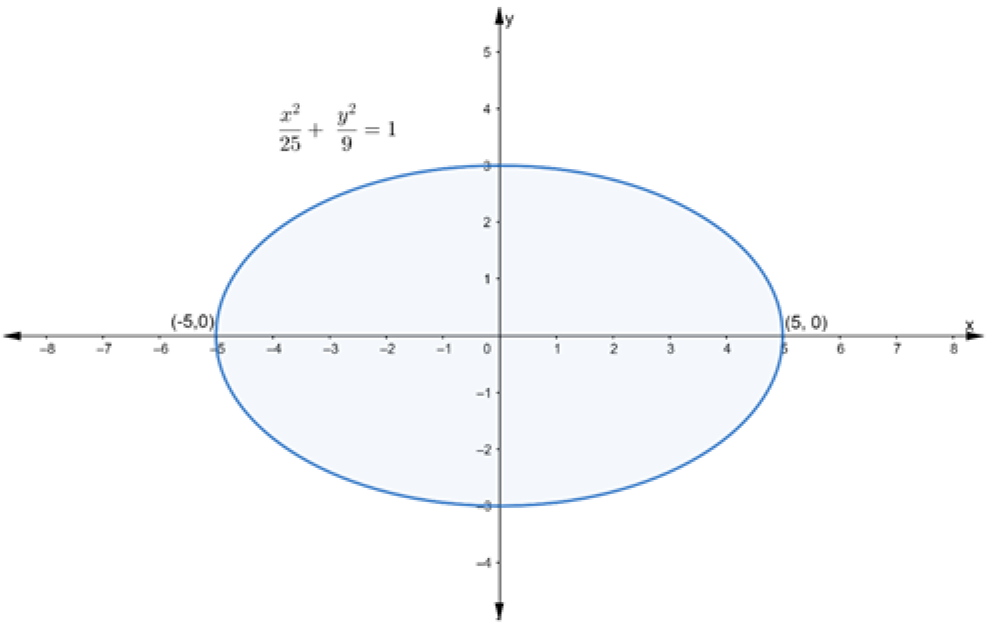
Ellipse

Exercise 23

Q. 1. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Answer :



Given:

$$\frac{x^2}{25}+\frac{y^2}{9}=1 \quad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$

Since, 25 > 9

So, above equation is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots(ii)$$

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 9$

⇒ $a = \sqrt{25}$ and $b = \sqrt{9}$

⇒ a = 5 and b = 3

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (5, 0) and (-5, 0)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 – b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

= 25 – 9

$c^2 = 16$

$c = \sqrt{16}$

$c = 4$ …(I)

∴ Coordinates of foci = (±4, 0)

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{4}{5} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

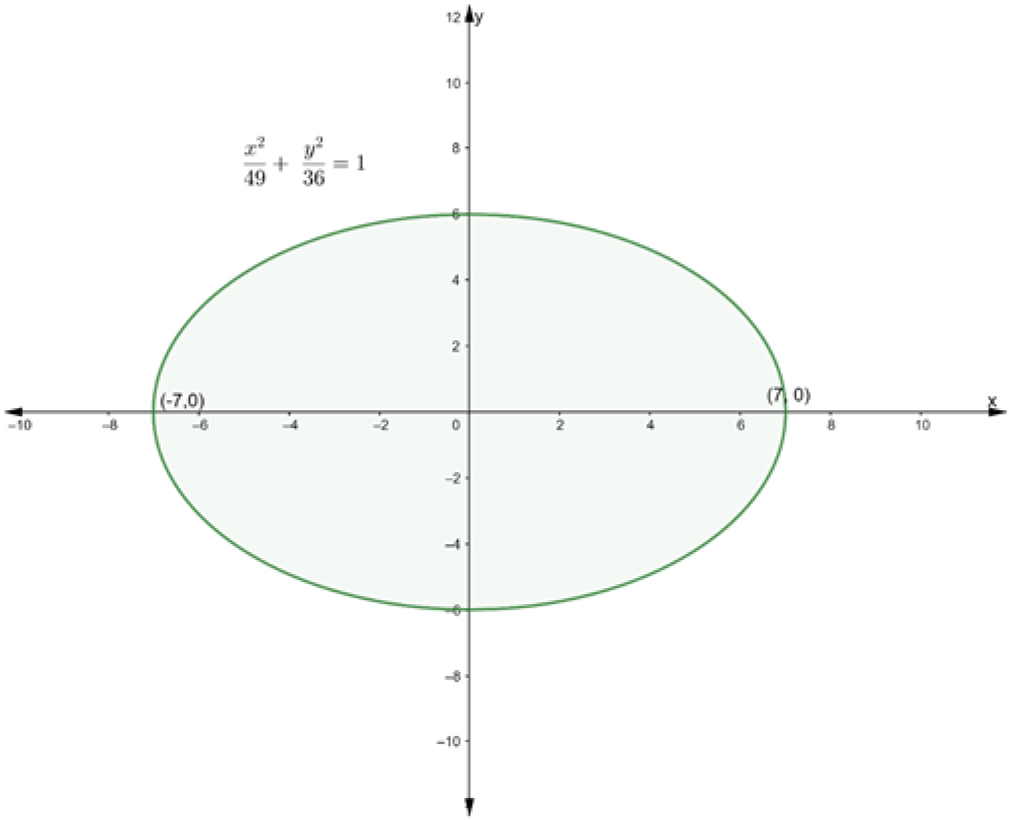
$$= \frac{2 \times (3)^2}{5}$$

$$= \frac{18}{5}$$

Q. 2. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$$\frac{x^2}{49} + \frac{y^2}{36} = 1$$

Answer :



Given:

$$\frac{x^2}{49}+\frac{y^2}{36}=1 \qquad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 49 > 36

So, above equation is of the form,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \ldots(ii)$$

Comparing eq. (i) and (ii), we get

$a^2 = 49$ and $b^2 = 36$

⇒ $a = \sqrt{49}$ and $b = \sqrt{36}$

⇒ $a = 7$ and $b = 6$

(i) <u>To find</u>: Length of major axes

Clearly, $a > b$, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

$= 2 \times 7$

= 14 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, $a > b$

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (7, 0) and (-7, 0)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = $(\pm c, 0)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

$= 49 - 36$

$c^2 = 13$

$c = \sqrt{13}$ …(I)

∴ Coordinates of foci = $(\pm\sqrt{13}, 0)$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$\Rightarrow e = \frac{\sqrt{13}}{7}$ [from (I)]

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

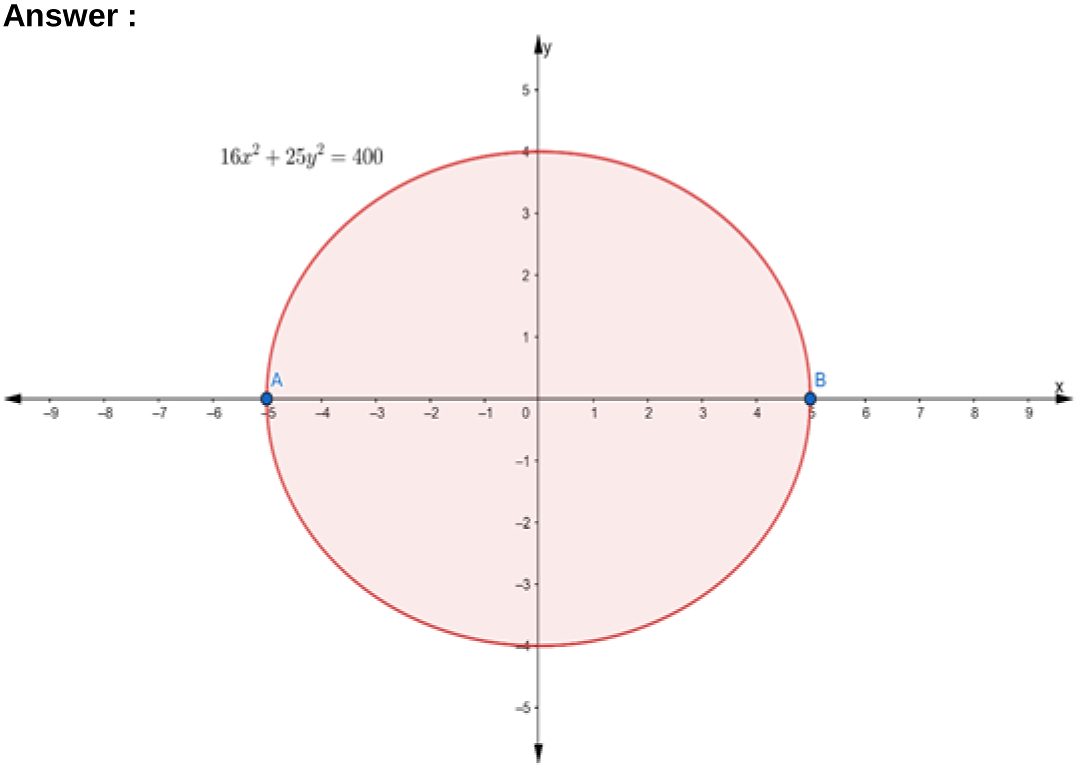
$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (6)^2}{7}$$

$$= \frac{72}{7}$$

Q. 3. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.
$16x^2 + 25y^2 = 400$

Answer :



Given:

$16x^2 + 25y^2 = 400$

Divide by 400 to both the sides, we get

$$\frac{16}{400}x^2 + \frac{25}{400}y^2 = 1$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1 \qquad \ldots\text{(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 25 > 4

So, above equation is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots\text{(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

$\Rightarrow a = \sqrt{25}$ and $b = \sqrt{4}$

$\Rightarrow a = 5$ and $b = 2$

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (5, 0) and (-5, 0)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 25 – 4

$c^2 = 21$

$c = \sqrt{21}$ …(I)

∴ Coordinates of foci = $(\pm\sqrt{21}, 0)$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{21}}{5} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

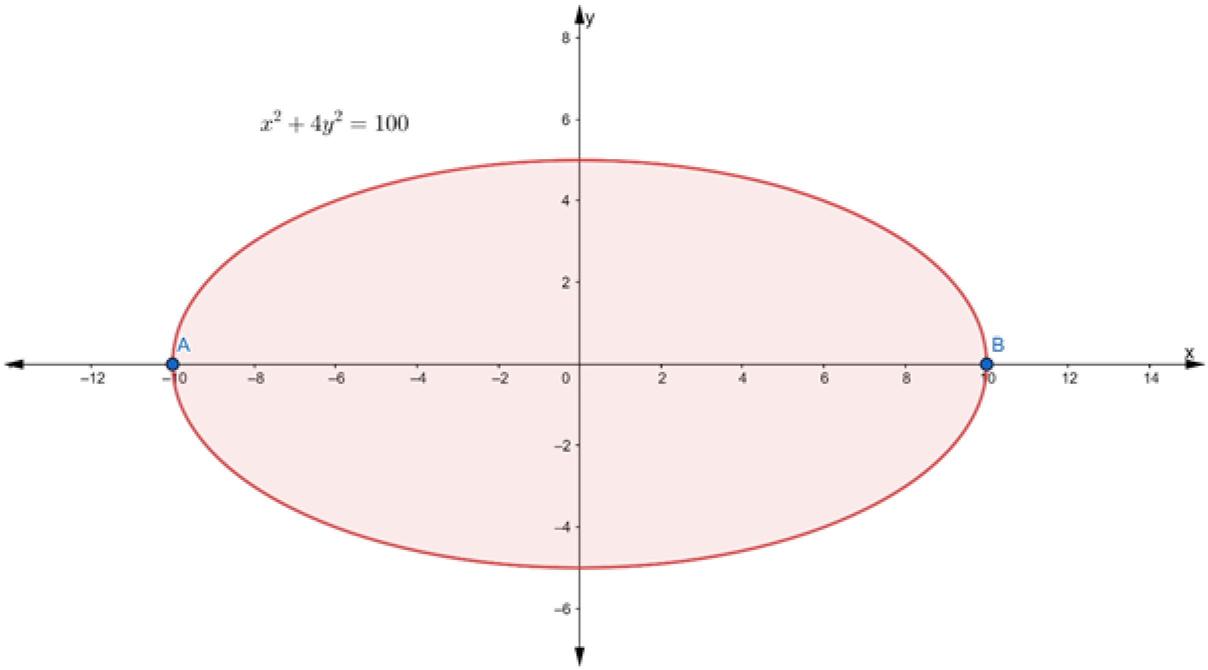
$$= \frac{2 \times (4)^2}{5}$$

$$= \frac{32}{5}$$

Q. 4. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$x^2 + 4y^2 = 100$

Answer :



Given:

$x^2 + 4y^2 = 100$

Divide by 100 to both the sides, we get

$$\frac{1}{100}x^2 + \frac{4}{100}y^2 = 1$$

$$\frac{x^2}{100} + \frac{y^2}{25} = 1 \quad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 100 > 25

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ …(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 100$ and $b^2 = 25$

⇒ $a = \sqrt{100}$ and $b = \sqrt{25}$

⇒ a = 10 and b = 5

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 10

= 20 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (10, 0) and (-10, 0)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 100 – 25

$c^2 = 75$

$c = \sqrt{75}$

$c = 5\sqrt{3}$ …(I)

∴ Coordinates of foci = $(\pm 5\sqrt{3}, 0)$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

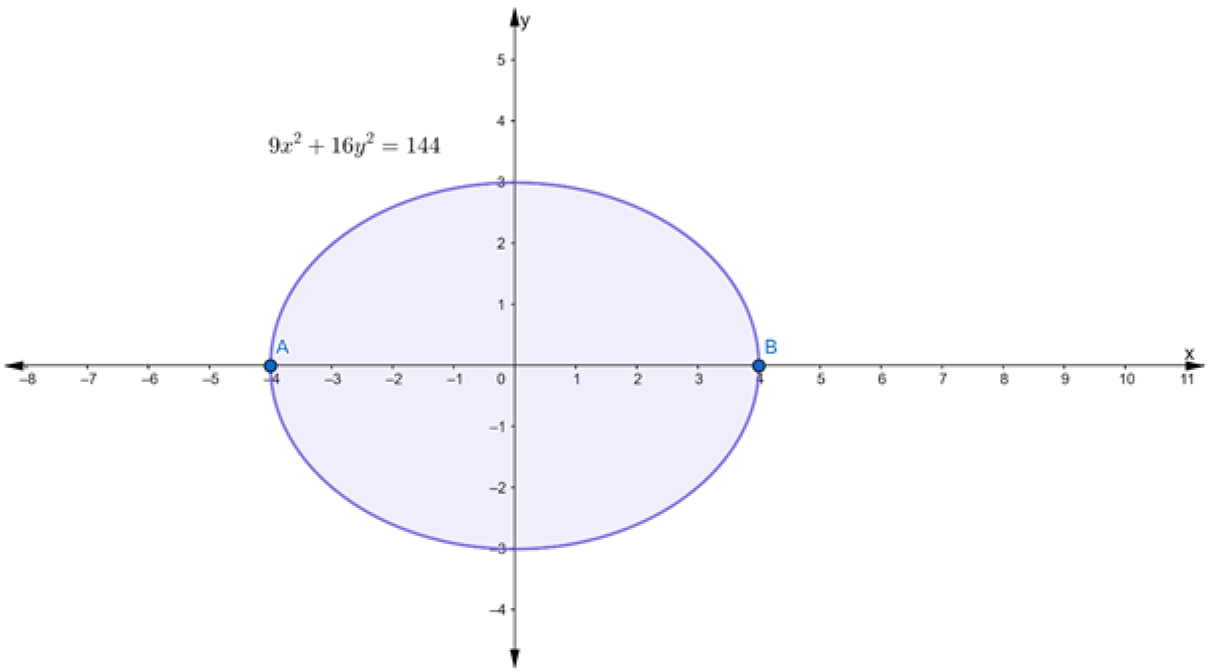
$$= \frac{2 \times (4)^2}{5}$$

$$= \frac{32}{5}$$

Q. 5. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$9x^2 + 16y^2 = 144$

Answer :



Given:

$9x^2 + 16y^2 = 144$

Divide by 144 to both the sides, we get

$$\frac{9}{144}x^2 + \frac{16}{144}y^2 = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad \text{…(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 16 > 9

So, above equation is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{…(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 9$

⇒ $a = \sqrt{16}$ and $b = \sqrt{9}$

⇒ a = 4 and b = 3

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (4, 0) and (-4, 0)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 – b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

= 16 – 9

$c^2 = 7$

$c = \sqrt{7}$ …(I)

∴ Coordinates of foci = $(\pm\sqrt{7}, 0)$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{7}}{4} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

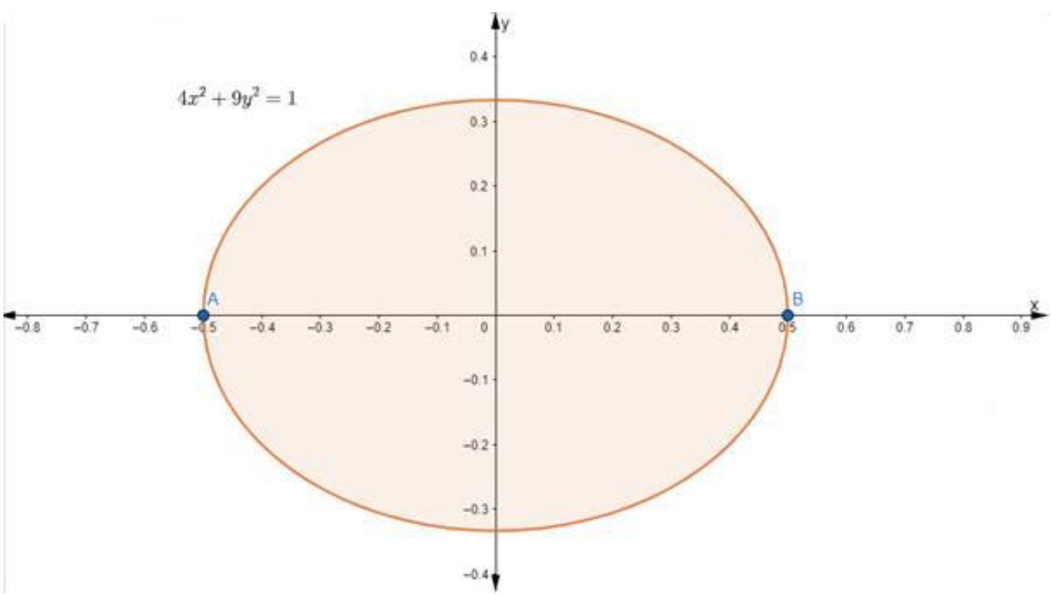
$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (3)^2}{4}$$

$$= \frac{9}{2}$$

Q. 6. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.
$4x^2 + 9y^2 = 1$

Answer :



Given:

$4x^2 + 9y^2 = 1$

$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	(± c, 0)	(± a, 0)	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	(0,± c)	(0,± a)	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since,

$\frac{1}{4} > \frac{1}{9}$

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = \frac{1}{4} \text{ and } b^2 = \frac{1}{9}$

$\Rightarrow a = \sqrt{\frac{1}{4}} \text{ and } b = \sqrt{\frac{1}{9}}$

$\Rightarrow a = \frac{1}{2} \text{ and } b = \frac{1}{3}$

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

$= 2 \times \frac{1}{2}$

= 1 unit

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

$= \left(\frac{1}{2}, 0\right) \text{and} \left(-\frac{1}{2}, 0\right)$

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 – b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

$= \frac{1}{4} - \frac{1}{9}$

$c^2 = \frac{9 - 4}{36}$

$c^2 = \frac{5}{36}$

$c = \frac{\sqrt{5}}{6}$ …(I)

$\therefore$ Coordinates of foci $= \left(\pm \frac{\sqrt{5}}{6}, 0\right)$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\frac{\sqrt{5}}{6}}{\frac{1}{2}} = \frac{\sqrt{5}}{6} \times 2 = \frac{\sqrt{5}}{3} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times \left(\frac{1}{3}\right)^2}{\frac{1}{2}}$$

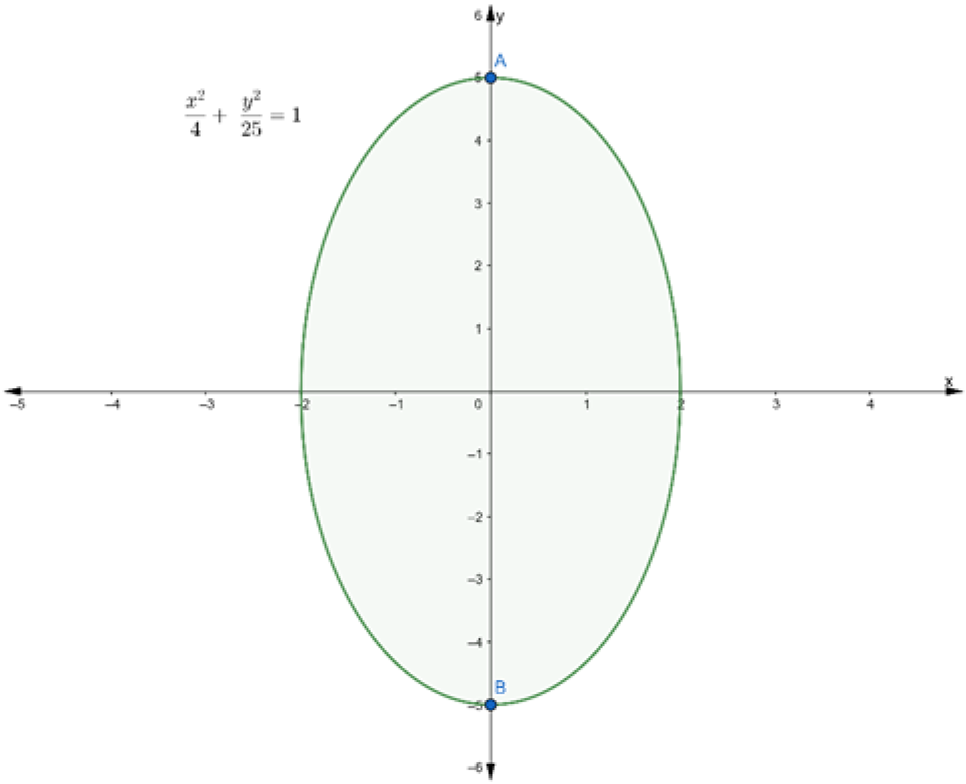
$$= \frac{\frac{2}{9}}{\frac{1}{2}}$$

$$= \frac{2}{9} \times 2$$

Q. 7. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

Answer :



Given:

$\frac{x^2}{4}+\frac{y^2}{25}=1$ …(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	(± c, 0)	(± a, 0)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	(0,± c)	(0,± a)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 25

So, above equation is of the form,

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ …(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

⇒ a = √25 and b = √4

$\Rightarrow$ a = 5 and b = 2

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 5) and (0, -5)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 25 – 4

$c^2 = 21$

c = √21 …(I)

∴ Coordinates of foci = (0, ±√21)

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$\Rightarrow e = \frac{\sqrt{21}}{5}$ [from (I)]

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

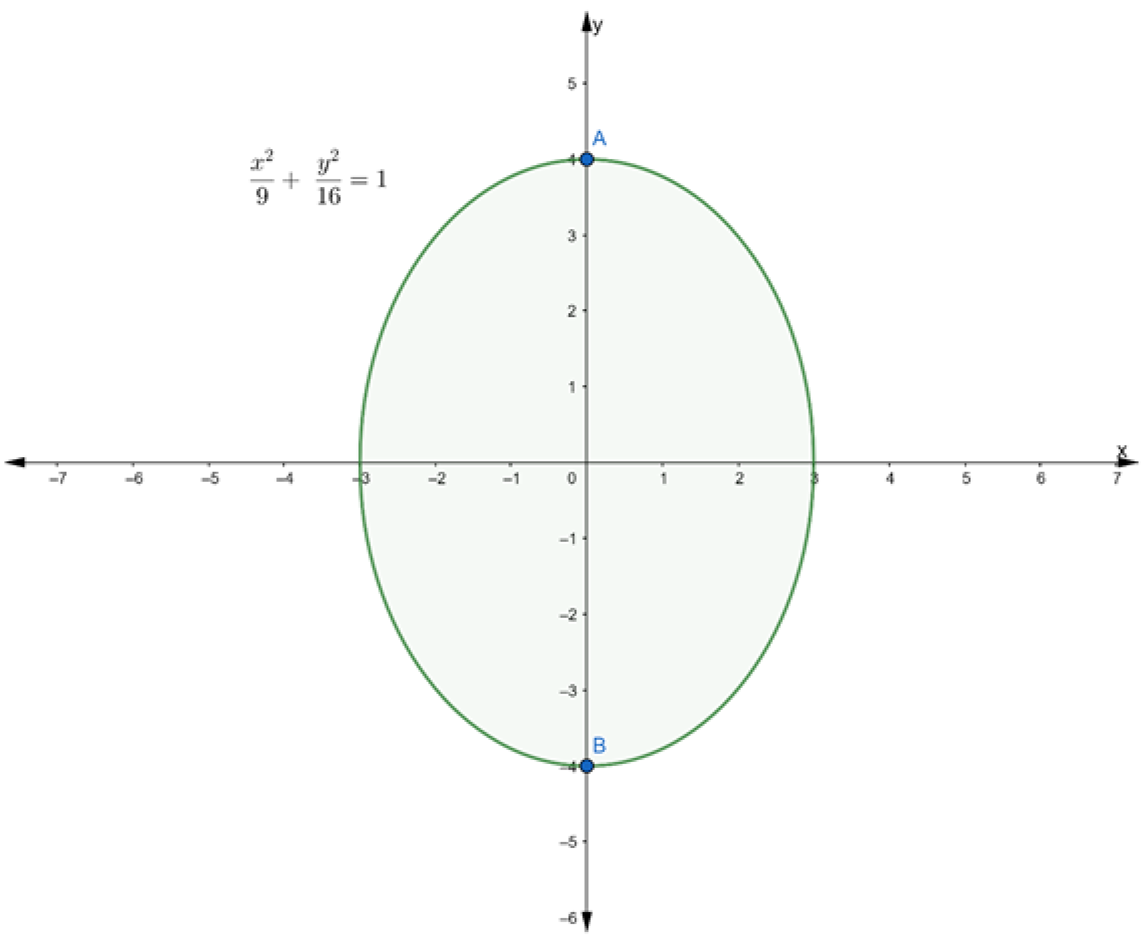
$$= \frac{2 \times (2)^2}{5}$$

$$= \frac{8}{5}$$

Q. 8. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Answer :



Given:

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \quad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	(± c, 0)	(± a, 0)	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	(0,± c)	(0,± a)	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 9 < 16

So, above equation is of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \ldots(ii)$$

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 9$

⇒ a = √16 and b = √9

⇒ a = 4 and b = 3

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 4) and (0, -4)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 16 – 9

$c^2 = 7$

$c = \sqrt{7}$ …(I)

∴ Coordinates of foci = $(0, \pm\sqrt{7})$

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{7}}{4} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

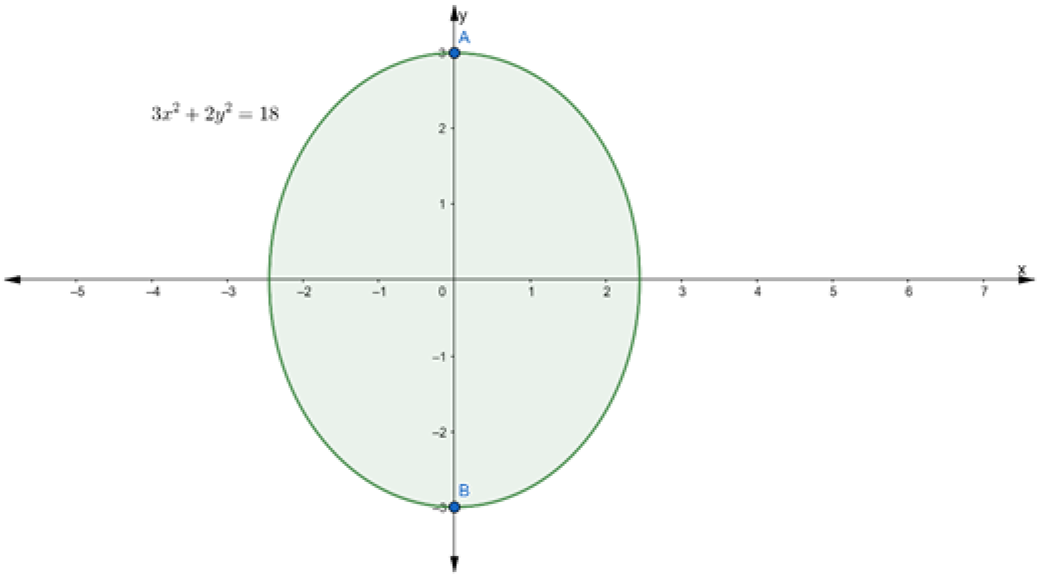
$$= \frac{2 \times (3)^2}{4}$$

$$= \frac{9}{2}$$

Q. 9. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$3x^2 + 2y^2 = 18$

Answer :



Given:

$3x^2 + 2y^2 = 18$

Divide by 18 to both the sides, we get

$$\frac{3}{18}x^2 + \frac{2}{18}y^2 = 1$$

$$\frac{x^2}{6} + \frac{y^2}{9} = 1 \quad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 6 < 9

So, above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ …(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 9$ and $b^2 = 6$

⇒ a = √9 and b = √6

⇒ a = 3 and b = √6

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 3

= 6 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 6) and (0, -6)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 – b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

= 9 – 6

$c^2 = 3$

c = √3 …(I)

∴ Coordinates of foci = (0, ±√3)

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{3}}{3} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times \left(\sqrt{6}\right)^2}{3}$$

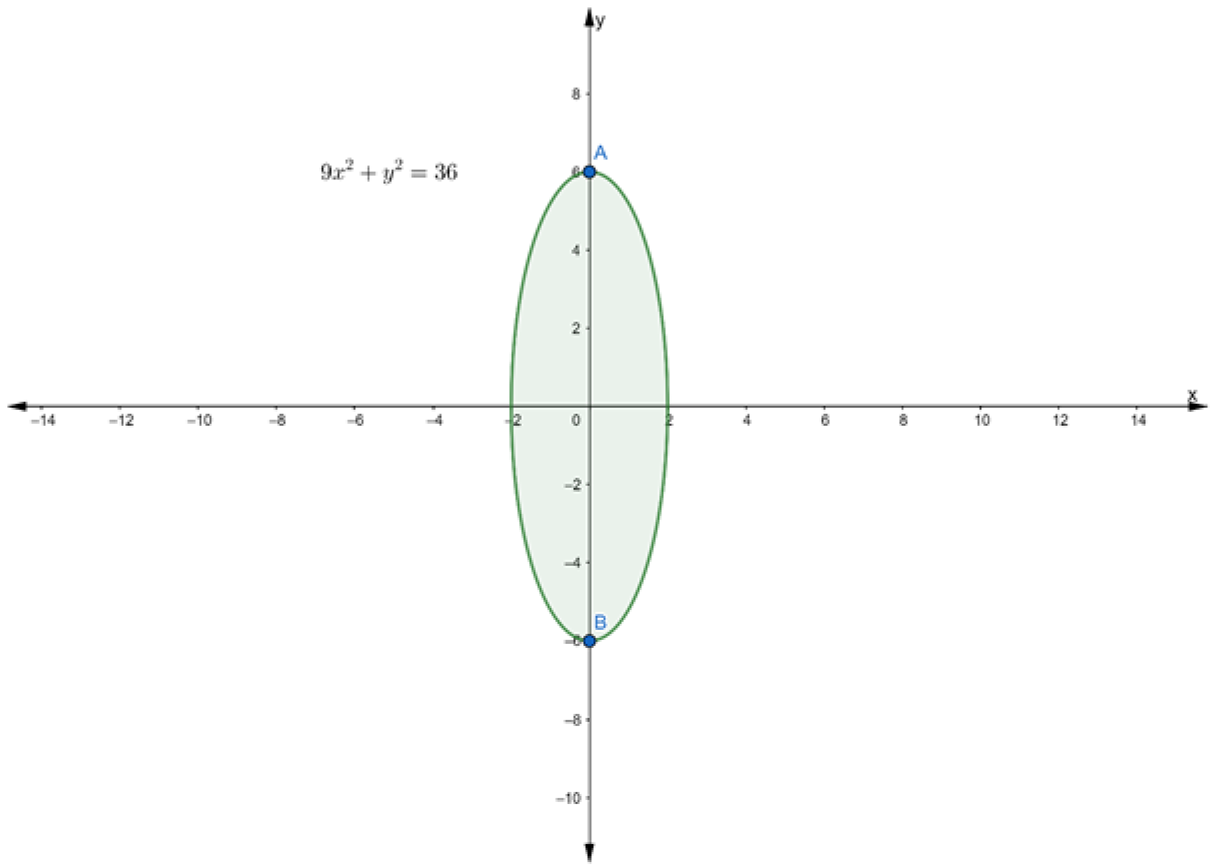
$$= \frac{2 \times 6}{3}$$

= 4

Q. 10. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$9x^2 + y^2 = 36$

Answer :



Given:

$9x^2 + y^2 = 36$

Divide by 36 to both the sides, we get

$$\frac{9}{36}x^2 + \frac{1}{36}y^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{36} = 1 \quad \ldots(i)$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 36

So, above equation is of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad \ldots(ii)$$

Comparing eq. (i) and (ii), we get

$a^2 = 36$ and $b^2 = 4$

$\Rightarrow a = \sqrt{36}$ and $b = \sqrt{4}$

$\Rightarrow a = 6$ and $b = 2$

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 6

= 12 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 6) and (0, -6)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = $(0, \pm c)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 36 – 4

$c^2 = 32$

$c = \sqrt{32}$ …(I)

∴ Coordinates of foci = (0, ±√32)

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{32}}{6} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (2)^2}{6}$$

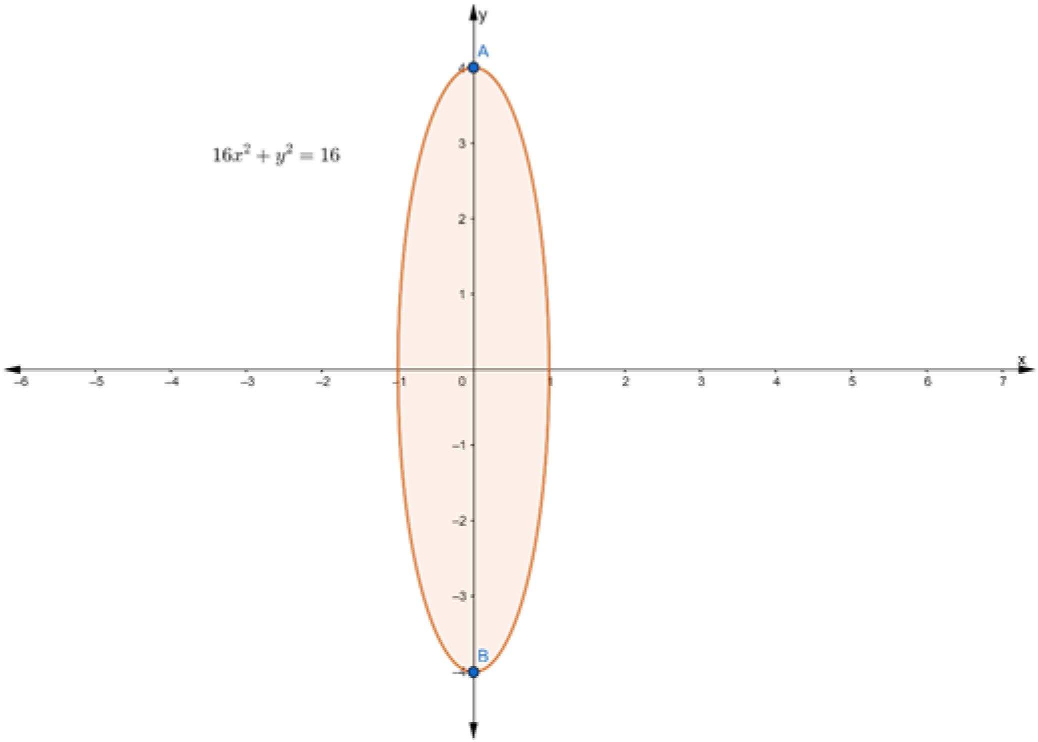
$$= \frac{8}{6}$$

$$= \frac{4}{3}$$

Q. 11. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$16x^2 + y^2 = 16$

Answer :



Given:

$16x^2 + y^2 = 16$

Divide by 16 to both the sides, we get

$$\frac{16}{16}x^2 + \frac{1}{16}y^2 = 1$$

$$\frac{x^2}{1} + \frac{y^2}{16} = 1$$ …(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	(± c, 0)	(± a, 0)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	(0,± c)	(0,± a)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$

Since, 1 < 16

So, above equation is of the form,

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad \text{...(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 1$

⇒ a = √16 and b = √1

⇒ a = 4 and b = 1

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 4) and (0, -4)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

= 16 – 1

$c^2 = 15$

c = √15 …(I)

∴ Coordinates of foci = (0, ±√15)

(iv) <u>To find</u>: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{15}}{4} \text{ [from (I)]}$$

(v) <u>To find</u>: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (1)^2}{4}$$

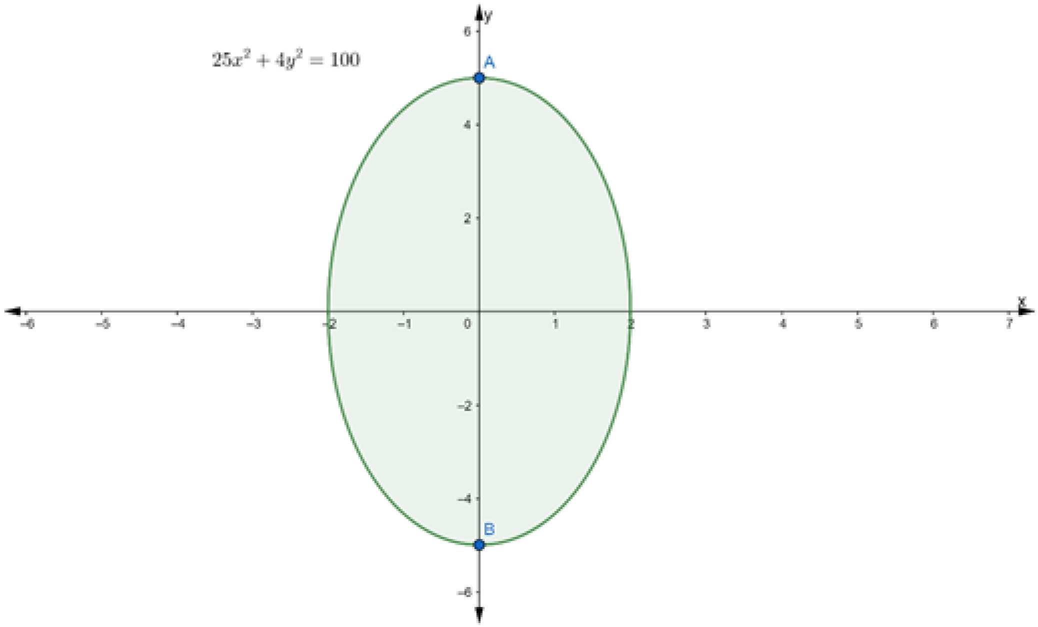
$$= \frac{2 \times 1}{4}$$

$$= \frac{1}{2}$$

Q. 12. Find the (i) lengths of major axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses.

$25x^2 + 4y^2 = 100$

Answer :



Given:

$25x^2 + 4y^2 = 100$

Divide by 100 to both the sides, we get

$$\frac{25}{100}x^2 + \frac{4}{100}y^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{25} = 1 \quad \text{…(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 25

So, above equation is of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{…(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

⇒ $a = \sqrt{25}$ and $b = \sqrt{4}$

⇒ a = 5 and b = 2

(i) **<u>To find</u>:** Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) **<u>To find</u>:** Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 5) and (0, -5)

(iii) **<u>To find</u>:** Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 – b^2$

So, firstly we find the value of c

$c^2 = a^2 – b^2$

= 25 – 4

$c^2 = 21$

$c = \sqrt{21}$ …(I)

∴ Coordinates of foci = $(0, \pm\sqrt{21})$

(iv) **<u>To find</u>:** Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{21}}{5} \text{ [from (I)]}$$

(v) **<u>To find</u>:** Length of the Latus Rectum

We know that,

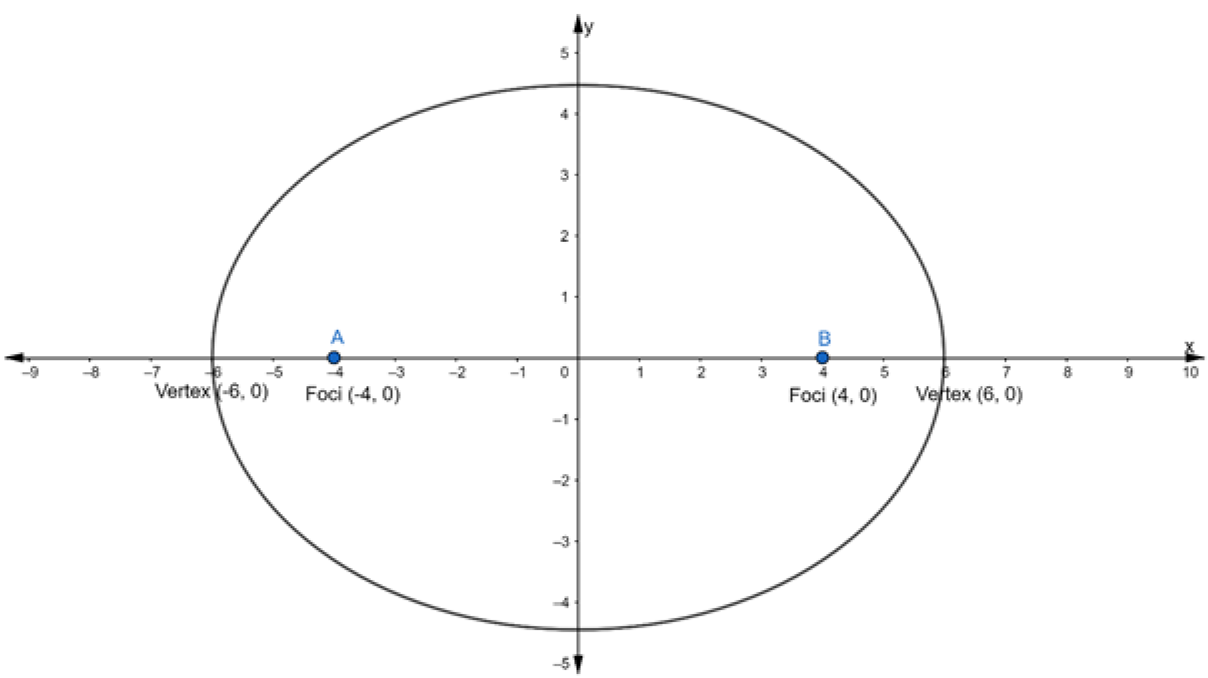
$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (2)^2}{5}$$

$$= \frac{8}{5}$$

Q. 13. Find the equation of the ellipse whose vertices are at (±6, 0) and foci at (±4, 0).

Answer :



Given: Vertices = (±6, 0) …(i)

The vertices are of the form = (±a, 0) …(ii)

Hence, the major axis is along x – axis

∴ From eq. (i) and (ii), we get

a = 6

⇒ $a^2 = 36$

and We know that, if the major axis is along x – axis then the equation of Ellipse is of the form of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Also, given coordinate of foci = (±4, 0) …(iii)

We know that,

Coordinates of foci = (±c, 0) …(iv)

∴ From eq. (iii) and (iv), we get

c = 4

We know that,

$c^2 = a^2 – b^2$

⇒ $(4)^2 = (6)^2 – b^2$

⇒ $16 = 36 – b^2$

⇒ $b^2 = 36 – 16$

⇒ $b^2 = 20$

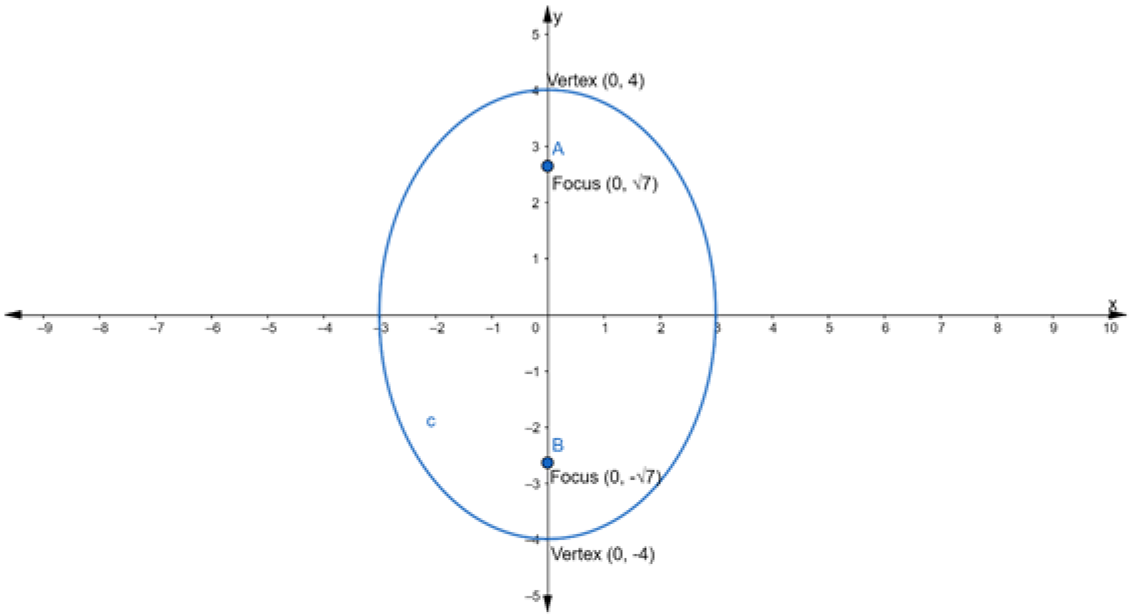
Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{36} + \frac{y^2}{20} = 1$$

Q. 14. Find the equation of the ellipse whose vertices are the (0, ±4) and foci at $(0, \pm\sqrt{7})$.

Answer :



Given: Vertices = (0, ±4) …(i)

The vertices are of the form = (0, ±a) …(ii)

Hence, the major axis is along y – axis

∴ From eq. (i) and (ii), we get

a = 4

⇒ $a^2 = 16$

and We know that, if the major axis is along y – axis then the equation of Ellipse is of the form of

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Also, given coordinate of foci = (0, ±√7) …(iii)

We know that,

Coordinates of foci = (0, ±c) …(iv)

∴ From eq. (iii) and (iv), we get

$c = \sqrt{7}$

We know that,

$c^2 = a^2 - b^2$

$\Rightarrow (\sqrt{7})^2 = (4)^2 - b^2$

$\Rightarrow 7 = 16 - b^2$

$\Rightarrow b^2 = 16 - 7$

$\Rightarrow b^2 = 9$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$$

Q. 15. Find the equation of the ellipse the ends of whose major and minor axes are (±4, 0) and (0, ±3) respectively.

Answer : Given:

Ends of Major Axis = (±4, 0)

and Ends of Minor Axis = (0, ±3)

Here, we can see that the major axis is along the x – axis.

∴ The Equation of Ellipse is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(i)$$

where, a is the semi – major axis and b is the semi – minor axis.

Accordingly, a = 4 and b = 3

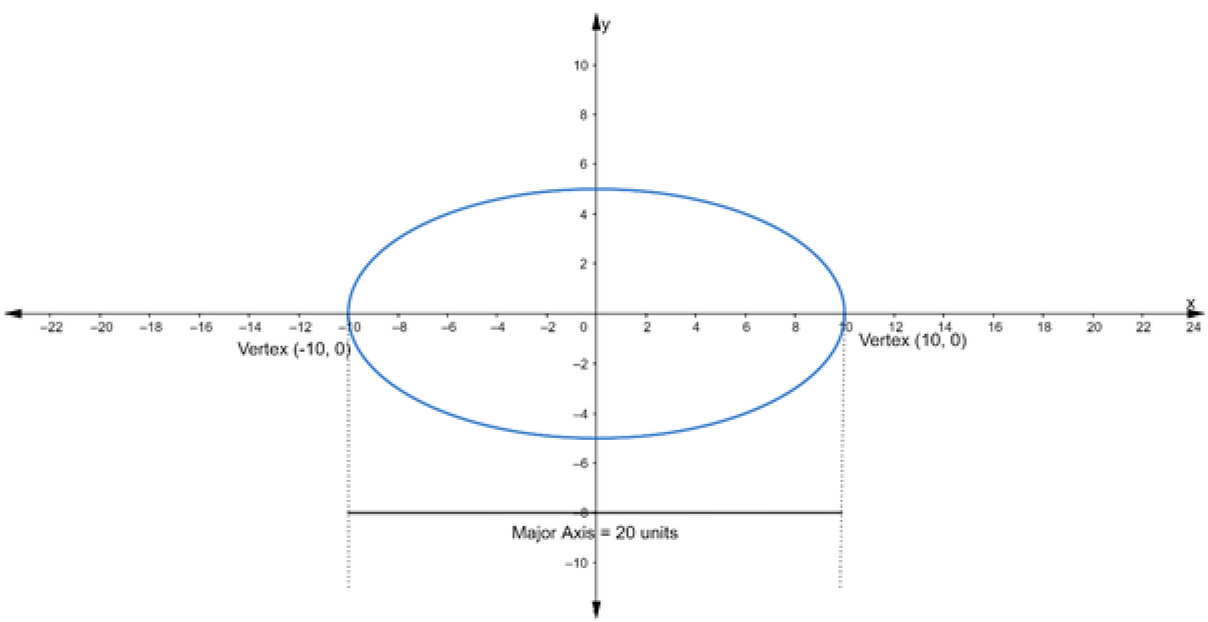
Substituting the value of a and b in eq. (i), we get

$$\frac{x^2}{(4)^2}+\frac{y^2}{(3)^2}=1$$

$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

Q. 16. The length of the major axis of an ellipse is 20 units, and its foci are $(\pm 5\sqrt{3}, 0)$. Find the equation of the ellipse.

Answer :



Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Given: Length of Major Axis = 20units …(i)

We know that,

Length of Major Axis = 2a …(ii)

∴ From eq. (i) and (ii), we get

$2a = 20$

$\Rightarrow a = 10$

It is also given that,

Coordinates of foci = $(\pm 5\sqrt{3}, 0)$ …(iii)

We know that,

Coordinates of foci = $(\pm c, 0)$ …(iv)

∴ From eq. (iii) and (iv), we get

$c = 5\sqrt{3}$

We know that,

$c^2 = a^2 - b^2$

$\Rightarrow (5\sqrt{3})^2 = (10)^2 - b^2$

$\Rightarrow 75 = 100 - b^2$

$\Rightarrow b^2 = 100 - 75$

$\Rightarrow b^2 = 25$

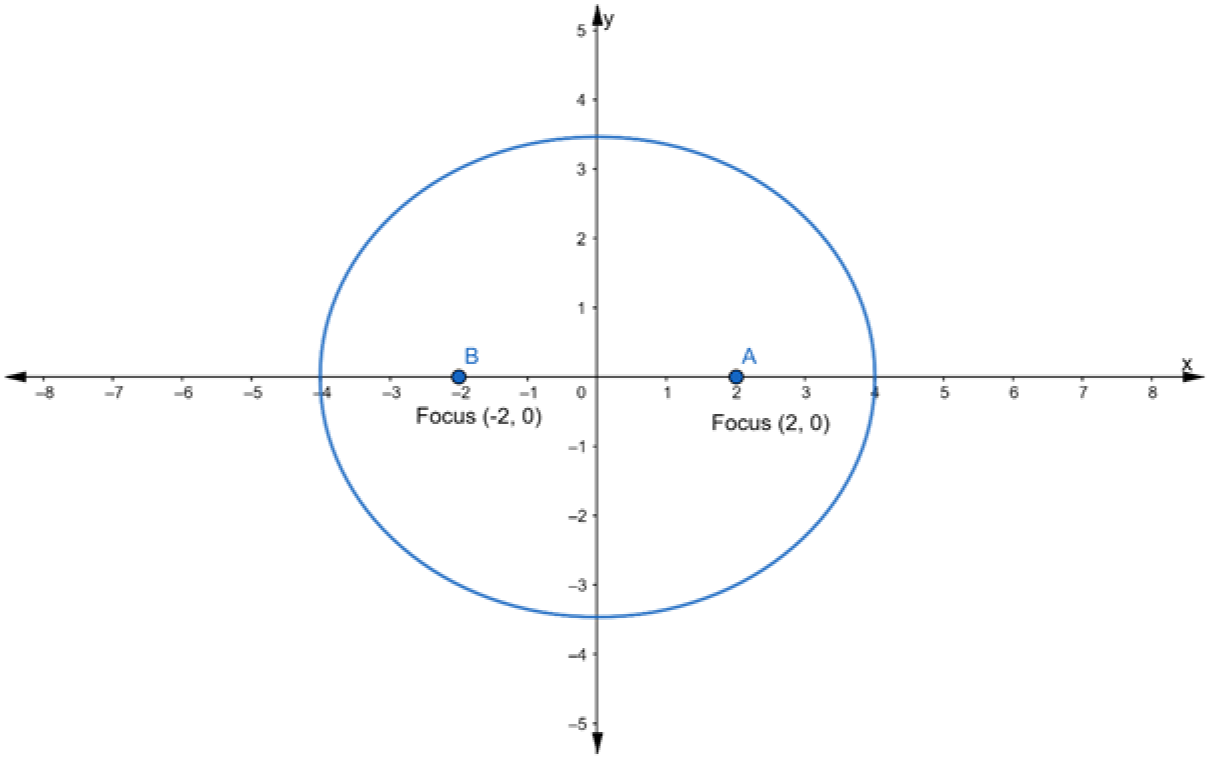
Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{100} + \frac{y^2}{25} = 1$$

Q. 17. Find the equation of the ellipse whose foci are (±2, 0) and the eccentricity is $\frac{1}{2}$.

Answer :



Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given:

Coordinates of foci = (±2, 0) …(iii)

We know that,

Coordinates of foci = (±c, 0) …(iv)

∴ From eq. (iii) and (iv), we get

c = 2

It is also given that

$$\text{Eccentricity} = \frac{1}{2}$$

we know that,

$\text{Eccentricity}, e = \frac{c}{a}$

$\Rightarrow \frac{1}{2} = \frac{2}{a}$ [∵ c = 2]

⇒ a = 4

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (2)^2 = (4)^2 - b^2$

$\Rightarrow 4 = 16 - b^2$

$\Rightarrow b^2 = 16 - 4$

$\Rightarrow b^2 = 12$

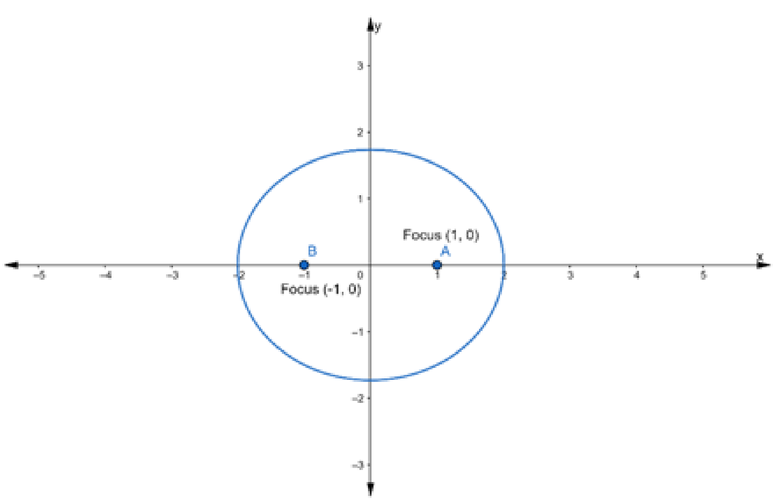
Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$$

Q. 18. Find the equation of the ellipse whose foci are at (±1, 0) and $e = \frac{1}{2}$.

Answer :



Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given:

Coordinates of foci = (±1, 0) …(i)

We know that,

Coordinates of foci = (±c, 0) …(ii)

∴ From eq. (i) and (ii), we get

c = 1

It is also given that

$$\text{Eccentricity} = \frac{1}{2}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$\Rightarrow \frac{1}{2} = \frac{1}{a}$ $[\because c = 1]$

⇒ a = 2

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (1)^2 = (2)^2 - b^2$

$\Rightarrow 1 = 4 - b^2$

$\Rightarrow b^2 = 4 - 1$

$\Rightarrow b^2 = 3$

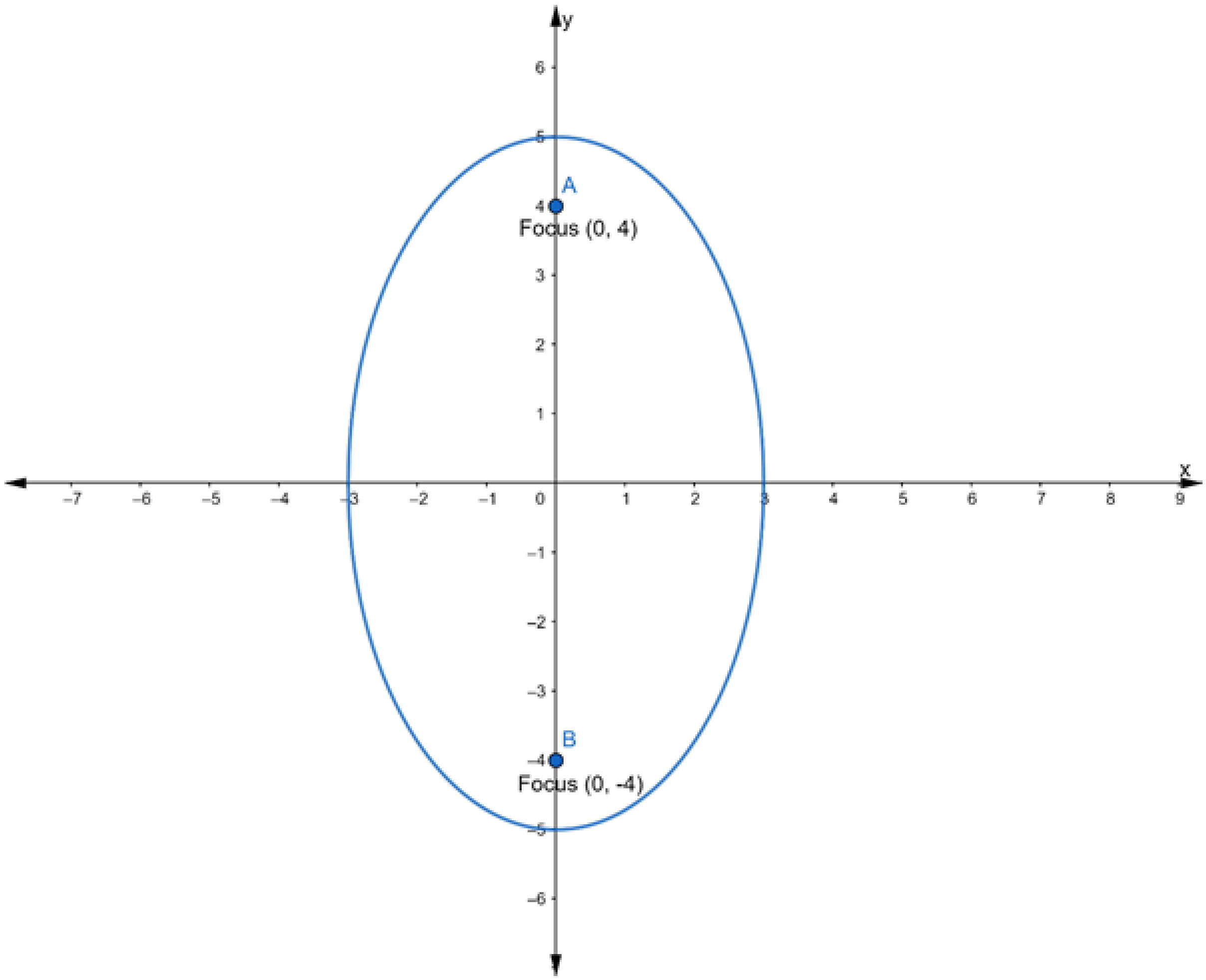
Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$$

Q. 19. Find the equation of the ellipse whose foci are at (0, ±4) and $e = \frac{4}{5}$.

Answer :



Given:

Coordinates of foci = (0, ±4) …(i)

We know that,

Coordinates of foci = (0, ±c) …(ii)

The coordinates of the foci are (0, ±4). This means that the major and minor axes are along y and x axes respectively.

∴ From eq. (i) and (ii), we get

$c = 4$

It is also given that

$$\text{Eccentricity} = \frac{4}{5}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$\Rightarrow \frac{4}{5} = \frac{4}{a} \; [\because c = 4]$$

$\Rightarrow a = 5$

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (4)^2 = (5)^2 - b^2$

$\Rightarrow 16 = 25 - b^2$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b^2 = 9$

Since, the foci of the ellipse are on y – axis. So, the Equation of Ellipse is

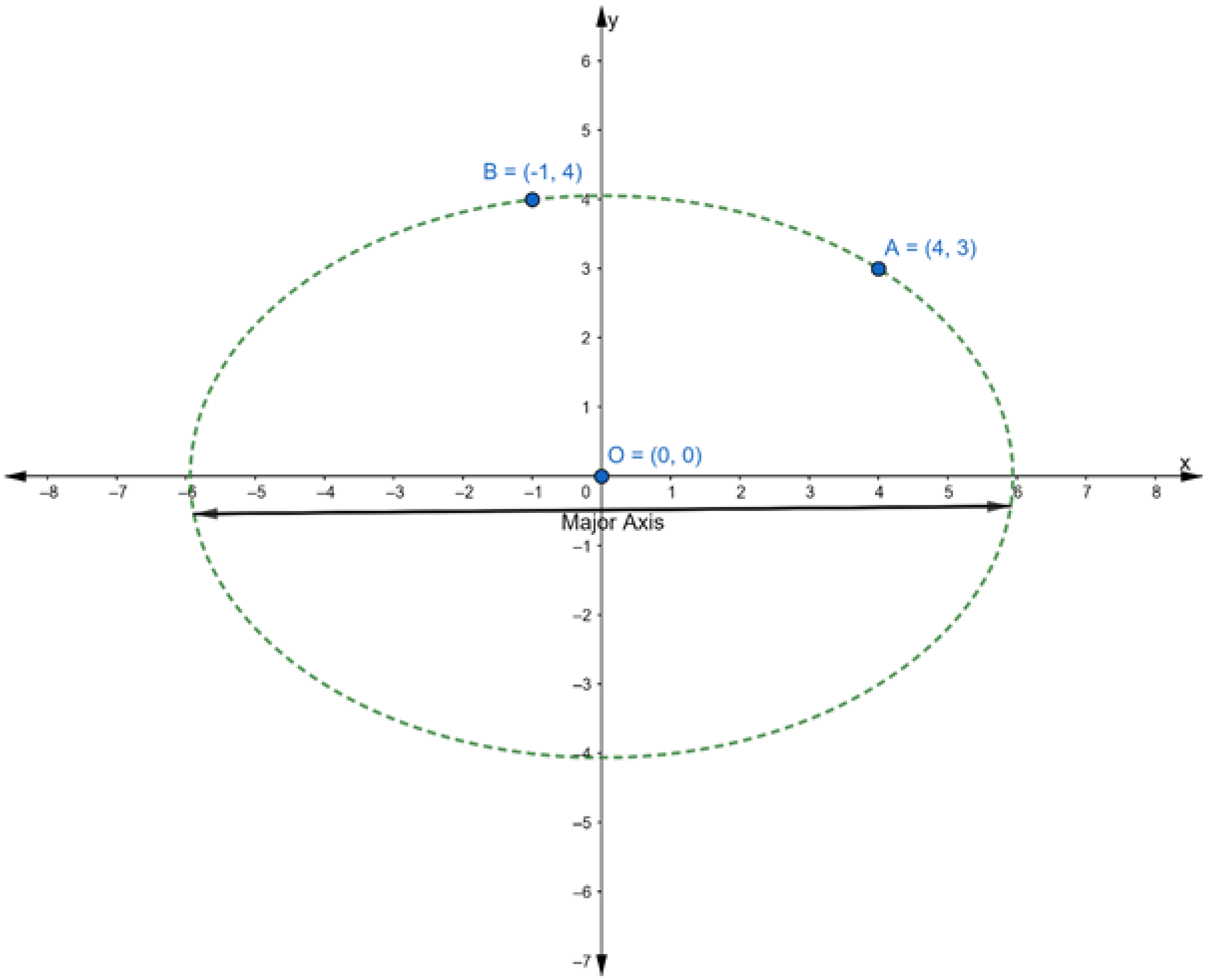
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Substituting the value of a^2 and b^2 , we get

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{25} = 1$$

Q. 20. Find the equation of the ellipse with center at the origin, the major axis on the x-axis and passing through the points (4, 3) and (-1, 4).

Answer :



Given: Center is at the origin

and Major axis is along x – axis

So, Equation of ellipse is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots(i)$$

Given that ellipse passing through the points (4, 3) and (-1, 4)

So, point (4, 3) and (-1, 4) will satisfy the eq. (i)

Taking point (4, 3) where x = 4 and y = 3

Putting the values in eq. (i), we get

$$\frac{(4)^2}{a^2}+\frac{(3)^2}{b^2}=1$$

$$\Rightarrow \frac{16}{a^2}+\frac{9}{b^2}=1 \qquad \ldots\text{(ii)}$$

Taking point (-1, 4) where x = -1 and y = 4

Putting the values in eq. (i), we get

$$\frac{(-1)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$$\Rightarrow \frac{1}{a^2}+\frac{16}{b^2}=1 \qquad \ldots\text{(iii)}$$

Now, we have to solve the above two equations to find the value of a and b

Multiply the eq. (iii) by 16, we get

$$\frac{16}{a^2}+\frac{16\times 16}{b^2}=1\times 16$$

$$\Rightarrow \frac{16}{a^2}+\frac{256}{b^2}=16 \qquad \ldots\text{(iv)}$$

Subtracting eq. (iv) from (ii), we get

$$\frac{16}{a^2}+\frac{9}{b^2}=1$$

$$\frac{16}{a^2}+\frac{256}{b^2}=16$$

$$-\quad -\quad -$$

$$\frac{9}{b^2}-\frac{256}{b^2}=1-16$$

$$\Rightarrow \frac{9-256}{b^2}=-15$$

$$\Rightarrow -\frac{247}{b^2} = -15$$

$$\Rightarrow b^2 = \frac{247}{15}$$

Substituting the value of b^2 in eq. (iii), we get

$$\frac{1}{a^2} + \frac{16}{\frac{247}{15}} = 1$$

$$\Rightarrow \frac{1}{a^2} + \frac{15 \times 16}{247} = 1$$

$$\Rightarrow \frac{1}{a^2} + \frac{240}{247} = 1$$

$$\Rightarrow \frac{1}{a^2} = 1 - \frac{240}{247}$$

$$\Rightarrow \frac{1}{a^2} = \frac{247 - 240}{247}$$

$$\Rightarrow \frac{1}{a^2} = \frac{7}{247}$$

$$\Rightarrow a^2 = \frac{247}{7}$$

Thus,

$$a^2 = \frac{247}{7} \ \& \ b^2 = \frac{247}{15}$$

Substituting the value of a^2 and b^2 in eq. (i), we get

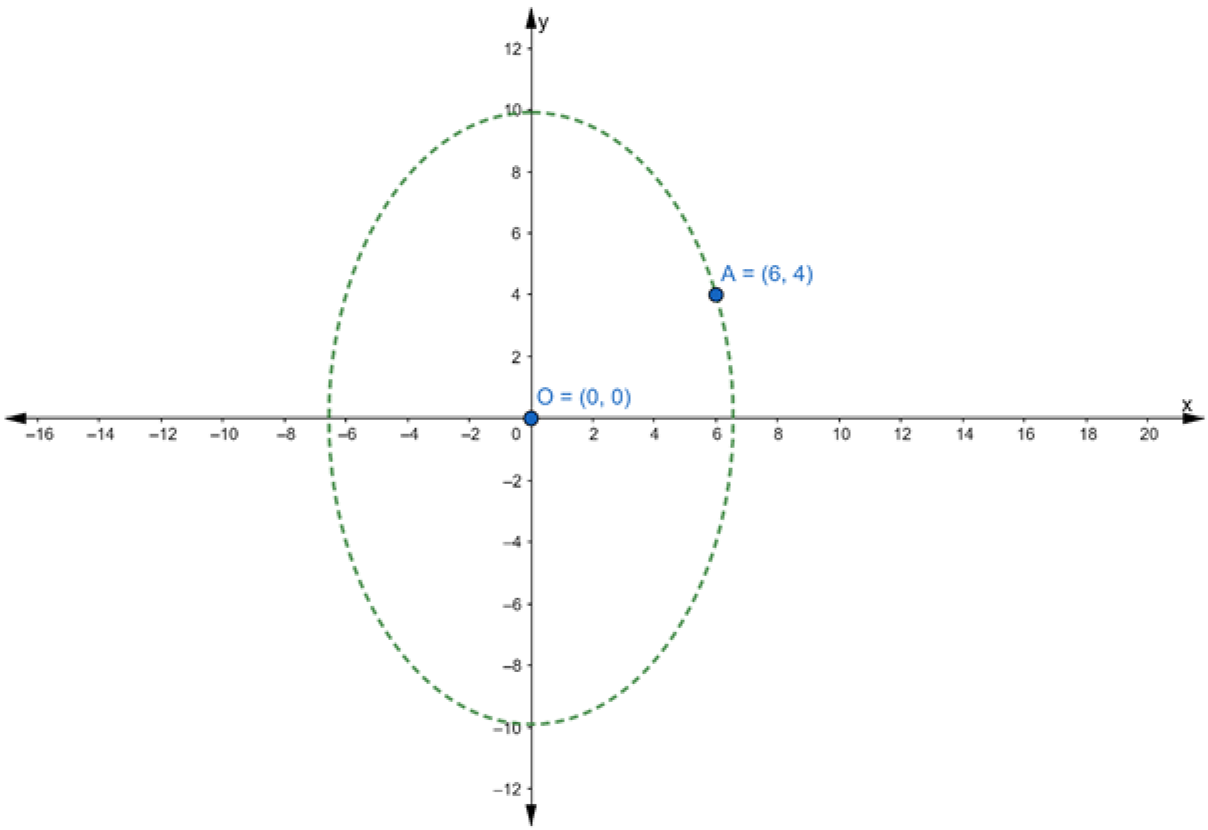
$$\frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1$$

$$\Rightarrow \frac{7x^2}{247} + \frac{15y^2}{247} = 1$$

$\Rightarrow 7x^2 + 15y^2 = 247$

Q. 21. Find the equation of the ellipse with eccentricity $\frac{3}{4}$, foci on the y-axis, center at the origin and passing through the point (6, 4).

Answer :



Given that

$$\text{Eccentricity} = \frac{3}{4}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$\Rightarrow \frac{3}{4} = \frac{c}{a}$$

$$\Rightarrow c = \frac{3}{4}a$$

We know that,

$c^2 = a^2 - b^2$

$$\Rightarrow \left(\frac{3a}{4}\right)^2 = a^2 - b^2$$

$$\Rightarrow \frac{9a^2}{16} = a^2 - b^2$$

$$\Rightarrow b^2 = a^2 - \frac{9a^2}{16}$$

$$\Rightarrow b^2 = \frac{16a^2 - 9a^2}{16}$$

$$\Rightarrow b^2 = \frac{7a^2}{16} \qquad \ldots(i)$$

It is also given that Coordinates of foci is on the y – axis

So, Equation of ellipse is of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Substituting the value of b^2 in above eq., we get

$$\frac{x^2}{\frac{7a^2}{16}} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \frac{16x^2}{7a^2} + \frac{y^2}{a^2} = 1 \qquad \ldots(ii)$$

Given that ellipse passing through the points (6, 4)

So, point (6, 4) will satisfy the eq. (ii)

Taking point (6, 4) where x = 6 and y = 4

Putting the values in eq. (ii), we get

$$\frac{16(6)^2}{7a^2}+\frac{(4)^2}{a^2}=1$$

$$\Rightarrow \frac{16\times 36}{7a^2}+\frac{16}{a^2}=1$$

$$\Rightarrow \frac{576+7\times 16}{7a^2}=1$$

$$\Rightarrow \frac{576+112}{7a^2}=1$$

$$\Rightarrow \frac{688}{7a^2}=1$$

$$\Rightarrow a^2=\frac{688}{7}$$

Substituting the value of a^2 in eq. (i), we get

$$b^2=\frac{7\times\frac{688}{7}}{16}$$

$$\Rightarrow b^2=\frac{688}{16}$$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

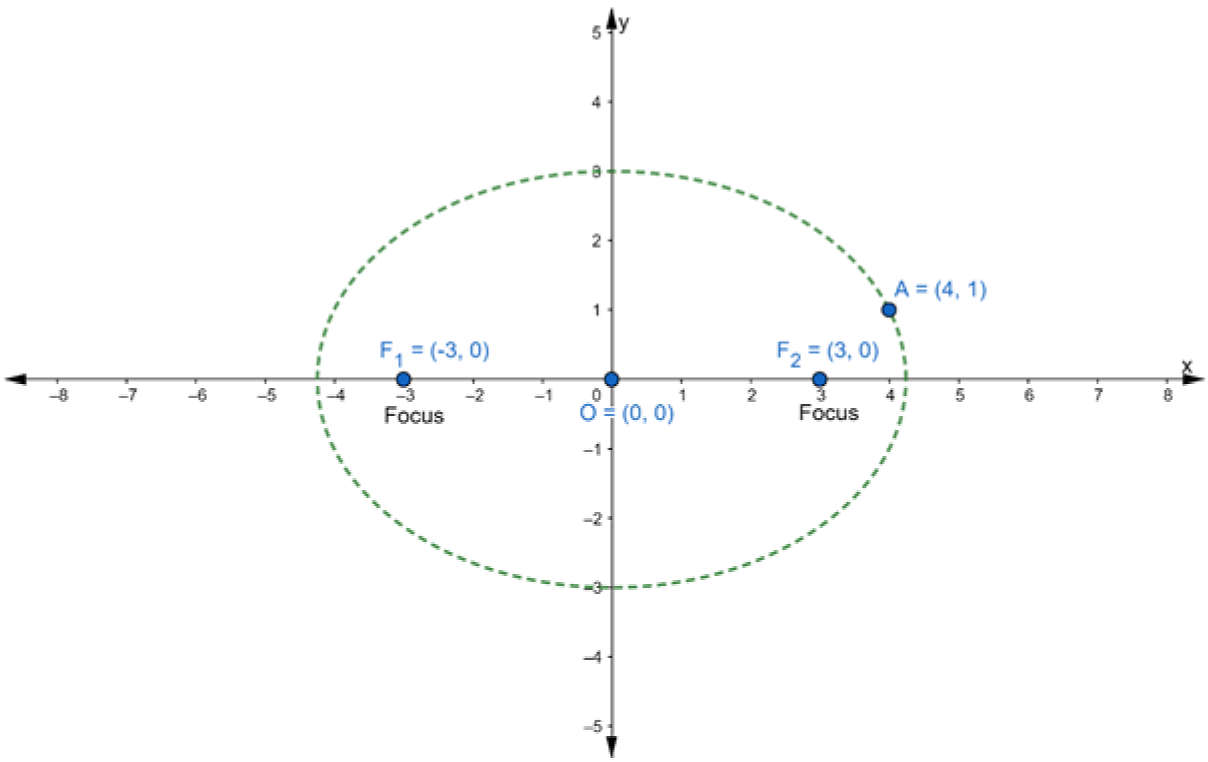
$$\Rightarrow \frac{x^2}{\frac{688}{16}}+\frac{y^2}{\frac{688}{7}}=1$$

$$\Rightarrow \frac{16x^2}{688} + \frac{7y^2}{688} = 1$$

or $16x^2 + 7y^2 = 688$

Q. 22. Find the equation of the ellipse which passes through the point (4, 1) and having its foci at (±3, 0).

Answer :



Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{...(i)}$$

Given:

Coordinates of foci = (±3, 0) …(ii)

We know that,

Coordinates of foci = (±c, 0) …(iii)

∴ From eq. (ii) and (iii), we get

$c = 3$

We know that,

$c^2 = a^2 – b^2$

$\Rightarrow (3)^2 = a^2 – b^2$

$\Rightarrow 9 = a^2 – b^2$

$\Rightarrow b^2 = a^2 – 9$ …(iv)

Given that ellipse passing through the points (4, 1)

So, point (4, 1) will satisfy the eq. (i)

Taking point (4, 1) where x = 4 and y = 1

Putting the values in eq. (i), we get

$$\frac{(4)^2}{a^2} + \frac{(1)^2}{b^2} = 1$$

$$\Rightarrow \frac{16}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow \frac{16}{a^2} + \frac{1}{a^2 - 9} = 1 \text{ [from (iv)]}$$

$$\Rightarrow \frac{16(a^2 - 9) + a^2}{(a^2)(a^2 - 9)} = 1$$

$\Rightarrow 16a^2 – 144 + a^2 = a^2(a^2 – 9)$

$\Rightarrow 17a^2 – 144 = a^4 – 9a^2$

$\Rightarrow a^4 – 9a^2 – 17a^2 + 144 = 0$

$\Rightarrow a^4 – 26a^2 + 144 = 0$

$\Rightarrow a^4 – 8a^2 – 18a^2 + 144 = 0$

$\Rightarrow a^2(a^2 – 8) – 18(a^2 – 8) = 0$

$\Rightarrow (a^2 - 8)(a^2 - 18) = 0$

$\Rightarrow a^2 - 8 = 0$ or $a^2 - 18 = 0$

$\Rightarrow a^2 = 8$ or $a^2 = 18$

If $a^2 = 8$ then

$b^2 = 8 - 9$

$= -1$

Since the square of a real number cannot be negative. So, this is not possible

If $a^2 = 18$ then

$b^2 = 18 - 9$

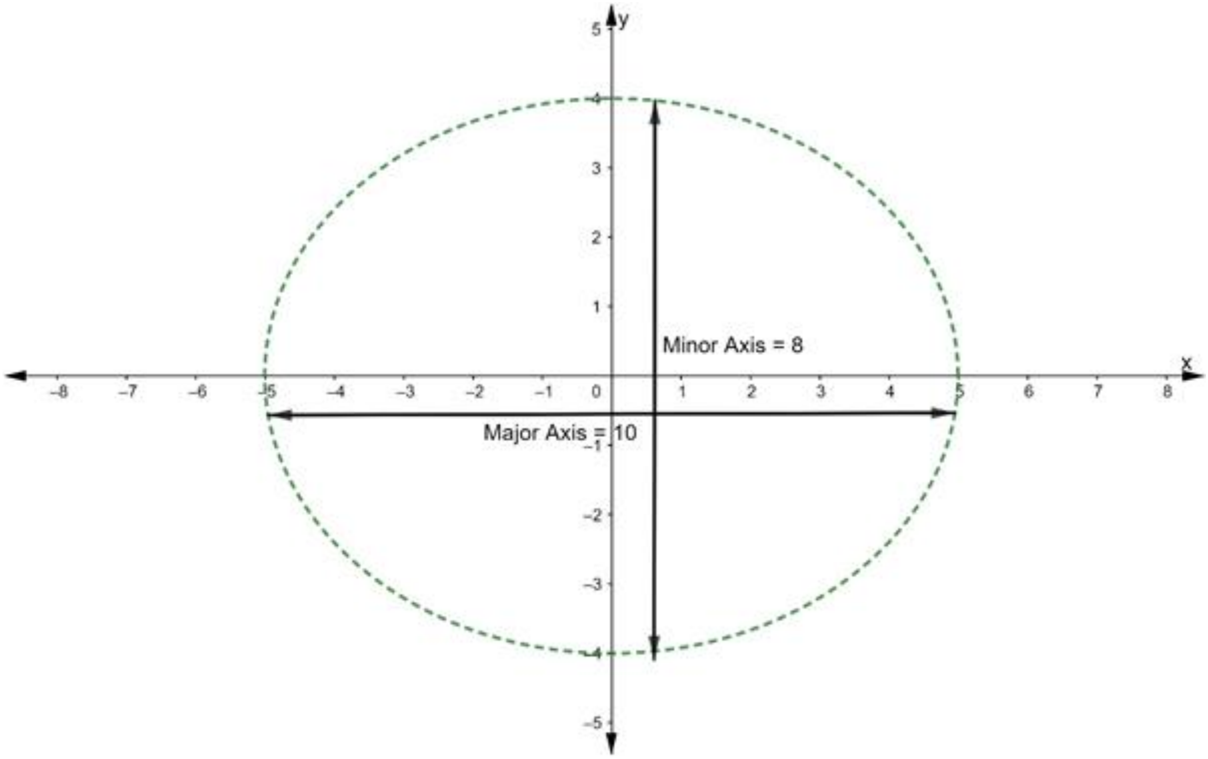
$= 9$

So, equation of ellipse if $a^2 = 18$ and $b^2 = 9$

$$\frac{x^2}{18} + \frac{y^2}{9} = 1$$

Q. 23. Find the equation of an ellipse, the lengths of whose major and mirror axes are 10 and 8 units respectively.

Answer :



Let the equation of required ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ …(A)

Given:

Length of Major Axis = 10units …(i)

We know that,

Length of major axis = 2a …(ii)

∴From eq. (i) and (ii), we get

2a = 10

⇒ a = 5

It is also given that

Length of Minor Axis = 8 units …(iii)

We know that,

Length of minor axis = 2b …(iv)

∴From eq. (iii) and (iv), we get

2b = 8

⇒ b = 4

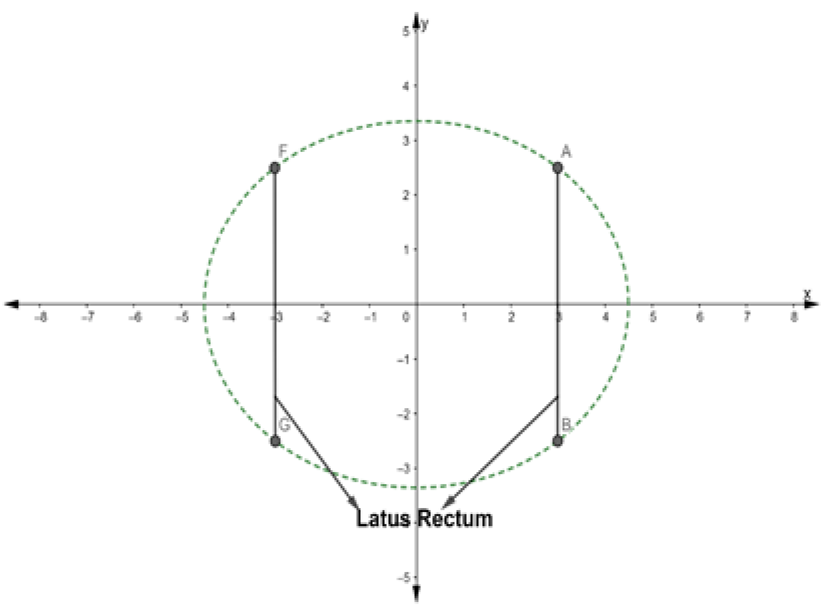
Substituting the value of a and b in eq. (A), we get

$$\frac{x^2}{(5)^2}+\frac{y^2}{(4)^2}=1$$

$$\Rightarrow \frac{x^2}{25}+\frac{y^2}{16}=1$$

Q. 24. Find the equation of an ellipse whose eccentricity is $\frac{2}{3}$, the latus rectum is 5, and the center is at the origin.

Answer :



Let the equation of the required ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{…(i)}$$

Given that

$$\text{Eccentricity} = \frac{2}{3}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$\Rightarrow \frac{2}{3} = \frac{c}{a}$$

$$\Rightarrow c = \frac{2}{3}a$$

We know that,

$c^2 = a^2 - b^2$

$$\Rightarrow \left(\frac{2a}{3}\right)^2 = a^2 - b^2$$

$$\Rightarrow \frac{4a^2}{9} = a^2 - b^2$$

$$\Rightarrow b^2 = a^2 - \frac{4a^2}{9}$$

$$\Rightarrow b^2 = \frac{9a^2 - 4a^2}{9}$$

$$\Rightarrow b^2 = \frac{5a^2}{9} \quad \text{...(ii)}$$

It is also given that, Latus Rectum = 5 …(iii)

We know that,

$$\text{Latus Rectum} = \frac{2b^2}{a}$$

$$\Rightarrow 5 = \frac{2 \times \left(\frac{5a^2}{9}\right)}{a}$$

$$\Rightarrow 5 = \frac{10a^2}{9a}$$

$$\Rightarrow 5 = \frac{10a}{9}$$

$$\Rightarrow a = \frac{5 \times 9}{10}$$

$$\Rightarrow a = \frac{9}{2}$$

$$\Rightarrow a^2 = \frac{81}{4}$$

Substituting the value of a in eq. (ii), we get

$$b^2 = \frac{5\left(\frac{9}{2}\right)^2}{9}$$

$$\Rightarrow b^2 = \frac{5 \times 9}{4}$$

$$\Rightarrow b^2 = \frac{45}{4}$$

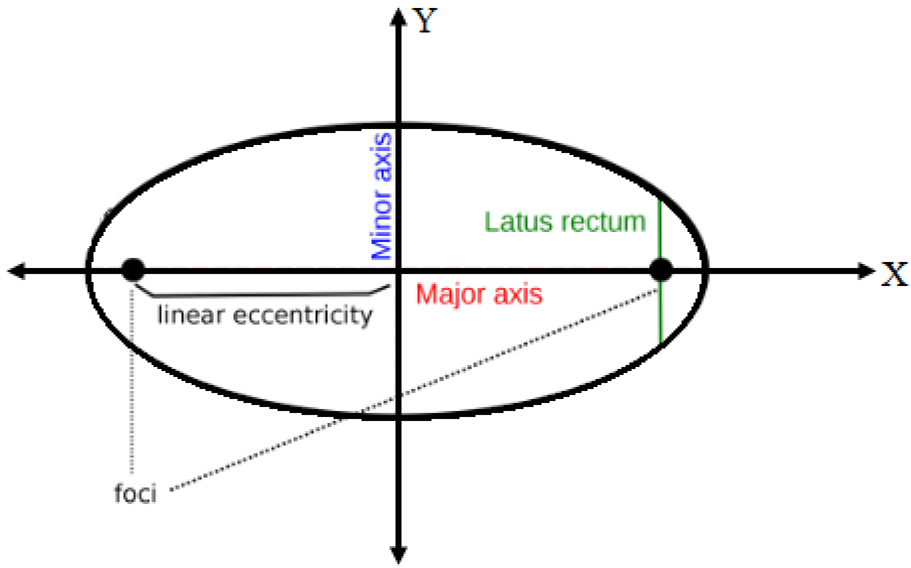
Substituting the value of a^2 and b^2 in eq. (i), we get

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

Q. 25. Find the eccentricity of an ellipse whose latus rectum is one half of its minor axis.

Answer :



Let the equation of the required ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(i)$$

It is given that,

$$\text{Length of Latus Rectum} = \frac{1}{2}\text{minor Axis}$$

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

and Length of Minor Axis = 2b

So, according to the given condition,

$$\frac{2b^2}{a} = \frac{1}{2} \times 2b$$

$$\Rightarrow \frac{2b^2}{a} = b$$

$$\Rightarrow \frac{2b^2}{b} = a$$

$\Rightarrow 2b = a$ …(ii)

Now, we have to find the eccentricity

We know that,

$$\text{Eccentricity}, e = \frac{c}{a} \quad \text{…(iii)}$$

where, $c^2 = a^2 - b^2$

So, $c^2 = (2b)^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = 4b^2 - b^2$

$\Rightarrow c^2 = 3b^2$

$\Rightarrow c = \sqrt{3b^2}$

$\Rightarrow c = b\sqrt{3}$

Substituting the value of c and a in eq. (iii), we get

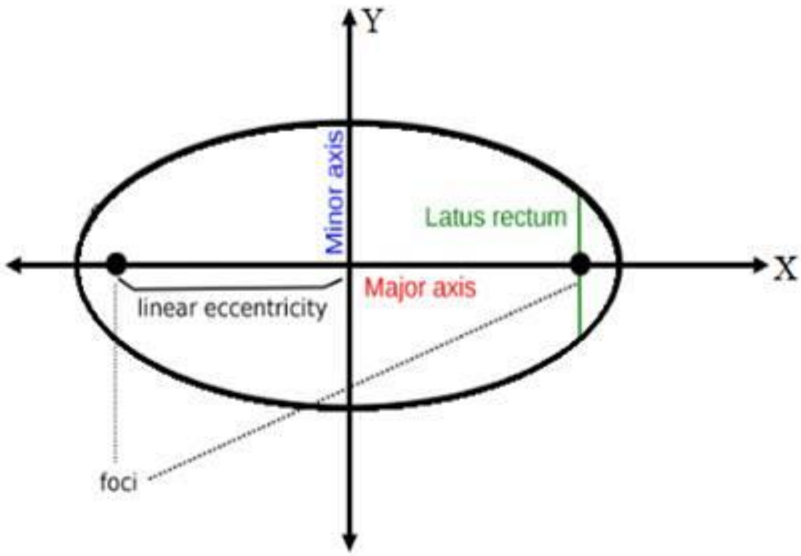
$$\text{Eccentricity}, e = \frac{c}{a}$$

$$= \frac{b\sqrt{3}}{2b}$$

$$\therefore e = \frac{\sqrt{3}}{2}$$

Q. 26. Find the eccentricity of an ellipse whose latus rectum is one half of its major axis.

Answer :



Let the equation of the required ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(i)$$

It is given that,

$$\text{Length of Latus Rectum} = \frac{1}{2}\text{major Axis}$$

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

and Length of Minor Axis = 2a

So, according to the given condition,

$$\frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow \frac{2b^2}{a} = a$$

$\Rightarrow 2b^2 = a^2$ …(ii)

$\Rightarrow a = \sqrt{2b^2}$

$\Rightarrow a = b\sqrt{2}$

Now, we have to find the eccentricity

We know that,

$$\text{Eccentricity}, e = \frac{c}{a} \qquad \ldots\text{(iii)}$$

where, $c^2 = a^2 - b^2$

So, $c^2 = 2b^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = b^2$

$\Rightarrow c = \sqrt{b^2}$

$\Rightarrow c = b$

Substituting the value of c and a in eq. (iii), we get

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$= \frac{b}{b\sqrt{2}}$$

$$\therefore e = \frac{1}{\sqrt{2}}$$