

RD SHARMA

Solutions

Class 8 Maths

Chapter 4

Ex 4.4

Q1. Find the cube roots of each of the following integers:

(i) -125

(ii) -5832

(iii) -2744000

(iv) -753571

(v) -32768

Answer:

(i) We have:

$$\sqrt[3]{-125} = -\sqrt[3]{125} = -\sqrt[3]{5 \times 5 \times 5} = -5$$

(ii) We have:

$$\sqrt[3]{-5832} = -\sqrt[3]{5832}$$

To find the cube root of 5832, we use the method of unit digits.

Let us consider the number 5832. The unit digit is 2; therefore the unit digit in the cube root of 5832 will be 8. After striking out the units, tens and hundreds of digits of the given number, we are left with 5.

Now, 1 is the largest number whose cube is less than or equal to 5. Therefore, the tens digit of the cube root of 5832 is 1.

$$\sqrt[3]{5832} = 18$$

$$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$$

(iii) We have:

$$\sqrt[3]{-2744000} = -\sqrt[3]{2744000}$$

To find the cube root of 2744000, we use the method of factorization.

On factorizing 2744000 into prime factors, we get:

$$2744000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$$

On grouping the factors in triples of equal factors, we get:

$$2744000 = (2 \times 2 \times 2)(2 \times 2 \times 2)(5 \times 5 \times 5)(7 \times 7 \times 7)$$

It is evident that the prime factors of 2744000 can be grouped into triples of equal factors and no factor is left over.

Now, collect one factor from each triplet and multiply; we get: $(2 \times 2 \times 5 \times 7) = 140$ This implies that 2744000 is a cube of 140.

Hence

$$\sqrt[3]{-2744000} = -\sqrt[3]{2744000} = -140$$

(iv) We have:

$$\sqrt[3]{-753571} = -\sqrt[3]{753571}$$

To find the cube root of 753571, we use the method of unit digits.

Let us consider the number 753571. The unit digit is 1; therefore the unit digit in the cube root of 753571 will be 1. After striking out the units, tens and hundreds digits of the given number, we are left with 753. Now, 9 is the largest number whose cube is less than or equal to 753 ($9^3 < 753 < 10^3$). Therefore, the tens digit of the cube root 753571 is 9.

$$\sqrt[3]{753571} = 91$$

$$\sqrt[3]{-753571} = -\sqrt[3]{753571} = -91$$

(v) We have:

$$\sqrt[3]{-32768} = -\sqrt[3]{32768}$$

To find the cube root of 32768, we use the method of unit digits.

Let us consider the number 32768.

The unit digit is 8; therefore, the unit digit in the cube root of 32768 will be 2. After striking out the units, tens and hundreds digits of the given number, we are left with 32. Now, 3 is the largest number whose cube is less than or equal to 32 ($3^3 < 32 < 4^3$). Therefore, the tens digit of the cube root 32768 is 3.

$$\sqrt[3]{32768} = 32$$

$$\sqrt[3]{-32768} = -\sqrt[3]{32768} = -32$$

Q2. Show that:

$$(i) \sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{27 \times 64}$$

$$(ii) \sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$$

$$(iii) \sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$$

$$(iv) \sqrt[3]{-125 \times -1000} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

Answer:

(i)

$$\text{LHS} = \sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{4 \times 4 \times 4} = 3 \times 4 = 12$$

$$\text{RHS} = \sqrt[3]{27 \times 64} = \sqrt[3]{3 \times 3 \times 3 \times 4 \times 4 \times 4} = 3 \times 4 = 12$$

Because LHS is equal to RHS, the equation is true.

(ii)

$$\text{LHS} = \sqrt[3]{64 \times 729} = \sqrt[3]{4 \times 4 \times 4 \times 9 \times 9 \times 9} = 4 \times 9 = 36$$

$$\text{RHS} = \sqrt[3]{64} \times \sqrt[3]{729} = \sqrt[3]{4 \times 4 \times 4} \times \sqrt[3]{9 \times 9 \times 9} = 4 \times 9 = 36$$

Because LHS is equal to RHS, the equation is true.

(iii)

$$\text{LHS} = \sqrt[3]{-125 \times 216} = \sqrt[3]{-5 \times -5 \times -5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3} = -5 \times 2 \times 3 = -30$$

$$\text{RHS} = \sqrt[3]{-125} \times \sqrt[3]{216} = \sqrt[3]{-5 \times -5 \times -5} \times \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = -5 \times 2 \times 3 = -30$$

Because LHS is equal to RHS, the equation is true.

(iv)

$$\text{LHS} = \sqrt[3]{-125 \times -1000} = \sqrt[3]{-5 \times -5 \times -5 \times -10 \times -10 \times -10} = -5 \times -10 = 50$$

$$\text{RHS} = \sqrt[3]{-125} \times \sqrt[3]{-1000} = \sqrt[3]{-5 \times -5 \times -5} \times \sqrt[3]{-10 \times -10 \times -10} = -5 \times -10 = 50$$

Because LHS is equal to RHS, the equation is true.

Q3. Find the cube root of the following numbers:

$$(i) 8 \times 125$$

$$(ii) -1728 \times 216$$

$$(iii) -27 \times 2744$$

$$(iv) -729 \times -15625$$

Answer:

(i) From the above property, we have:

$$\sqrt[3]{8 \times 125} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5} = 2 \times 5 = 10$$

(ii) From the above property, we have:

$$\begin{aligned} & \sqrt[3]{-1728 \times 216} \\ &= \sqrt[3]{-1728} \times \sqrt[3]{216} \\ &= -\sqrt[3]{1728} \times \sqrt[3]{216} \end{aligned}$$

Cube root using units digit: Let us consider the number 1728. The unit digit is 8; therefore, the unit digit in the cube root of 1728 will be 2. After striking out the units, tens and hundreds digits of the given number, we are left with 1. Now, 1 is the largest number whose cube is less than or equal to 1. Therefore, the tens digit of the cube root of 1728 is 1.

$$\sqrt[3]{-1728} = -12$$

On factorizing 216 into prime factors, we get:

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

On grouping the factors in triples of equal factors, we get:

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Now, taking one factor from each triple, we get:

$$216 = 2 \times 3$$

Thus

$$\sqrt[3]{-1728} \times \sqrt[3]{216} = -12 \times 6 = -72$$

(iii) From the above property, we have:

$$\begin{aligned} & \sqrt[3]{-27 \times 2744} \\ &= \sqrt[3]{-27} \times \sqrt[3]{2744} \\ &= -\sqrt[3]{27} \times \sqrt[3]{2744} \end{aligned}$$

Cube root using units digit:

Let us consider the number 2744.

The unit digit is 4; therefore, the unit digit in the cube root of 2744 will be 4. After striking out the units, tens, and hundreds digits of the given number, we are left with 2. Now, 2 is the largest number whose cube is less than or equal to 2. Therefore, the tens digit of the cube root of 2744 is 2.

$$\dots 12/2744 = 14$$

Thus

$$\sqrt[3]{-27} \times \sqrt[3]{2744} = -3 \times 14 = -42$$

(iv) From the above property, we have:

$$\begin{aligned} & \sqrt[3]{-729 \times -15625} \\ &= \sqrt[3]{-729} \times \sqrt[3]{-15625} \\ &= -\sqrt[3]{729} \times -\sqrt[3]{15625} \end{aligned}$$

Cube root using units digit:

Let us consider the number 15625.

The unit digit is 5; therefore, the unit digit in the cube root of 15625 will be 5. After striking out the units, tens and hundreds digits of the given number, we are left with 15. Now, 2 is the largest number whose cube is less than or equal to 15 ($2^3 < 15 < 3^3$). Therefore, the tens digit of the cube root of 15625 is 2.

$$\sqrt[3]{15625} = 25$$

Also

$$\sqrt[3]{729}=9$$

Thus

$$\sqrt[3]{-729} \times \sqrt[3]{-15625} = -9 \times 25 = 225$$

Q4. Evaluate:

(i) $\sqrt[4]{4^3 \text{ times } 6^3}$

(ii) $\sqrt[3]{8 \text{ times } 17 \times 17 \times 17}$

(iii) $\sqrt[3]{700 \text{ times } 2 \times 49 \times 5}$

(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$

Answer:

For any two integers a and b,

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

(i) From the above property, we have:

$$\sqrt[4]{4^3 \text{ times } 6^3} = \sqrt[4]{4^3} \times \sqrt[4]{6^3} = 4 \times 6 = 24$$

(ii) Use above property and proceed as follows:

$$\sqrt[3]{8 \text{ times } 17 \times 17 \times 17} = \sqrt[3]{2^3} \times \sqrt[3]{17^3} = 2 \times 17 = 34$$

(iii) From the above property, we have:

$$\begin{aligned} & \sqrt[3]{700 \text{ times } 2 \times 49 \times 5} \\ &= \sqrt[3]{2 \times 2 \text{ times } 5 \times 5 \times 7 \times 2 \times 7 \times 7 \times 5} \\ &= \sqrt[3]{2^3 \text{ times } 5^3 \times 7^3} \\ &= \sqrt[3]{700 \text{ times } 2 \times 49 \times 5} \\ &= 2 \times 5 \times 7 = 70 \end{aligned}$$

(iv) From the above property, we have:

$$\begin{aligned} & 125\sqrt[3]{a^6} - \sqrt[3]{125a^6} \\ &= 125\sqrt[3]{a^6} - (\sqrt[3]{125} \times \sqrt[3]{a^6}) \\ &= 125 \times a^2 - (5 \times a^2) \\ & \sqrt[3]{a^6} = \sqrt[3]{(a \times a \times a)(a \times a \times a)} = a \times a = a^2 \text{ and} \\ & \sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} \\ &= 125a^2 - 5a^2 \\ &= 120a^2 \end{aligned}$$

Q5. Find the cube root of each of the following rational numbers:

(i) $\frac{-125}{729}$

(ii) $\frac{10648}{12167}$

$$(iii) \frac{-19683}{24389}$$

$$(iv) \frac{686}{-3456}$$

$$(v) \frac{-393304}{-42875}$$

Answer:

(i) Let us consider the following rational number:

$$\frac{-125}{729}$$

Now

$$\sqrt[3]{\frac{-125}{729}} = \frac{\sqrt[3]{-125}}{\sqrt[3]{729}} = \frac{-\sqrt[3]{125}}{\sqrt[3]{729}} = -\frac{5}{9}$$

(ii) Let us consider the following rational number:

$$\frac{10648}{12167}$$

Now

$$\sqrt[3]{\frac{10648}{12167}} = \frac{\sqrt[3]{10648}}{\sqrt[3]{12167}}$$

Cube root by factors:

On factorizing 10648 into prime factors, we get: $10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

On grouping the factors in triples of equal factors, we get: $10648 = (2 \times 2 \times 2)(11 \times 11 \times 11)$

Now, taking one factor from each triple, we get: $\sqrt[3]{10648} = 2 \times 11$

Also

On factorizing

12167 into prime factors, we get: $12167 = 23 \times 23 \times 23$

On grouping the factors in triples of equal factors, we get: $12167 = (23 \times 23 \times 23)$

Now, taking one factor from the triple, we get: 23

Now

$$\sqrt[3]{\frac{10648}{12167}} = \frac{\sqrt[3]{10648}}{\sqrt[3]{12167}} = \frac{22}{23}$$

(iii) Let us consider the following rational number:

$$\frac{-19683}{24389}$$

Now,

$$\sqrt[3]{\frac{-19683}{24389}}$$

$$\frac{\sqrt[3]{-19683}}{\sqrt[3]{24389}}$$

$$\frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}}$$

Cube root by factors:

On factorizing 19683 into prime factors, we get: $19683 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

On grouping the factors in triples of equal factors, we get: $19683 = (3 \times 3 \times 3)(3 \times 3 \times 3)(3 \times 3 \times 3)$

Now, taking one factor from each triple, we get: $\sqrt[3]{19683} = 3 \times 3 \times 3 = 27$

Also

On factorising 24389 into prime factors, we get:

$$\sqrt[3]{24389} = 29 \times 29 \times 29$$

On grouping the factors in triples of equal factors, we get: $\sqrt[3]{24389} = 29 \times 29 \times 29$

Now, taking one factor from each triple, we get: $\sqrt[3]{24389} = 29$

Now

$$\sqrt[3]{\frac{-19683}{24389}} = \sqrt[3]{\frac{-19683}{24389}} = \frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}} = \frac{-27}{29}$$

(iv)

Let us consider the following rational number:

$$\frac{686}{-3456}$$

Now

$$\sqrt[3]{\frac{686}{-3456}} = \sqrt[3]{\frac{2 \times 7^3}{2^7 \times 3^3}} = \sqrt[3]{\frac{7^3}{2^6 \times 3^3}} = \frac{-\sqrt[3]{7^3}}{\sqrt[3]{2^6 \times 3^3}} = \frac{-7}{2 \times 2 \times 3} = \frac{-7}{12}$$

(v) Let us consider the following rational number:

$$\frac{-393304}{-42875}$$

Now

$$\sqrt[3]{\frac{-393304}{-42875}} = \sqrt[3]{\frac{-393304}{-42875}} = \frac{\sqrt[3]{-393304}}{\sqrt[3]{-42875}}$$

Cube root by factors:

On factorizing 39304 into prime factors, we get:

$$39304 = 2 \times 2 \times 2 \times 17 \times 17 \times 17$$

On grouping the factors in triples of equal factors, we get:

$$39304 = (2 \times 2 \times 2)(17 \times 17 \times 17)$$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{393304} = 2 \times 17 = 34$$

Also

On factorizing 42875 into prime factors, we get:

$$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$$

On grouping the factors in triples of equal factors, we get:

$$42875 = (5 \times 5 \times 5)(7 \times 7 \times 7)$$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{42875} = 5 \times 7 = 35$$

Now

$$\sqrt[3]{\frac{-393304}{-42875}} = \frac{-\sqrt[3]{-393304}}{\sqrt[3]{-42875}} = \frac{-34}{-35} = \frac{34}{35}$$

Q6. Find the cube root of each of the following rational numbers

(i) 0.001728

(ii) 0.003375

(iii) 0.001

(iv) 1.331

Answer:

(i)

We have:

$$0.001728 = \frac{1728}{1000000}$$

$$\sqrt[3]{\frac{1728}{1000000}} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}}$$

Now

On factorizing 1728 into prime factors, we get:

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

On grouping the factors in triples of equal factors, we get:

$$1728 = (2 \times 2 \times 2)(2 \times 2 \times 2)(3 \times 3 \times 3)$$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$$

Also

$$\sqrt[3]{1000000} = 100 \times 100 \times 100 = 100$$

$$0.001728 = \frac{1728}{1000000}$$

$$\sqrt[3]{\frac{1728}{1000000}} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}}$$

$$\frac{12}{100} = 0.12$$

(ii)

We have:

$$0.003375 = \frac{3375}{1000000}$$

$$\sqrt[3]{\frac{3375}{1000000}} = \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$$

Now

On factorizing 3375 into prime factors, we get:

$$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

On grouping the factors in triples of equal factors, we get:

$$3375 = (3 \times 3 \times 3)(5 \times 5 \times 5)$$

Now, taking one factor from each triple, we get:

$$\sqrt[3]{3375} = 3 \times 5 = 15$$

Also

$$\sqrt[3]{1000000} = 100 \times 100 \times 100 = 100$$

$$0.003375 = \frac{3375}{1000000}$$

$$\sqrt[3]{\frac{3375}{1000000}} = \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$$

$$\frac{15}{100}=0.15$$

(iii)

We have:

$$0.001 = \frac{1}{1000}$$

$$\sqrt[3]{\frac{1}{1000}} = \frac{\sqrt[3]{1}}{\sqrt[3]{1000}}$$

$$\frac{1}{10}=0.1$$

(iv)

We have:

$$1.331 = \frac{1331}{1000}$$

$$\sqrt[3]{\frac{1331}{1000}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}}$$

$$\frac{11}{10}=1.1$$

Q7. Evaluate each of the following:

(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$

(iii) $\sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$

(iv) $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt[3]{\frac{0.09}{0.04}}$

(v) $\sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13}$

To evaluate the value of the given expression, we need to proceed as follows:

$$\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064} =$$

$$= \sqrt[3]{3 \times 3 \times 3} + \sqrt[3]{\frac{8}{1000}} + \sqrt[3]{\frac{64}{1000}}$$

$$= \sqrt[3]{3 \times 3 \times 3} + \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} + \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$$

$$= \sqrt[3]{3 \times 3 \times 3} + \frac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{1000}} + \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{1000}}$$

$$= 3 + \frac{2}{10} + \frac{4}{10}$$

$$= 3 + 0.2 + 0.4 = 3.6$$

Thus, the answer is 3.6.

(ii) To evaluate the value of the given expression, we need to proceed as follows:

$$\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125} =$$

$$= \sqrt[3]{10 \times 10 \times 10} + \sqrt[3]{\frac{8}{1000}} - \sqrt[3]{\frac{125}{1000}}$$

$$= \sqrt[3]{10 \times 10 \times 10} + \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{125}}{\sqrt[3]{1000}}$$

$$= \sqrt[3]{10 \times 10 \times 10} + \frac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{1000}}$$

$$= 10 + \frac{2}{10} - \frac{5}{10}$$

$$= 10 + 0.2 + 0.5 = 9.7$$

Thus, the answer is 9.7.

(iii) To evaluate the value of the given expression, we need to proceed as follows:

$$\begin{aligned} & \sqrt[3]{\frac{729}{216}} \times \frac{6}{9} \\ & \sqrt[3]{\frac{9 \times 9 \times 9}{2 \times 2 \times 2 \times 3 \times 3 \times 3}} \times \frac{6}{9} \\ & \frac{9}{2 \times 3} \times \frac{6}{9} = 1 \end{aligned}$$

Thus, the answer is 1.

(iv) To evaluate the value of the expression, we need to proceed as follows:

$$\begin{aligned} & \sqrt[3]{\frac{0.027}{0.008}} \div \sqrt[3]{\frac{0.09}{0.04}} - 1 \\ & \sqrt[3]{\frac{27}{8}} \div \sqrt[3]{\frac{9}{4}} - 1 \\ & \sqrt[3]{\frac{27}{8}} \div \sqrt[3]{94} - 1 \\ & \frac{\sqrt[3]{27}}{\sqrt[3]{8}} \div \frac{\sqrt[3]{9}}{\sqrt[3]{4}} - 1 \\ & \frac{3}{2} \div \frac{3}{2} - 1 \\ & \frac{3}{2} \times \frac{2}{3} - 1 \\ & 1 - 1 = 0 \end{aligned}$$

(v) To evaluate the value of the expression, we need to proceed as follows:

$$\begin{aligned} & \sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13} \\ & \sqrt[3]{\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times 13 \times 13 \times 13} \\ & \frac{13}{10} \\ & = 1.3 \end{aligned}$$

Thus, the answer is 1.3.

Q9. Fill in the blanks:

(i) $\sqrt[3]{125 \times 27} = 3 \times \dots\dots$

(ii) $\sqrt[3]{8 \times \dots}$

(iii) $\sqrt[3]{1728} = 12 = 4 \times \dots\dots$

(iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{\dots}$

(v) $\sqrt[3]{\dots} = \sqrt[3]{7} \times \sqrt[3]{8}$

(vi) $\sqrt[3]{\dots} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{5}$

$$(vii) \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\dots}{5}$$

$$(viii) \frac{\sqrt[3]{729}}{\sqrt[3]{1331}} = \frac{9}{\dots}$$

$$(ix) \frac{\sqrt[3]{512}}{\sqrt[3]{\dots}} = \frac{8}{13}$$

Answer:

(i) 5

$$\begin{aligned} \sqrt[3]{125 \times 27} &= 3 \times 5 \\ \sqrt[3]{125 \times 27} &= \sqrt[3]{125} \times \sqrt[3]{27} = \sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{3 \times 3 \times 3} = 5 \times 3 \\ &= 3 \times 5 \end{aligned}$$

(ii)

$$8 \times 8 = 64 \quad \sqrt[3]{8 \times 8 \times 8} = 8$$

(iii)

3

$$\sqrt[3]{1728} = 12 = 4 \times 3$$

(iv)

$$\sqrt[3]{480} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5} = 2\sqrt[3]{3} \times \sqrt[3]{5 \times 2 \times 2} = \sqrt[3]{3} \times 2 \times \sqrt[3]{20}$$

(v)

$$7 \times 8 = 56 \quad \sqrt[3]{7 \times 8} = \sqrt[3]{7} \times \sqrt[3]{8}$$

(vi)

$$7 \times 8 = 56 \quad \sqrt[3]{7 \times 8} = \sqrt[3]{7} \times \sqrt[3]{8}$$

Q10. The volume of a cubical box is 474.552 cubic meters. Find the length of each side of the box.

Answer:

Volume of a cube is given by:

$$V = s^3, \text{ where } s = \text{side of the cube}$$

$$\text{Now } s^3 = 474.552 \text{ cubic metres}$$

$$= \sqrt[3]{474.552} = \sqrt[3]{\frac{474552}{1000}} = \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}}$$

To find the cube root of 474552, we need to proceed as follows:

On factorising 474552 into prime factors, we get:

$$474552 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13$$

On grouping the factors in triples of equal factors, we get:

$$474552 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (13 \times 13 \times 13)$$

Now, taking one factor from each triple, we get:

$$= \sqrt[3]{474552} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13} = 2 \times 3 \times 13 = 78$$

$$\text{Also } \sqrt[3]{1000} = 10$$

$$s = \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}} = \frac{78}{10} = 7.8$$

$$= 2 \times 2 \times 2 \times 3 = 24$$

Thus, the answer is 24.

(ii)

96 and 122 are not perfect cubes; therefore, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

$$= \sqrt[3]{96} \times \sqrt[3]{144}$$

$$= \sqrt[3]{96 \times 144}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3} \quad (\text{By prime factorisation})$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$$

$$= 2 \times 2 \times 2 \times 3 = 24$$

Thus, the answer is 24.

(iii) 100 and 270 are not perfect cubes; therefore, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

$$= \sqrt[3]{100} \times \sqrt[3]{270}$$

$$= \sqrt[3]{100 \times 270}$$

$$= \sqrt[3]{2 \times 2 \times 5 \times 5 \times 2 \times 3 \times 3 \times 3 \times 5} \quad (\text{By prime factorisation})$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5}$$

$$= 2 \times 3 \times 5 = 30$$

Thus, the answer is 30.

(iv) 121 and 297 are not perfect cubes; therefore, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

$$= \sqrt[3]{121} \times \sqrt[3]{297}$$

$$= \sqrt[3]{121 \times 297}$$

$$= \sqrt[3]{11 \times 11 \times 3 \times 3 \times 3 \times 11} \quad (\text{By prime factorisation})$$

$$= \sqrt[3]{11 \times 11 \times 11 \times 3 \times 3 \times 3} = 11 \times 3 = 33$$

Thus, the answer is 33.

Q14. Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that:

(i) $3048625 = 3375 \times 729$

(ii) $20346417 = 9261 \times 2197$

(iii) $210644875 = 42875 \times 4913$

(iv) $57066625 = 166375 \times 343$

Answer:

(i) To find the cube root, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

Now

$$\begin{aligned}
& \sqrt[3]{3048625} \\
&= \sqrt[3]{3375 \times 729} \\
&= \sqrt[3]{3375} \times \sqrt[3]{729} \text{(By the above property)} \\
&= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9} \text{(By prime factorisation)} \\
&= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9} \\
&= 3 \times 5 \times 9 = 135
\end{aligned}$$

Thus, the answer is 135.

(ii) To find the cube root, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

Now

$$\begin{aligned}
& \sqrt[3]{20346417} \\
&= \sqrt[3]{9261 \times 2197} \\
&= \sqrt[3]{9261} \times \sqrt[3]{2197} \text{(By the above property)} \\
&= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 13 \times 13 \times 13} \text{(By prime factorisation)} \\
&= 3 \times 7 \times 13 = 273
\end{aligned}$$

Thus, the answer is 273.

(iii) To find the cube root, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

Now

$$\begin{aligned}
& \sqrt[3]{210644875} \\
&= \sqrt[3]{42875 \times 4913} \\
&= \sqrt[3]{42875} \times \sqrt[3]{4913} \text{(By the above property)} \\
&= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 17 \times 17 \times 17} \text{(By prime factorisation)} \\
&= 5 \times 7 \times 17 = 595
\end{aligned}$$

Thus, the answer is 595.

(iv) To find the cube root, we use the following property:

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ for any two integers a and b}$$

Now

$$\begin{aligned}
& \sqrt[3]{57066625} \\
&= \sqrt[3]{166375 \times 343} \\
&= \sqrt[3]{166375} \times \sqrt[3]{343} \text{(By the above property)} \\
&= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11} \text{(By prime factorisation)} \\
&= 5 \times 7 \times 11 = 385
\end{aligned}$$

Thus, the answer is 385.

Q15. Find the units digit of the cube root of the following numbers:

(i) 226981

(ii) 13824

(iii) 571787

(iv) 175616

Answer:

(i) Cube root using units digit:

Let us consider the number 226981. The unit digit is 1; therefore, the unit digit of the cube root of 226981 is 1.

(ii) Cube root using units digit:

Let us consider the number 13824. The unit digit is 4; therefore, the unit digit of the cube root of 13824 is 4.

(iii) Cube root using units digit:

Let us consider the number 571787. The unit digit is 7; therefore, the unit digit of the cube root of 571787 is 3.

(iv) Cube root using units digit:

Let us consider the number 175616. The unit digit is 6; therefore, the unit digit of the cube root of 175616 is 6.

Q16. Find the tens digit of the cube root of the each of the numbers in Q. No. 15.

Answer:

(i) Let us consider the number 226981.

The unit digit is 1; therefore, the unit digit of the cube root of 226981 is 1.

After striking out the units, tens and hundreds of digits of the given number, we are left with 226.

Now, 6 is the largest number, whose cube is less than or equal to 226 ($6^3 < 226 < 7^3$).

Therefore, the tens digit of the cube root of 226981 is 6.

(ii) Let us consider the number 13824.

The unit digit is 4; therefore, the unit digit of the cube root of 13824 is 4.

striking out the units, tens and hundreds of digits of the given number, we are left with 13.

Now, 2 is the largest number, whose cube is less than or equal to 13 ($2^3 < 13 < 3^3$).

Therefore, the tens digit of the cube root of 13824 is 2.

(iii) Let us consider the number 571787.

The unit digit is 7; therefore, the unit digit of the cube root of 571787 is 3.

After striking out the units, tens and hundreds of digits of the given number, we are left with 571.

Now, 8 is the largest number, whose cube is less than or equal to 571 ($8^3 < 571 < 9^3$).

Therefore, the tens digit of the cube root of 571787 is 8.

(iv) Let us consider the number 175616.

The unit digit is 6; therefore, the unit digit of the cube root of 175616 is 6.

After striking out the units, tens and hundreds of digits of the given number, we are left with 175.

Now, 5 is the largest number, whose cube is less than or equal to 175 ($5^3 < 175 < 6^3$).

Therefore, the tens digit of the cube root of 175616 is 5.