

RD SHARMA

Solutions

Class 8 Maths

Chapter 6

Ex 6.6

Q1 Write the following squares of binomials as trinomials

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and

$$(a - b)^2 = a^2 - 2ab + b^2$$

(i) $(x + 2)^2$

Sol:

$(x + 2)^2$ is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

here, $a = x$, $b = 2$

$$\Rightarrow x^2 + 2 \times x \times 2 + b^2$$

$$\Rightarrow x^2 + 4x + b^2$$

(ii) $(8a + 3b)^2$

Sol:

$(8a + 3b)^2$ is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

here, $a = 8a$, $b = 3b$

$$\Rightarrow (8a)^2 + 2 \times (8a) \times (3b) + (3b)^2$$

$$\Rightarrow 64^2 + 48ab + 36b^2$$

(iii) $(2m + 1)^2$

Sol:

$(2m + 1)^2$ is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

here, $a = 2m$, $b = 1$

$$\Rightarrow (2m)^2 + 2 \times (2m) \times (1) + (1)^2$$

$$\Rightarrow 4m^2 + 4m + 1$$

(iv) $(9a + \frac{1}{6})^2$

Sol:

$(9a + \frac{1}{6})^2$ is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

here, $a = 9a$, $b = \frac{1}{6}$

$$\Rightarrow (9a)^2 + 2 \times (9a) \times (\frac{1}{6}) + (\frac{1}{6})^2$$

$$\Rightarrow 81a^2 + 3a + \frac{1}{36}$$

(v) $(x + \frac{x^2}{2})^2$

Sol:

$(x + \frac{x^2}{2})^2$ is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

here, $a = x$, $b = \frac{x^2}{2}$

$$\Rightarrow x^2 + 2 \times x \times \left(\frac{x^2}{2}\right) + \left(\frac{x^2}{2}\right)^2$$

$$\Rightarrow x^2 + x^3 + \frac{x^4}{4}$$

$$\text{(vi)} \left(\frac{x}{4} - \frac{y}{3}\right)^2$$

Sol:

$$\left(\frac{x}{4} - \frac{y}{3}\right)^2 \text{ is in the form of } (a-b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = \frac{x}{4}, b = \frac{y}{3}$$

$$\Rightarrow \left(\frac{x}{4}\right)^2 - 2 \times \left(\frac{x}{4}\right) \times \left(\frac{y}{3}\right) + \left(\frac{y}{3}\right)^2$$

$$\Rightarrow \frac{x^2}{16} - \frac{1}{6}xy + \frac{y^2}{9}$$

$$\text{(vii)} \left(3x - \frac{1}{3x}\right)^2$$

Sol:

$$\left(3x - \frac{1}{3x}\right)^2 \text{ is in the form of } (a-b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = 3x, b = \frac{1}{3x}$$

$$\Rightarrow (3x)^2 - 2 \times 3x \times \left(\frac{1}{3x}\right) + \left(\frac{1}{3x}\right)^2$$

$$\Rightarrow 9x^2 - 2 + \frac{1}{9x^2}$$

$$\text{(viii)} \left(\frac{x}{y} - \frac{y}{x}\right)^2$$

Sol:

$$\left(\frac{x}{y} - \frac{y}{x}\right)^2 \text{ is in the form of } (a-b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = \frac{x}{y}, b = \frac{y}{x}$$

$$\Rightarrow \left(\frac{x}{y}\right)^2 - 2 \times \left(\frac{x}{y}\right) \times \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$$

$$\Rightarrow \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$$

$$\text{(ix)} \left(\frac{3a}{2} - \frac{5b}{4}\right)^2$$

Sol:

$$\left(\frac{3a}{2} - \frac{5b}{4}\right)^2 \text{ is in the form of } (a-b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = \frac{3a}{2}, b = \frac{5b}{4}$$

$$\Rightarrow \left(\frac{3a}{2}\right)^2 - 2 \times \left(\frac{3a}{2}\right) \times \left(\frac{5b}{4}\right) + \left(\frac{5b}{4}\right)^2$$

$$\Rightarrow \frac{9a^2}{4} - \frac{15ab}{4} + \frac{25b^2}{16}$$

$$\text{(x)} (a^2b - bc^2)^2$$

Sol:

$$(a^2b - bc^2)^2 \text{ is in the form of } (a-b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = a^2b, b = bc^2$$

$$\Rightarrow (a^2b)^2 - 2 \times (a^2b) \times (bc^2) + (bc^2)^2$$

$$\Rightarrow a^4b^2 - 2a^2b^2c^2 + b^2c^4$$

Q2 Find the product of the following binomials

(i) $(2x + y)(2x + y)$

sol:

$$(2x + y)(2x + y) \text{ can be written as } (2x + y)^2$$

$$\Rightarrow (2x + y)^2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow (2x)^2 + 2 \times (2x) \times (y) + y^2$$

$$\Rightarrow 4x^2 + 4xy + y^2$$

(ii) $(a + 2b)(a - 2b)$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = a, b = 2b$$

$$\Rightarrow a^2 - (2b)^2$$

$$\Rightarrow a^2 - 4b^2$$

(iii) $(a^2 + bc)(a^2 - bc)$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = a^2, b = bc$$

$$\Rightarrow (a^2)^2 - (bc)^2$$

$$\Rightarrow a^4 - b^2c^2$$

(iv) $(\frac{4x}{5} - \frac{3y}{4})(\frac{4x}{5} + \frac{3y}{4})$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = \frac{4x}{5}, b = \frac{3y}{4}$$

$$\Rightarrow (\frac{4x}{5})^2 - (\frac{3y}{4})^2$$

$$\Rightarrow \frac{16x^2}{25} - \frac{9y^2}{16}$$

(v) $(2x + \frac{3}{y})(2x - \frac{3}{y})$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = 2x, b = \frac{3}{y}$$

$$\Rightarrow (2x)^2 - (\frac{3}{y})^2$$

$$\Rightarrow 4x^2 - \frac{9}{y^2}$$

$$(vi) (2a^3 + b^3)(2a^3 - b^3)$$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = 2a^3, b = b^3$$

$$\Rightarrow (2a^3)^2 - (b^3)^2$$

$$\Rightarrow 4a^6 - b^6$$

$$(vii) (x^4 + \frac{2}{x^2})(x^4 - \frac{2}{x^2})$$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = x^4, b = \frac{2}{x^2}$$

$$\Rightarrow (x^4)^2 - (\frac{2}{x^2})^2$$

$$\Rightarrow x^8 - \frac{4}{x^4}$$

$$(viii) (x^3 + \frac{1}{x^3})(x^3 - \frac{1}{x^3})$$

sol:

$$\text{we know that } (a + b)(a - b) = a^2 - b^2$$

$$\text{here, } a = x^3, b = \frac{1}{x^3}$$

$$\Rightarrow (x^3)^2 - (\frac{1}{x^3})^2$$

$$\Rightarrow x^6 - \frac{1}{x^6}$$

Q3 Using the formula for squaring a binomial, evaluate the following

$$(i) (102)^2$$

sol:

$$(102)^2 \text{ can be written as } (100 + 2)^2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{here, } a = 100, b = 2$$

$$\Rightarrow (100 + 2)^2$$

$$\Rightarrow (100)^2 + 2 \times (100) \times 2 + 2^2$$

$$\Rightarrow 10000 + 400 + 4$$

$$\Rightarrow 10404$$

$$(ii) (99)^2$$

sol:

$$(99)^2 \text{ can be written as } (100 - 1)^2$$

$$\text{we know that, } (a - b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = 100, b = 1$$

$$\Rightarrow (100 - 1)^2$$

$$\Rightarrow (100)^2 - 2 \times (100) \times 1 + 1^2$$

$$\Rightarrow 10000 - 200 + 1$$

$$\Rightarrow 9801$$

(iii) $(1001)^2$

sol:

$$(1001)^2 \text{ can be written as } (1000 + 1)^2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{here, } a = 1000, b = 1$$

$$\Rightarrow (1000 + 1)^2$$

$$\Rightarrow (1000)^2 + 2 \times (1000) \times 1 + 1^2$$

$$\Rightarrow 1000000 + 2000 + 1$$

$$\Rightarrow 1002001$$

(iv) $(999)^2$

sol:

$$(999)^2 \text{ can be written as } (1000 - 1)^2$$

$$\text{we know that, } (a - b)^2 = a^2 - 2ab + b^2$$

$$\text{here, } a = 1000, b = 1$$

$$\Rightarrow (1000 - 1)^2$$

$$\Rightarrow (1000)^2 - 2 \times (1000) \times 1 + 1^2$$

$$\Rightarrow 1000000 - 2000 + 1$$

$$\Rightarrow 998001$$

(v) $(703)^2$

sol:

$$(703)^2 \text{ can be written as } (700 + 3)^2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{here, } a = 700, b = 3$$

$$\Rightarrow (700 + 3)^2$$

$$\Rightarrow (700)^2 + 2 \times (700) \times 3 + 3^2$$

$$\Rightarrow 490000 + 4200 + 9$$

$$\Rightarrow 494209$$

Q4 Simplify the following using the formula: $(a + b)(a - b) = a^2 - b^2$

(i) $(82)^2 - (18)^2$

sol:

$$(82)^2 - (18)^2$$

$$\text{here, } a = 82, b = 18$$

$$\Rightarrow (82 + 18)(82 - 18)$$

$$\Rightarrow 100 \times 64$$

$$\Rightarrow 6400$$

(ii) $(467)^2 - (33)^2$

sol:

$$(467)^2 - (33)^2$$

here, $a = 467$, $b = 33$

$$\Rightarrow (467 + 33)(467 - 33)$$

$$\Rightarrow 500 \times 434$$

$$\Rightarrow 217000$$

$$\text{(iii) } (79)^2 - (69)^2$$

sol:

$$(79)^2 - (69)^2$$

here, $a = 79$, $b = 69$

$$\Rightarrow (79 + 69)(79 - 69)$$

$$\Rightarrow 148 \times 10$$

$$\Rightarrow 1480$$

$$\text{(iv) } 197 \times 203$$

sol:

$$\text{Since, } \frac{197+203}{2} = \frac{400}{2} = 200$$

197×203 can be written as $(200 + 3)(200 - 3)$

$$\Rightarrow (200 + 3)(200 - 3)$$

$$\Rightarrow (200)^2 - (3)^2$$

$$\Rightarrow 40000 - 9$$

$$\Rightarrow 39991$$

$$\text{(v) } 113 \times 87$$

sol:

$$\text{Since, } \frac{113+87}{2} = \frac{200}{2} = 100$$

113×87 can be written as $(100 + 13)(100 - 13)$

$$\Rightarrow (100 + 13)(100 - 13)$$

$$\Rightarrow (100)^2 - (13)^2$$

$$\Rightarrow 10000 - 169$$

$$\Rightarrow 9831$$

$$\text{(vi) } 95 \times 105$$

sol:

$$\text{Since, } \frac{95+105}{2} = \frac{200}{2} = 100$$

95×105 can be written as $(100 + 5)(100 - 5)$

$$\Rightarrow (100 + 5)(100 - 5)$$

$$\Rightarrow (100)^2 - (5)^2$$

$$\Rightarrow 10000 - 25$$

$$\Rightarrow 9975$$

$$\text{(vii) } 1.8 \times 2.2$$

sol:

$$\text{Since, } \frac{1.8+2.2}{2} = \frac{4}{2} = 2$$

1.8×2.2 can be written as $(2 + 0.2)(2 - 0.2)$

$$\Rightarrow (2 + 0.2)(2 - 0.2)$$

$$\Rightarrow (2)^2 - (0.2)^2$$

$$\Rightarrow 4 - 0.04$$

$$\Rightarrow 3.96$$

(viii) 9.8×10.2

sol:

$$\text{Since, } \frac{9.8+10.2}{2} = \frac{20}{2} = 10$$

$$9.8 \times 10.2 \text{ can be written as } (10 + 0.2)(10 - 0.2)$$

$$\Rightarrow (10 + 0.2)(10 - 0.2)$$

$$\Rightarrow (10)^2 - (0.2)^2$$

$$\Rightarrow 100 - 0.04$$

$$\Rightarrow 99.96$$

Q5 Simplify the following using identities

(i) $\frac{(58)^2 - (42)^2}{16}$

sol:

The numerator is in the form of $(a + b)(a - b) = a^2 - b^2$

$$\frac{(58)^2 - (42)^2}{16} = \frac{(58+42)(58-42)}{16}$$

$$\Rightarrow \frac{(58)^2 - (42)^2}{16} = \frac{100 \times 16}{16}$$

$$\Rightarrow \frac{(58)^2 - (42)^2}{16} = 100$$

(ii) $(178 \times 178) - (22 \times 22)$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow (178 \times 178) - (22 \times 22) = (178)^2 - (22)^2$$

$$\Rightarrow (178 \times 178) - (22 \times 22) = (178 + 22)(178 - 22)$$

$$\Rightarrow (178 \times 178) - (22 \times 22) = 200 \times 156$$

$$\Rightarrow (178 \times 178) - (22 \times 22) = 31200$$

(iii) $\frac{(198 \times 198) - (102 \times 102)}{96}$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow \frac{(198 \times 198) - (102 \times 102)}{96} = \frac{(198)^2 - (102)^2}{96}$$

$$\Rightarrow \frac{(198 \times 198) - (102 \times 102)}{96} = \frac{(198+102)(198-102)}{96}$$

$$\Rightarrow \frac{(198 \times 198) - (102 \times 102)}{96} = \frac{300 \times 96}{96}$$

$$\Rightarrow \frac{(198 \times 198) - (102 \times 102)}{96} = 300$$

(iv) $(1.73 \times 1.73) - (0.27 \times 0.27)$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow (1.73 \times 1.73) - (0.27 \times 0.27) = (1.73)^2 - (0.27)^2$$

$$\Rightarrow (1.73 \times 1.73) - (0.27 \times 0.27) = (1.73 + 0.27)(1.73 - 0.27)$$

$$\Rightarrow (1.73 \times 1.73) - (0.27 \times 0.27) = 2 \times 1.46$$

$$\Rightarrow (1.73 \times 1.73) - (0.27 \times 0.27) = 2.92$$

$$(v) \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726}$$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726} = \frac{(8.63)^2 - (1.37)^2}{0.726}$$

$$\Rightarrow \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726} = \frac{(8.63 + 1.37)(8.63 - 1.37)}{0.726}$$

$$\Rightarrow \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726} = \frac{10 \times 7.26}{0.726}$$

$$\Rightarrow \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726} = 10 \times 10$$

$$\Rightarrow \frac{(8.63 \times 8.63) - (1.37 \times 1.37)}{0.726} = 100$$

Q6 Find the value of x, if:

$$(i) 4x = (52)^2 - (48)^2$$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow 4x = (52)^2 - (48)^2$$

$$\Rightarrow 4x = (52 + 48)(52 - 48)$$

$$\Rightarrow 4x = 100 \times 4$$

$$\Rightarrow 4x = 400$$

$$\Rightarrow x = \frac{400}{4}$$

$$\Rightarrow x = 100$$

$$(ii) 14x = (47)^2 - (33)^2$$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow 14x = (47)^2 - (33)^2$$

$$\Rightarrow 14x = (47 + 33)(47 - 33)$$

$$\Rightarrow 14x = 80 \times 14$$

$$\Rightarrow 14x = 1120$$

$$\Rightarrow x = \frac{1120}{14}$$

$$\Rightarrow x = 80$$

$$(iii) 5x = (50)^2 - (40)^2$$

sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow 5x = (50)^2 - (40)^2$$

$$\Rightarrow 5x = (50 + 40)(50 - 40)$$

$$\Rightarrow 5x = 90 \times 10$$

$$\Rightarrow 5x = 900$$

$$\Rightarrow x = \frac{900}{5}$$

$$\Rightarrow x = 180$$

Q7 If $x + \frac{1}{x} = 20$, find the value of $x^2 + \frac{1}{x^2}$

sol:

Given that,

$$x + \frac{1}{x} = 20$$

squaring on both sides

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (20)^2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 400$$

we know that, $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 400$$

$$\Rightarrow x^2 + 2 + \left(\frac{1}{x}\right)^2 = 400$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 400 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 398$$

hence, $x^2 + \frac{1}{x^2} = 398$

Q8 If $x - \frac{1}{x} = 3$, find the values of $x^2 + \frac{1}{x^2}$, $x^4 + \frac{1}{x^4}$

sol:

Given that,

$$x - \frac{1}{x} = 3$$

squaring on both sides

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (3)^2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 9$$

we know that, $(a - b)^2 = a^2 - 2ab + b^2$

$$\Rightarrow x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9$$

$$\Rightarrow x^2 - 2 + \left(\frac{1}{x}\right)^2 = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 11$$

Again, squaring on both sides

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (11)^2$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 121$$

$$\Rightarrow x^2 + 2 \times (x^2) \times \left(\frac{1}{x^2}\right) + \left(\frac{1}{x^2}\right)^2 = 121$$

$$\Rightarrow x^4 + 2 + \frac{1}{x^4} = 121$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 121 - 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 119$$

$$\text{hence, } x^4 + \frac{1}{x^4} = 119$$

Q9 If $x^2 + \frac{1}{x^2} = 18$, find the values of $x + \frac{1}{x}$, $x - \frac{1}{x}$

sol:

Given that,

$$x^2 + \frac{1}{x^2} = 18, \text{ find the values of } x + \frac{1}{x}, x - \frac{1}{x}$$

$$\text{consider, } x + \frac{1}{x}$$

squaring the above equation

$$(x + \frac{1}{x})^2 = x^2 + 2 \times x \times \frac{1}{x} + (\frac{1}{x})^2$$

$$= x^2 + 2 + \frac{1}{x^2}$$

$$\Rightarrow (x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2}$$

$$\Rightarrow (x + \frac{1}{x})^2 = 18 + 2$$

$$\Rightarrow (x + \frac{1}{x})^2 = 20$$

$$\Rightarrow x + \frac{1}{x} = \pm\sqrt{20}$$

$$\text{consider, } x - \frac{1}{x}$$

squaring the above equation

$$(x - \frac{1}{x})^2 = x^2 - 2 \times x \times \frac{1}{x} + (\frac{1}{x})^2$$

$$= x^2 - 2 + \frac{1}{x^2}$$

$$\Rightarrow (x - \frac{1}{x})^2 = x^2 - 2 + \frac{1}{x^2}$$

$$\Rightarrow (x - \frac{1}{x})^2 = 18 - 2$$

$$\Rightarrow (x - \frac{1}{x})^2 = 16$$

$$\Rightarrow x - \frac{1}{x} = \pm\sqrt{16}$$

$$\Rightarrow x - \frac{1}{x} = \pm 4$$

Q10 If $x + y = 4$ and $xy = 2$, find the value of $x^2 + y^2$

sol:

Given that,

$$x + y = 4 \text{ and } xy = 2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow x^2 + y^2 = (x + y)^2 - 2xy$$

$$\Rightarrow x^2 + y^2 = 4^2 - (2 \times 2)$$

$$\Rightarrow x^2 + y^2 = 16 - 4$$

$$\Rightarrow x^2 + y^2 = 12$$

Q11 If $x - y = 7$ and $xy = 9$, find the value of $x^2 + y^2$

sol:

Given that,

$$x - y = 7 \text{ and } xy = 9$$

$$\text{we know that, } (a-b)^2 = a^2 - 2ab + b^2$$

$$\Rightarrow x^2 - y^2 = (x-y)^2 + 2xy$$

$$\Rightarrow x^2 - y^2 = 7^2 + (2 \times 9)$$

$$\Rightarrow x^2 - y^2 = 49 + 18$$

$$\Rightarrow x^2 - y^2 = 67$$

Q12 If $3x + 5y = 11$ and $xy = 2$, find the value of $9x^2 + 25y^2$

sol:

Given that,

$$3x + 5y = 11 \text{ and } xy = 2$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$(3x + 5y)^2 = (3x)^2 + 2 \times (3x) \times (5y) + (5y)^2$$

$$\Rightarrow (3x + 5y)^2 = 9x^2 + 30xy + 25y^2$$

$$\Rightarrow 9x^2 + 25y^2 = (3x + 5y)^2 - 10xy$$

$$\Rightarrow 9x^2 + 25y^2 = (11)^2 - (30 \times 2)$$

$$\Rightarrow 9x^2 + 25y^2 = 121 - 60$$

$$\Rightarrow 9x^2 + 25y^2 = 61$$

Q13 Find the values of the following expressions

(i) $16x^2 + 24x + 9$ when $x = \frac{7}{4}$

Sol:

$$\text{Given, } 16x^2 + 24x + 9 \text{ and } x = \frac{7}{4}$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$16x^2 + 24x + 9 = (4x + 3)^2$$

$$\Rightarrow 16x^2 + 24x + 9 = (4(\frac{7}{4}) + 3)^2$$

$$\Rightarrow 16x^2 + 24x + 9 = (7 + 3)^2$$

$$\Rightarrow 16x^2 + 24x + 9 = (10)^2$$

$$\Rightarrow 16x^2 + 24x + 9 = 100$$

(ii) $64x^2 + 81y^2 + 144xy$ when $x = 11$ and $y = \frac{4}{3}$

sol:

$$\text{Given, } 64x^2 + 81y^2 + 144xy \text{ and } x = 11, y = \frac{4}{3}$$

$$\text{we know that, } (a + b)^2 = a^2 + 2ab + b^2$$

$$64x^2 + 81y^2 + 144xy = (8x + 9y)^2$$

$$\Rightarrow 64x^2 + 81y^2 + 144xy = (8(11) + 9(\frac{4}{3}))^2$$

$$\Rightarrow 64x^2 + 81y^2 + 144xy = (88 + 12)^2$$

$$\Rightarrow 64x^2 + 81y^2 + 144xy = (100)^2$$

$$\Rightarrow 64x^2 + 81y^2 + 144xy = 10000$$

(iii) $81x^2 + 16y^2 - 72xy$ when $x = \frac{2}{3}$

and $y = \frac{3}{4}$

sol:

Given that, $81x^2 + 16y^2 - 72xy$ and $x = \frac{2}{3}$,

$$y = \frac{3}{4}$$

we know that, $(a-b)^2 = a^2 - 2ab + b^2$

$$81x^2 + 16y^2 - 72xy = (9x - 4y)^2$$

$$\Rightarrow 81x^2 + 16y^2 - 72xy = (9(\frac{2}{3}) - 4(\frac{3}{4}))^2$$

$$\Rightarrow 81x^2 + 16y^2 - 72xy = (6 - 3)^2$$

$$\Rightarrow 81x^2 + 16y^2 - 72xy = 3^2$$

$$\Rightarrow 81x^2 + 16y^2 - 72xy = 9$$

Q14 If $x + \frac{1}{x} = 9$, find the value of $x^4 + \frac{1}{x^4}$

sol:

Given,

$$x + \frac{1}{x} = 9$$

squaring on both sides

$$(x + \frac{1}{x})^2 = 9^2$$

$$\Rightarrow (x + \frac{1}{x})^2 = 81$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{x} + (\frac{1}{x})^2 = 81$$

$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 81$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 81 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 79$$

Again, squaring on both sides

$$(x^2 + \frac{1}{x^2})^2 = (79)^2$$

$$\Rightarrow (x^2 + \frac{1}{x^2})^2 = 6241$$

$$\Rightarrow (x^2)^2 + 2 \times (x^2) \times (\frac{1}{x^2}) + (\frac{1}{x^2})^2 = 6241$$

$$\Rightarrow x^4 + 2 + \frac{1}{x^4} = 6241$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 6241 - 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 6239$$

Q15 If $x + \frac{1}{x} = 12$, find the value of $x - \frac{1}{x}$

sol:

Given that,

$$x + \frac{1}{x} = 12$$

squaring on both sides

$$\begin{aligned} \left(x + \frac{1}{x}\right)^2 &= (12)^2 \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= 144 \\ \Rightarrow x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 &= 144 \\ \Rightarrow x^2 + 2 + \frac{1}{x^2} &= 144 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 144 - 2 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 142 \end{aligned}$$

Here,

$$\begin{aligned} \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 \\ &= x^2 - 2 + \frac{1}{x^2} \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 + \frac{1}{x^2} \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= 142 - 2 \\ \Rightarrow \left(x - \frac{1}{x}\right)^2 &= 140 \\ \Rightarrow x - \frac{1}{x} &= \pm\sqrt{140} \end{aligned}$$

Q16 If $2x + 3y = 14$ and $2x - 3y = 2$, find the value of xy

Sol:

we know that, $(a + b)(a - b) = a^2 - b^2$

Given, $2x + 3y = 14$ and $2x - 3y = 2$

squaring $(2x + 3y)$ and $(2x - 3y)$ and then subtracting them, we get

$$\begin{aligned} (2x + 3y)^2 - (2x - 3y)^2 &= [(2x + 3y) + (2x - 3y)][(2x + 3y) - (2x - 3y)] \\ \Rightarrow (2x + 3y)^2 - (2x - 3y)^2 &= 4x \times 6y \\ \Rightarrow (2x + 3y)^2 - (2x - 3y)^2 &= 24xy \\ \Rightarrow (14)^2 - (2)^2 &= 24xy \\ \Rightarrow (14 + 2)(14 - 2) &= 24xy \\ \Rightarrow 16 \times 12 &= 24xy \\ \Rightarrow 24xy &= 192 \\ \Rightarrow xy &= \frac{192}{24} \\ \Rightarrow xy &= 8 \end{aligned}$$

hence, $xy = 8$

Q17 If $x^2 + y^2 = 29$ and $xy = 2$, Find the value of

(i) $x + y$

sol

Given,

$$x^2 + y^2 = 29 \text{ and } xy = 2$$

squaring the $(x + y)$

$$\begin{aligned} (x + y)^2 &= x^2 + 2 \times x \times y + y^2 \\ \Rightarrow (x + y)^2 &= 29 + (2 \times 2) \\ \Rightarrow (x + y)^2 &= 29 + 4 \\ \Rightarrow (x + y)^2 &= 33 \end{aligned}$$

$$\Rightarrow x + y = \pm\sqrt{33}$$

(ii) $x - y$

sol:

Given,

$$x^2 + y^2 = 29 \text{ and } xy = 2$$

squaring the $(x - y)$

$$(x - y)^2 = x^2 - 2 \times x \times y + y^2$$

$$\Rightarrow (x - y)^2 = 29 - (2 \times 2)$$

$$\Rightarrow (x - y)^2 = 29 - 4$$

$$\Rightarrow (x - y)^2 = 25$$

$$\Rightarrow x - y = \pm\sqrt{25}$$

$$\Rightarrow x + y = \pm 5$$

(iii) $x^4 + y^4$

Sol:

Given,

$$x^2 + y^2 = 29 \text{ and } xy = 2$$

$$(x^2 + y^2)^2 = x^4 + 2x^2y^2 + y^4$$

$$\Rightarrow x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2$$

$$\Rightarrow x^4 + y^4 = (x^2 + y^2)^2 - 2(xy)^2$$

$$\Rightarrow x^4 + y^4 = (29)^2 - 2(2)^2$$

$$\Rightarrow x^4 + y^4 = 841 - 8$$

$$\Rightarrow x^4 + y^4 = 833$$