RD SHARMA

Solutions

Class 8 Maths

Chapter 21

Ex 21.4

Q 1. Find the length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.

Soln:

Length of the room = 12 m

Breadth = 9 m

Height = 8 m

Since the room is cuboidal in shape, the length of the longest rod that can be placed in the room will be equal to the length of the diagonal between opposite vertices.

Length of the diagonal of the floor using the Pythagorus theorem

$= \sqrt{(l^2 + b^2)}$

$= \sqrt{(12^2 + 9^2)}$

$= \sqrt{(144 + 81)}$

$= \sqrt{(225)}$ = 15 m

i.e. , the length of the longest rod would be equal to the length of the diagonal of the right angle triangle of base 15 m and altitude 8 m.

Similarly, using the Pythagorus theorem, length of the diagonal

$= \sqrt{(15^2 + 8^2)}$

$= \sqrt{(225 + 64)}$ = 17 m

Therefore, the length of the longest rod that can be placed in the room is 17 m.

Q 2. If V is the volume of a cuboid of dimension a, b , c and S is its surface area, then prove that :

$$\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

Soln:

It is given that V is the volume of a cuboid of length = a, breadth = b and height = c.

Also, S is the surface area of a cuboid.

Then, V = a x b x c

Surface area of the cuboid = 2 x (length x breadth + breadth x height + length x height)

= S = 2 x (a x b + b x c + a x c)

Let us take the right — hand side of the equation to be proven.

$$\frac{2}{S}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= \frac{2}{2x(axb+bxc+axc)} x \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= \frac{2}{2x(axb+bxc+axc)} x \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= \frac{1}{(axb+bxc+axc)} x \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

Now, multiplying the numerator and the denominator with a x b x c, we get :

$$= \frac{1}{(axb+bxc+axc)} x \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) x \frac{axbxc}{axbxc}$$

$$= \frac{1}{(axb+bxc+axc)} x \left(\frac{axbxc}{a} + \frac{axbxc}{b} + \frac{axbxc}{c}\right) x \frac{1}{axbxc}$$

$$= \frac{1}{(axb+bxc+axc)} x(bxc + axc + axb) x \frac{1}{axbxc}$$

$$= \frac{1}{(axb+bxc+axc)} x(axb + bxc + axc) x \frac{1}{axbxc}$$

$= \frac{1}{axbxc} = \frac{1}{V}$

Therefore, $\frac{1}{V}$

$= \frac{2}{S}(\frac{1}{a} + \frac{1}{b} + \frac{1}{c})$

Q 3. The area of three adjacent faces of a cuboid are x, y and z. If the volume is V, Prove that V^2 = x y z.

Soln:

The areas of three adjacent faces of a cuboid are x, y and z.

Volume of the cuboid = V

Observe, that x = length x breadth

y = breadth x height,

z = length x height

Since volume of cuboid V = length x breadth x height, we have :

V^2 = V x V

= (length x breadth x height) x (length x breadth x height) = (length x breadth) x (breadth x height) x (length x height)

= (x) x (y) x (z)

= x y z

Therefore, V^2 = x y z

Q 4. A rectangular water reservoir contains 105 m^3 of water. Find the depth of the water in the reservoir if its base measures 12 m by 3.5 m.

Soln:

Length of the rectangular water reservoir = 12 m

Breadth = 3.5 m

Suppose that the height of the reservoir = h m

Also, it contains 105 m^3 of water,

i.e. , its volume = 105 m^3

Volume of the cuboidal water reservoir = length x breadth x-height

105 = 12 x 3.5 x h

105 = 42 x h

$h = \frac{105}{42} = 2.5$ m

Therefore, the depth of the water in the reservoir is 2.5 m.

Q 5. Cubes A, B, C, having edges 18 cm, 24 cm, and 30 cm respectively are melted and molded into a new cube D. Find the edge and bigger cube D.

Soln:

We have the following :

Length of the edge of cube A = 18 cm

Length of the edge of cube B = 24 cm

Length of the edge of cube C = 30 cm

The given cubes are melted and molded into a new cube D.

Hence, volume of cube D = volume of cube A + volume of cube B + volume of cube C

= $(\text{side of cube A})^3 + (\text{side of cube B})^3 + (\text{side of cube C})^3$

= $18^3 + 24^3 + 30^3$

= 5832 + 13824 + 27000

= 46656 cm^3

Suppose that the edge of the new cube D = x

$x^3 = 46656$

x = 36 cm

Therefore, the edge of the bigger cube D is 36 cm.

Q 6. The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu Dm. Find its dimension.

Soln:

Suppose that the breadth of the room = x dm

Since breadth is twice the height, breadth = 2 x height

So, height of the room = breadth / 2 ;

Also, it is given that the breadth is half the length.

So, breadth = a x length

i.e , length = 2 x breadth = 2 x x

Since volume of the room = 512 cu dm, we have :

Volume of a cuboid = length x breadth x height

$512 = 2 \times (x) \times (x) \times \frac{x}{2}$

$x^3 = 512$

x = 8 dm

$512 = x^3$

x = 8 dm

Hence, length of the room = 2 x x = 2 x 8 = 16 dm

Breadth of the room = x = 8 dm

Height of the room = $\frac{x}{2} = \frac{8}{2} = 4$ dm

Q 7. A closed iron tank 12 m long, 9m wide and 4 m deep is to be made. Determine the cost of an iron sheet used at the rate of Rs. 5 per meter sheet, the sheet being 2 m wide.

Soln:

A closed iron tank of dimensions 12 m long, 9 m wide and 4 m deep is to be made.

Surface area of the cuboidal tank = 2 x (length x breadth + breadth x height + length x height)

= 2 x (12 x 9 + 9 x 4 + 12 x 4)

= 2 x (108 + 36 + 48)

= 384 m^2

Also, the cost of an iron sheet is Rs 5 per metre and the sheet is 2 metres wide.

i.e. , area of a sheet = 1 m x 2 m = 2 m^2

So, the cost of 2 m^2 of iron sheet = Rs 5

i.e. , the cost of 1 m^2 of iron sheet = Rs $\frac{5}{2}$

Therefore, Cost of 384 m^2 of iron sheet = $384 \times \frac{5}{2}$

= Rs 960

Q 8. A tank open at the top is made of an iron sheet 4 m wide. If the dimensions of the tank are 12 m x 8 m x 6 m, find the cost of an iron sheet at Rs. 17.50 per meter.

Soln:

An open iron tank of dimensions 12 m x 8 m x 6 m is to be made.

Surface area of the open tank = (area of the base) + (total area of the 4 walls)

$= (12 \times 8) + 2 \times (8 \times 6 + 12 \times 6) = (96) + 2 \times (48 + 72) = 336\ m^2$

Also, it is given that the cost of the iron sheet that is 4 m wide is Rs 17.50 per metre.

i.e. , the area of the iron sheet = $1\ m \times 4\ m = 4\ m^2$

So, the cost of $4\ m^2$ of iron sheet = Rs 17.50

The cost of iron sheet required to an iron tank of surface area $336\ m^2 = 336 \times \frac{17.50}{4}$ = Rs. 1470.

Q 9. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Soln:

Suppose that the side of the cube = x cm

Surface area of the cube = $6 \times (side)^2 = 6 \times x^2 = 6x^2\ cm^2$

i.e. , the sum of the surface areas of three such cubes = $6x^2 + 6x^2 + 6x^2 = 18\ x^2\ cm^2$

Now, these three cubes area placed together to form a cuboid.

Then the length of the new cuboid will be 3 times the edge of the cube = 3 x x = 3 x cm

Breadth of the cuboid = x cm

Height of the cuboid = x cm

Therefore, Total surface area of the cuboid = 2 x (length x breadth + breadth x height + length x height)

$= 2 \times (3x \times (x) + (x) \times (x) + 3x \times (x))$

$= 2 \times (3x^2 + x^2 + 3x^2)$

$= 2 \times (7 x^2)$

$= 14\ x^2$ cm

i.e., the ratio of the total surface area cuboid to the sum of the surface areas of the three cubes

$= 14\ x^2\ cm^2 : 18\ x^2\ cm^2 = 7 : 9$

Hence, the ratio is 7 : 9.

Q 10. The dimensions of a room are 12.5 m by 9 m by 7 m. there are 2 doors and 4 windows in the room; each door measures 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of painting the walls at Rs. 3.50 per square meter.

Soln :

The dimensions of the room are 12.5 m x 9 m x 7 m.

Hence, the surface area of walls = 2 x (length x height + breadth x height)

$= 2 \times (12.5 \times 7 + 9 \times 7) = 301\ m^2$

Also, there are 2 doors and 4 windows in the room.

The dimensions of door are 2.5 m x 1.2 m.

2. e. , area of a door = $2.5 \times 1.2 = 3\ m^2$

Therefore, Total area of 2 doors = $2 \times 3 = 6\ m^2$

The dimensions of a window are 1.5 m x 1 m.

1. e. , area of a window = $1.5 \times 1 = 1.5\ m^2$

Total area of 4 windows = $4 \times 1.5 = 6\ m^2$

Hence, the total area to be painted = $301 – (6 + 6) = 289\ m^2$

The rate of painting $1\ m^2$ of wall = Rs 3.50

Therefore, the total cost of painting $289\ m^2$ of wall

= Rs 289 x 3.50 = Rs 1011.50

Q 11. A field is 150 m long and 100 m wide. A plot (outside the field) 50 m long and 30 m wide is dug to a depth of 8 m and the earth taken out from the plot is spread evenly in the field. By how much is the level of field raised?

Soln:

The dimensions of the plot dug outside the field are 50 m x 30 m x 8 m.

Hence, volume of the earth dug – out from the plot = 50 x 30 x 8 = 12000 m^3

Suppose that the level of the earth rises by hm.

When we spread this dug – out earth on the field of length 150 m, breadth 100 m and height h m, we have :

Volume of earth dug – out = 150 x 100 x h

12000 = 15000 x h

h = $\frac{12000}{15000}$ = 0.8 m

h = 80 cm (Because 1 m = 100 cm)

Therefore, the level of the field will rise by 80 cm.

Q 12. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Soln:

Two cubes each of volume 512 cm^3 are joined end to end.

Now, volume of a cube = $(side)^3$

512 = $(side)^3$

Side of the cube = 8 cm

If the cubes are joined side by side, then the length of the resulting cuboid is

2 x 8 cm = 16 cm.

Breadth = 8 cm

Height = 8 cm

Surface area of the cuboid = 2 x (length x breadth + breadth x height + length x height)

= 2 x (16 x 8 + 8 x 8 + 16 x 8) = 2 x (128 + 64 + 128) = 640 cm^2

Q 13. Three cubes, each whose edges measure 3 cm, 4 cm and 5 cm respectively are melted to form a new cube. Find the surface area of the new cube formed.

Soln:

Three cubes of edges 3 cm, 4 cm and 5 cm are melted and molded to form a new cube.

i.e. , volume of the new cube = sum of the volumes of the three cubes

= $(3)^3 + (4)^3 + (5)^3$

= 27 + 64 + 125

= 216 cm^3

We know that volume of a cube = $(side)^3$

216 = $(side)^3$

Side of the new cube = 6 cm

Therefore, Surface area of the new cube = 6 x $(side)^2$ = 6 x $(6)^2$ = 216 cm^2

Q 14. The cost of preparing the walls of a room 12 m long at the rate of Rs. 1.35 per square meter is Rs. 304.20 and the cost of matting the floor at 85 paise per square meter is Rs. 91.80. Find the height of the room.

Soln:

The cost of preparing 4 walls of a room whose length is 12 m is Rs. 340.20 at a rate of Rs. 1.35 / m^2.

Area of the four walls of the room = $\frac{totalcost}{rate} = \frac{Rs.340.20}{Rs.1.35}$ = 252 m^2

Also, the cost of matting the floor at 85 paise /m^2 is Rs 91.80.

Therefore, Area of the floor = $\frac{totalcost}{rate} = \frac{Rs.91.80}{Rs.0.85}$ = 108 m^2

Hence, breadth of the room = $\frac{areaofthefloor}{length} = \frac{108}{12}$ = 9 m^2

Suppose that the height of the room is h m.

Then, we have :

Area of four walls = 2 x (length x height + breadth x height)

252 = 2 x (12 x h+ 9 x h)

252 = 2 x (21 h)

21 h = $\frac{252}{21}$ = 126

h = $\frac{126}{21}$ = 6 m

Therefore, the height of the room is 6 m.

Q 15. The length of a hall is 18 m and the width 12 m. the sum of the area of the floor and the flat roof is equal to the sum of the area of the four walls. Find the height of the wall.

Soln:

Length of the hall = 18 m

Its width = 12 m

Suppose that the height of the wall is hm.

Also, sum of the areas of the floor and the flat roof

= sum of the areas of the four walls

= 2 x (length x breadth)

= 2 x (length + breadth) x height

= 2 x (18 x 12) = 2 x (18 + 12) x h

432 = 60 x h

h = $\frac{432}{60}$ = 7. 2 m

Therefore, the height of wall is 7.2 m

Q 16. A metal cube edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Soln:

Let the edge of the third cube be x cm.

Three small cubes are formed by melting the cube of edge 12 cm.

Edges of two small cubes are 6 cm and 8 cm.

Now, volume of a cube = $(\text{side})^3$

Volume of the big cube = sum of the volumes of the three small cubes

$(12)^3 = (6)^3 + (8)^3 + (x)^3$

$1728 = 216 + 512 + x^3$

$x^3 = 1728 – 728 = 1000$

x = 10 cm

Therefore, the edge of the third cube is 10 cm.

Q 17. The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many people can sit in the hall, if each person requires 150 m^3 of air?

Soln:

The dimensions of a cinema hall are 100 m x 50 m x 18 m.

i.e. , volume of air in the cinema hall = 100 x 50 x 18 = 90000 m^3

It is given that each person requires 150 m^3 of air.

Therefore, The number of persons that can sit in the cinema hall

$= \frac{volume of air in hall}{Volume of air required by 1 person}$

$= \frac{9000}{150} = 600$

Therefore, the number of persons that can sit in the cinema hall is 600

Q 18. The external dimensions of a closed wooden box are 48 cm, 36 cm, 30 cm. the box is made of 1.5 cm thick wood. How many bricks of size 6 cm x 3 cm x 0.75 cm can be put in this box?

Soln:

The outer dimensions of the closed wooden box are 48 cm x 36 cm x 30 cm.

Also, the box is made of a 1.5 cm thick wood, so the inner dimensions of the box will be (2 x 1.5 = 3) cm less.

i.e. , the inner dimensions of the box are 45 cm x 33 cm x 27 cm

Therefore, Volume of the box = 45 x 33 x 27 = 40095 cm^3

Also, the dimensions of a brick are 6 cm x 3 cm x 0.75 cm.

Volume of a brick = 6 x 3 x 0.75 = 13.5 cm^3

Therefore, The number of bricks that can be put in the box $= \frac{40095}{13.5} = 2970$

Q 19. The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with the sheet of paper at the rates of Rs. 8 and Rs . 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Soln:

Suppose that the dimensions be x multiple of each other.

The dimensions are in the ratio 2 : 3 : 4.

Hence, length = 2x m

Breadth = 3x m

Height = 4x m

So, total surface area of the rectangular box = 2 x (length x breadth + breadth x height + length x height)

= 2 x (2x x 3x + 3x x 4x + 2x x 4x)

= 2 x (6 x^2 + 12 x^2 + 8 x^2)

= 2 x (26 x^2)

= 52 x^2 m^2

Also, the cost of covering the box with paper at the rate Rs 8 /m^2 and Rs 9.50 / m^2 is Rs. 1248.

Here, the total cost of covering the box at a rate of its 8 /m^2 = 8 x 52 x^2= Rs. 416 x^2

And the total cost of covering the box at a rate of Rs 9.50 /m^2 = 9.50 x 52 x^2 = Rs. 494 x^2

Now, total cost of covering the box at the rate Rs. 9.50 /m^2 – total cost of covering the box at the rate Rs 8/m^2 = 1248

494 x^2 – 416 x^2 = 1248

78 x^2 = 1248

$x^2 = \frac{1248}{78} = 16$

x = 4 m

hence, length of the rectangular box = 2 x x = 2 x 4 = 8 m

Breadth = 3 x x = 3 x 4 = 12 m

height = 4 x x = 4 x 4 = 16 m