x 7 x 7) x (4 x 4 x 4) =7^3 x 4^3

42875 = 35^3 = 35 x 35 x 35 = (7 x 5) x (7 x 5) x (7 x 5) = (7 x 7 x 7) x (5 x 5 x 5) = 7^3 x 5^3

Hence, the cube of multiple of 7 is a multiple of 7^3.

Q8. Which of the following are perfect cubes?

(i) 64 (ii) 216 (iii) 243 (iv) 1000 (v) 1728 (vi) 3087 (vii) 4608 (viii) 106480 (ix) 166375 (x) 456533

Answer:

(i) On factorising 64 into prime factors, we get

64 = 2 x 2 x 2 x 2 x 2 x 2

Group the factors in triples of equal factors as:

64 = {2 x 2 x 2} x {2 x 2 x 2}

It is evident that the prime factors of 64 can be grouped into triples of equal factors and no factor is left over.

Therefore, 64 is a perfect cube.

(ii) On factorising 216 into prime factors, we get:

216 = 2 x 2 x 2 x 3 x 3 x 3

Group the factors in triples of equal factors as:

216 = {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 216 can be grouped into triples of equal factors and no factor is left over.

Therefore, 216 is a perfect cube.

(iii) On factorizing 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

Group the factors in triples of equal factors as:

243 = {3 x 3 x 3} x 3 x3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is a not perfect cube.

(iv) On factorising 1000 into prime factors, we get:

1000 = 2 x 2 x 2 x 5 x 5 x 5

Group the factors in triples of equal factors as:

1000 = {2 x 2 x 2} x {5 x 5 x 5}

It is evident that the prime factors of 1000 can be grouped into triples of equal factors and no factor is left over. Therefore, 1000 is a perfect cube.

(v) On factorising 1728 into prime factors, we get:

1728 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3

Group the factors in triples of equal factors as:

1728 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 1728 can be grouped into triples of equal factors and no factor is left over.

Therefore, 1728 is a perfect cube.

(vi) On factorizing 3087 into prime factors, we get:

3087 = 3 x 3 x 7 x 7 x 7

Group the factors in triples of equal factors as:

3087 = 3 x 3 x {7 x 7 x 7}

It is evident that the prime factors of 3087 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is a not perfect cube.

(vii) On factorising 4608 into prime factors, we get:

4608 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3

Group the factors in triples of equal factors as:

4608 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x 3 x 3

It is evident that the prime factors of 4608 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 4608 is a not perfect cube.

(viii) On factorising 106480 into prime factors, we get:

106480 = 2 x 2 x 2 x 2 x 5 x 11 x 11 x 11

Group the factors in triples of equal factors as:

106480 = {2 x 2 x 2} x 2 x 5 x {11 x 11 x 11}

It is evident that the prime factors of 106480 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 106480 is a not perfect cube.

(ix) On factorising 166375 into prime factors, we get:

166375 = 5 x 5 x 5 x 11 x 11 x 11

Group the factors in triples of equal factors as:

166375 = {5 x 5 x 5} x {11 x 11 x 11}

It is evident that the prime factors of 166375 can be grouped into triples of equal factors and no factor is left over.

Therefore, 166375 is a perfect cube.

(x) On factorizing 456533 into prime factors, we get:

456533 = 7 x 7 x 7 x 11 x 11 x11

Group the factors in triples of equal factors as:

456533={7 x 7 x 7} x {11 x 11 x 11}

It is evident that the prime factors of 456533 can be grouped into triples of equal factors and no factor is left over.

Therefore, 456533 is a perfect cube.

Q9. Which of the following are cubes of even natural numbers?

216, 512, 729, 1000, 3375, 13824

Answer :

We know that the cubes of all even natural numbers are even.

The numbers 216, 512, 1000 and 13824 are cubes of even natural numbers.

The numbers 216, 512, 1000 and 13824 are even and it could be verified by divisibility test of 2, i.e., a number is divisible by 2 if it ends with 0, 2, 4, 6 or 8.

Thus, the cubes of even natural numbers are 216, 512, 1000 and 13824.

Q10. Which of the following are cubes of odd natural numbers?

125, 343, 1728, 4096, 32768, 6859

Answer :

We know that the cubes of all odd natural numbers are odd.

The numbers 125, 343, and 6859 are cubes of odd natural numbers.

Any natural numbers could be either even or odd.

Therefore, if a natural number is not even, it is odd.

Now, the numbers 125, 343 and 6859 are odd (It could be verified by divisibility test of 2, i.e., a number is divisible by 2 if it ends with 0, 2, 4, 6 or 8).

None of the three numbers 125, 343 and 6859 are divisible by 2. Therefore, they are not even, they are odd. The numbers 1728, 4096 and 32768 are even.

Thus, cubes of odd natural numbers are 125, 343 and 6859.

Q11. What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?

(i) 675 (ii) 1323 (iii) 2560 (iv) 7803 (v) 107811 (vi) 35721

Answer :

(i) On factorising 675 into prime factors, we get:

675 = 3 x 3 x 3 x 5 x 5

On grouping the factors in triples of equal factors, we get:

675 = {3 x 3 x 3} x 5 x 5

It is evident that the prime factors of 675 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 675 is a not perfect cube. However, if the number is multiplied by 5, the factors can be grouped into triples of equal factors and no factor will be left over.

Thus, 675 should be multiplied by 5 to make it a perfect cube.

(ii) On factorising 1323 into prime factors, we get:

1323 = 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

675 = {3 x 3 x 3} x 5 x 5

It is evident that the prime factors of 1323 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 1323 is a not perfect cube.

However, if the number is multiplied by 7, the factors can be grouped into triples of equal factors and no factor will be left over.

Thus, 1323 should be multiplied by 7 to make it a perfect cube.

(iii) On factorising 2560 into prime factors, we get:

2560 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 5

On grouping the factors in triples of equal factors, we get:

2560 = {2 x 2 x 2} x {2 x 2 x 2} x {2 x 2 x 2} x 5

It is evident that the prime factors of 2560 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 2560 is a not perfect cube.

However, if the number is multiplied by 5 x 5 = 25, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 2560 should be multiplied by 25 to make it a perfect cube.

(iv) On factorising 7803 into prime factors, we get:

7803 = 3 x 3 x 3 x 17 x 17

On grouping the factors in triples of equal factors, we get:

7803 = {3 x 3 x 3} x 17 x 17

It is evident that the prime factors of 7803 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 7803 is a not perfect cube.

However, if the number is multiplied by 17, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 7803 should be multiplied by 17 to make it a perfect cube.

(v) On factorising 107811 into prime factors, we get:

107811 = 3 x 3 x 3 x 3 x 11 x 11 x 11

On grouping the factors in triples of equal factors, we get:

107811 ={3 x 3 x 3} x 3 x{11 x 11 x 11}

It is evident that the prime factors of 107811 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 107811 is a not perfect cube.

However, if the number is multiplied by 3 x 3 = 9, the factors be grouped into triples of equal factors such that no factor is left over.

Thus, 107811 should be multiplied by 9 to make it a perfect cube.

(vi) On factorising 35721 into prime factors, we get:

35721 = 3 x 3 x 3 x 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

35721 = {3 x 3 x 3} x {3 x 3 x 3} x 7 x 7

It is evident that the prime factors of 35721 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 35721 is a not perfect cube.

However, if the number is multiplied by 7, the factors be grouped into triples of equal factors such that no factor is left over.

Thus, 35721 should be multiplied by 7 to make it a perfect cube.

Q12. By which smallest number must the following numbers be divided so that the quotient is a perfect cube?

(i) 675 (ii) 8640 (iii) 1600 (iv) 8788 (v) 7803 (vi) 107811 (vii) 35721 (viii) 243000

Answer :

(i) On factorising 675 into prime factors, we get: 675 = 3 x 3 x 3 x 5 x 5

On grouping the factors in triples of equal factors, we get:

675 = {3 x 3 x 3 x 5 x 5}

It is evident that the prime factors of 675 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 675 is a not perfect cube.

However, if the number is divided by 5 x 5 = 25, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 675 should be divided by 25 to make it a perfect cube.

(ii) On factorising 8640 into prime factors, we get:

8640 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5

On grouping the factors in triples of equal factors, we get:

8640 = {2 x 2 x 2} x {2 x 2 x 2 } x {3 x 3 x 3} x 5

It is evident that the prime factors of 8640 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8640 is a not perfect cube.

However, if the number is divided by 5, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 8640 should be divided by 5 to make it a perfect cube.

(iii) On factorising 1600 into prime factors, we get:

1600 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5

On grouping the factors in triples of equal factors, we get:

1600 = {2 x 2 x 2} x {2 x 2 x 2} x 5 x 5

It is evident that the prime factors of 1600 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 1600 is a not perfect cube.

However, if the number is divided by (5 x 5= 25), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 1600 should be divided by 25 to make it a perfect cube.

(iv) On factorising 8788 into prime factors, we get:

8788 = 2 x 2 x 13 x 13 x 13

On grouping the factors in triples of equal factors, we get:

8788 = 2 x 2 x {13 x 13 x 13}

It is evident that the prime factors of 8788 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8788 is a not perfect cube. However, if the number is divided by (2 x 2= 4), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 8788 should be divided by 4 to make it a perfect cube.

(v) On factorising 7803 into prime factors, we get:

7803 = 3 x 3 x 3 x 17 x 17

On grouping the factors in triples of equal factors, we get:

7803 = {3 x 3 x 3} x 17 x 17

It is evident that the prime factors of 7803 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 7803 is a not perfect cube. However, if the number is divided by 17 x 17 = 289), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 7803 should be divided by 289 to make it a perfect cube.

(vi) On factorising 107811 into prime factors, we get:

107811 = 3 x 3 x 3 x 3 x 11 x 11 x 11

On group the factors in triples of equal factors, we get: 107811 = {3 x 3 x 3} x 3 x {11 x 11 x 11}

It is evident that the prime factors of 107811 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 107811 is a not perfect cube.

However, if the number is divided by 3, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 107811 should be divided by 3 to make it a perfect cube.

(vii) On factorising 35721 into prime factors, we get:

35721 = 3 x 3 x 3 x 3 x 3 x 3 x 7 x 7

On grouping the factors in triples of equal factors, we get:

35721 = {3 x 3 x 3} x {3 x 3 x 3} x 7 x7

It is evident that the prime factors of 35721 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 35721 is a not perfect cube.

However, if the number is divided by (7 x 7= 49), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 35721 should be divided by 49 to make it a perfect cube.

(viii) On factorising 243000 into prime factors, we get:

243000 = 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3 x 5 x 5 x 5

On grouping the factors in triples of equal factors, we get:

243000 = {2 x 2 x 2} x {3 x 3 x 3} x 3 x 3 x {5 x 5 x 5}

It is evident that the prime factors of 243000 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243000 is a not perfect cube. However, if the number is divided by (3 x 3= 9), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243000 should be divided by 9 to make it a perfect cube.

Q13. Prove that if a number is trebled they its cube is 27 times the cube of the given number.

Answer:

Let us consider a number n. Then its cube be n^3.

If the number n is trebled, i.e., 3n, we get:

$(3n)^3 = 3^n \times n^3 = 27\ n^3$

It is evident that the cube of 3n is 27 times of the cube of n.

Hence, the statement is proved.

Q14. What happens to the cube of a number if the number is multiplied by

(i) 3? (ii) 4? (iii) 5?

Answer:

(i) Let us consider a number n. Its cube would be n^3. If n is multiplied by 3, it becomes 3n.

Let us now find the cube of 3n, we get: $(3n)^3 = 3^3 \times n^3 = 27n^3$ Therefore, the cube of 3n is 27 times of the cube of n.

Thus, if a number is multiplied by 3, its cube is 27 times of the cube of that number.

(ii) Let us consider a number n. Its cube would be n^3. If n is multiplied by 4, it becomes 4n. Let us now find the cube of 4n, we get: $(4n)^3 = 4^3 \times n^3 = 64n^3$ Therefore, the cube of 4n is 64 times of the cube of n.

Thus, if a number is multiplied by 4, its cube is 64 times of the cube of that number.

(iii) Let us consider a number n. Its cube would be n3. If the number n is multiplied by 5, it becomes 5n. Let us now find the cube of 4n, we get: $(5n)^3 = 5^3$ x $n^3 = 125n^3$ Therefore, the cube of 5n is 125 times of the cube of n.

Thus, if a number is multiplied by 5, its cube is 125 times of the cube of that number.

Q15. Find the volume of a cube, one face of which has an area of 64 m^2.

Answer :

Area of a face of cube is given by:

$A = s^2$, where s = Side of the cube

Further, volume of a cube is given by:

$V = s^3$, where s = Side of the cube

It is given that the area of one face of the cube is 64 m^2. Therefore we have:

$s^2 = 64$

=> $s = \sqrt{64} = 8$ m

Now, volume is given by:

$V = s^3 = 8^3$

V = 8 x 8 x 8 = 512 m^3

Thus, the volume of the cube is 512 m^3.

Q16. Find the volume of a cube whose surface area is 384 m^2.

Answer :

Surface area of a cube is given by: SA = 682, where s = Side of the cube

Further, volume of a cube is given by: V = s3, where s = Side of the cube

It is given that the surface area of the cube is 384 m2. Therefore, we have: 6s2 = 384 s = a = 1,/64 = 8 m Now, volume is given by: V = s3 = 83 V= 8 x 8 x 8 =512 m3 Thus, the required volume is 512 m3.

Q17. Evaluate the following:

(i) $\{(5^2 + 12^2)^{1/2}\}3$ (ii) $\{(6^2 + 8^2)^{1/2}\}^3$

Answer:

(i) To evaluate the value of the given expression, we can proceed as follows:

$\{(5^2 + 12^2)^{1/2}\}^3$

$= \{(25 + 144)^{1/2}\}^3 = \{(169)^{1/2}\}^3$

=

$(\sqrt{169})^3$
$(\sqrt{13 \times 13})^3$
13^3
$13 \times 13 \times 13$
$= 2197$

(ii) To evaluate the value of the given expression, we can proceed as follows:

$\{(6^2 + 8^2)^{1/2}\}^3$

$= \{(36 + 64)^{1/2}\}^3$

$= \{(100)^{1/2}\}^3$

=

$(\sqrt{100})^3$

$= (\sqrt{10 \times 10})^3$

$= 10^3$

$= 10 \times 10 \times 10$

$= 1000$

Q18. Write the units digit of the cube of each of the following numbers: 31, 109, 388, 833, 4276, 5922, 77774, 44447, 125125125

Answer :

Properties:

If numbers end with digits 1, 4, 5, 6 or 9, its cube will have the same ending digit.

If a number ends with 2, its cube will end with 8.

If a number ends with 8, its cube will end with 2.

If a number ends with 3, its cube will end with 7.

If a number ends with 7, its cube will end with 3.

From the above properties, we get:

Cube of the number 31 will end with 1.

Cube of the number 109 will end with 9.

Cube of the number 388 will end with 2.

Cube of the number 833 will end with 7.

Cube of the number 4276 will end with 6.

Cube of the number 5922 will end with 8.

Cube of the number 77774 will end with 4.

Cube of the number 44447 will end with 3.

Cube of the number 125125125 will end with 5.

Q19. Find the cubes of the following numbers by column method:

(i) 35 (ii) 56 (iii) 72

Answer:

(i) We have to find the cube of 35 using column method. We have:

a = 3 and b = 5

Column I a^3	**Column II** $3 \times a^2 \times b$	**Column III** $3 \times a \times b^2$	**Column IV** b^3
$3^3 = 27$	$3 \times a^2 \times b = 3 \times 3^2 \times 5 = 135$	$3 \times a \times b^2 = 3 \times 3 \times 5^2 = 225$	$5^3 = 125$
+ 15	+ 23	+ 12	12<u>5</u>
<u>42</u>	15<u>8</u>	23<u>7</u>	
42	8	7	5

Thus, the cube of 35 is 42875.

(ii) We have to find the cube of 56 using the column method. We have:

a = 5 and b = 6

Column I **a^3**	**Column II** **$3 \times a^2 \times b$**	**Column III** **$3 \times a \times b^2$**	**Column IV** **b^3**
$5^3 = 125$	$3 \times a^2 \times b = 3 \times 5^2 \times 6 = 450$	$3 \times a \times b^2 = 3 \times 5 \times 6^2 = 540$	$6^3 = 216$
+ 50	+ 56	+ 21	21<u>6</u>
<u>175</u>	50<u>6</u>	56<u>1</u>	
175	6	1	6

(iii) We have to find the cube of 72 using the column method. We have:

a = 7 and b = 2

Column I **a^3**	**Column II** **$3 \times a^2 \times b$**	**Column III** **$3 \times a \times b^2$**	**Column IV** **b^3**
7^3	$3 \times a^2 \times b = 3 \times 7^2 \times 2 = 294$	$3 \times a \times b^2 = 3 \times 7 \times 2^2 = 84$	$2^3 = 8$
+ 30	+ 8	+ 0	<u>8</u>
<u>373</u>	30<u>2</u>	8<u>4</u>	
373	2	4	8

Q20. Which of the following numbers are not perfect cubes?

(i) 64 (ii) 216 (iii) 243 (iv) 1728

Answer :

(i) On factorising 64 into prime factors, we get:

64 = 2 x 2 x 2 x 2 x 2 x 2

On grouping the factors in triples of equal factors, we get: 64 = {2 x 2 x 2} x {2 x 2 x 2}

It is evident that the prime factors of 64 can be grouped into triples of equal factors and no factor is left over.

Therefore, 64 is a perfect cube.

(ii) On factorising 216 into prime factors, we get:

216 = 2 x 2 x 2 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

216 = {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 216 can be grouped into triples of equal factors and no factor is left over.

Therefore, 216 is a perfect cube.

(iii) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 = {3 x 3 x 3} x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 243 is a not perfect cube.

(iv) On factorising 1728 into prime factors, we get:

1728 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

1728 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3}

It is evident that the prime factors of 1728 can be grouped into triples of equal factors and no factor is left over.

Therefore, 1728 is a perfect cube.

Thus, (iii) 243 is the required number, which is not a perfect cube.

Q21. For each of the non-perfect cubes in Q. No. 20 find the smallest number by which it must be:

(a) multiplied so that the product is a perfect cube.

(b) divided so that the quotient is a perfect cube.

Answer :

The only non-perfect cube in question number 20 is 243.

(a) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 ={3 x 3 x 3}x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is not a perfect cube.

However, if the number is multiplied by 3, the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243 should be multiplied by 3 to make it a perfect cube.

(b) On factorising 243 into prime factors, we get:

243 = 3 x 3 x 3 x 3 x 3

On grouping the factors in triples of equal factors, we get:

243 = {3 x 3 x 3} x 3 x 3

It is evident that the prime factors of 243 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 243 is not a perfect cube.

However, if the number is divided by 3 x 3 = 9), the factors can be grouped into triples of equal factors such that no factor is left over.

Thus, 243 should be divided by 9 to make it a perfect cube.

Q.22: By taking three different values of n verify the truth of the following statements:

(i) If n is even, then n^3 is also even.

(ii) if n is odd, then n^3 is also odd.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as the remainder when divided by 3.

(iv) If a natural number n is of the form 3p + 2 then n^3 also a number of the same type.

Answer :

(i) Let the three even natural numbers be 2, 4 and 8.

Cubes of these numbers are:

$2^3 = 8$, $4^3 = 64$, $8^3 = 512$

By divisibility test, it is evident that 8, 64 and 512 are divisible by 2.

Thus, they are even. This verifies the statement.

(ii) Let the three odd natural numbers be 3, 9 and 27. Cubes of these numbers are: $3^3 = 27$, $9^3 = 729$, $27^3 = 19683$

By divisibility test, it is evident that 27, 729 and 19683 are divisible by 3.

Thus, they are odd.

This verifies the statement.

(iii) Three natural numbers of the form (3n + 1) can be written by choosing n = 1,2,3… etc.

Let three such numbers be 4,7 and 10. Cubes of the three chosen numbers are: $4^3 = 64$, $7^3 = 343$ and $10^3 = 1000$

Cubes of 4,7 and 10 can expressed as:

64 = 3 x 21 + 1, which is of the form (3n + 1) for n = 21

343 = 3 x 114 + 1, which is of the form (3n + 1) for n = 114

1000 = 3 x 333 + 1, which is of the form (3n + 1) for n = 333

Cubes of 4, 7, and 10 can be expressed as the natural numbers of the form (3n + 1) for some natural number n. Hence, the statement is verified.

(iv) Three natural numbers of the form (3p + 2) can be written by choosing p = 1,2,3… etc.

Let three such numbers be 5, 8 and 11.

Cubes of the three chosen numbers are: $5^3 = 125$, $8^3 = 512$ and $11^3 = 1331$

Cubes of 5, 8, and 11 can be expressed as:

125 = 3 x 41 + 2, which is of the form (3p + 2) for p = 41

512 = 3 x 170 + 2, which is of the form (3p + 2) for p = 170

1331 = 3 x 443 + 2, which is of the form (3p + 2) for p = 443

Cubes of 5, 8, and 11 could be expressed as the natural numbers of the form (3p + 2) for some natural number p. Hence, the statement is verified.

Q23. Write true (T) or false (F) for the following statements:

(i) 392 is a perfect cube.

(ii) 8640 is not a perfect cube.

(iii) No cube can end with exactly two zeros.

(iv) There is no perfect cube which ends in 4.

(v) For an integer a, a^3 is always greater than a^2.

(vi) If a and b are integers such that $a^2 > b^2$, then $a^3 > b^3$.

(vii) if a divides b, then a^3 divides b^3

(viii) If a^2 ends in 9, then a^3 ends in 7.

(ix) If a^2 ends in 5, then a^3 ends in 25.

(x) I a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

Answer:

(i) False

On factorising 392 into prime factors, we get:

392 = 2 x 2 x 2 x 7 x 7

On grouping the factors in triples of equal factors, we get:

392 = {2 x 2 x 2} x 7 x 7

It is evident that the prime factors of 392 cannot be grouped into triples of equal factors such that no factor is left over. Therefore, 392 is not a perfect cube.

(ii) True

On factorising 8640 into prime factors, we get:

8640 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5

On grouping the factors in triples of equal factors, we get:

8640 = {2 x 2 x 2} x {2 x 2 x 2} x {3 x 3 x 3} x 5

It is evident that the prime factors of 8640 cannot be grouped into triples of equal factors such that no factor is left over.

Therefore, 8640 is not a perfect cube.

(iii) True

Because a perfect cube always ends with multiples of 3 zeros, e.g., 3 zeros, 6 zeros etc.

(iv) False.

64 is a perfect cube, and it ends with 4.

(v) False

It is not true for a negative integer Example: $(-5)^2 = 25$; $(-5)^3 = -125$

=> $(-5)^3 < (-5)^2$

(vi) False

It is not true for negative integers. Example: $(-5)^2 > (-4)^2$ but $(-5)^3 < (-4)^3$

(vii) True

a divides b

$$\frac{b^3}{a^3} = \frac{b \times b \times b}{a \times a \times a} = \frac{(ak) \times (ak) \times (ak)}{a \times a \times a}$$

a divides b

b = ak for some k

$$\frac{b^3}{a^3} = \frac{(ak) \times (ak) \times (ak)}{a \times a \times a} = k^3$$

$$\Rightarrow b^3 = a^3(k^3)a^3 \text{ divides } b^3$$

(viii) False

a^3 ends in 7 if a ends with 3.

But for every a^2 ending in 9, it is not necessary that a is 3.

E.g., 49 is a square of 7 and cube of 7 is 343.

(ix) False

$35^2 = 1225$ but $35^3 = 42875$

(x) False

$100^2 = 10000$ and $100^3 = 100000$