Exercise – 4A

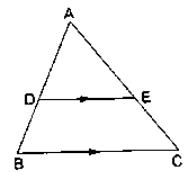
1. D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC.

(i) If AD = 3.6cm, AB = 10cm and AE = 4.5cm, find EC and AC.

(ii) If AB = 13.3cm, AC = 11.9cm and EC = 5.1cm, find AD.

(iii) If $\frac{AD}{DB} = \frac{4}{7}$ and AC = 6.6cm, find AE.

(iv) If $\frac{AD}{AB} = \frac{8}{15}$ and EC = 3.5cm, find AE.



Sol:

(i) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

∵ AD = 3.6 cm , AB = 10 cm, AE = 4.5cm

∴ DB = 10 − 3.6 = 6.4cm

Or, $\frac{3.6}{6.4} = \frac{4.5}{EC}$

Or, EC = $\frac{6.4 \times 4.5}{3.6}$

Or, EC= 8 cm

Thus, AC = AE + EC

= 4.5 + 8 = 12.5 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' Theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 to both sides, we get :

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

$\Rightarrow \frac{13.3}{DB} = \frac{11.9}{5.1}$

$\Rightarrow DB = \frac{13.3 \times 5.1}{11.9} = 5.7$ cm

Therefore, AD=AB-DB=13.5-5.7=7.6 cm

(iii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{7} = \frac{AE}{EC}$

Adding 1 to both the sides, we get :

$\frac{11}{7} = \frac{AC}{EC}$

$\Rightarrow$ EC = $\frac{6.6 \times 7}{11}$ = 4.2 cm

Therefore,

AE = AC – EC = 6.6 – 4.2 = 2.4 cm

(iv) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+EC}$

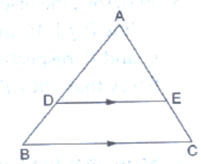
$\Rightarrow \frac{8}{15} = \frac{AE}{AE+3.5}$

⟹ 8AE + 28 = 15AE

⟹ 7AE = 28

⟹ AE = 4cm

2. D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC. Find the value of x, when



(i) AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm.

(ii) AD = 4cm, DB = (x – 4) cm, AE = 8cm and EC = (3x – 19) cm.

(iii) AD = (7x – 4) cm, AE = (5x – 2) cm, DB = (3x + 4) cm and EC = 3x cm.

Sol:

(i) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

⟹X(x-1) = (x-2) (x+2)

$\Rightarrow x^2 - x = x^2 - 4$

⟹x= 4 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

⟹ 4 (3x-19) = 8 (x-4)

⟹12x – 76 = 8x – 32

⟹4x = 44

⟹ x = 11 cm

(iii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

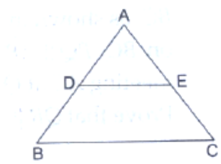
$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

⟹3x (7x-4) = (5x-2) (3x+4)

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$
$\Rightarrow 6x^2 - 26x + 8 = 0$
$\Rightarrow (x-4)(6x-2) = 0$
$\Rightarrow x = 4, \frac{1}{3}$
$\because x \neq \frac{1}{3}$ (as if $x = \frac{1}{3}$ *then AE will become negative*)
$\therefore x = 4$ cm

3. D and E are points on the sides AB and AC respectively of a ΔABC. In each of the following cases, determine whether DE ‖ BC or not.



(i) AD = 5.7cm, DB = 9.5cm, AE = 4.8cm and EC = 8cm.
(ii) AB = 11.7cm, AC = 11.2cm, BD = 6.5cm and AE = 4.2cm.
(iii) AB = 10.8cm, AD = 6.3cm, AC = 9.6cm and EC = 4cm.
(iv) AD = 7.2cm, AE = 6.4cm, AB = 12cm and AC = 10cm.

Sol:

(i) We have:
$\frac{AD}{DE} = \frac{5.7}{9.5} = 0.6\ cm$
$\frac{AE}{EC} = \frac{4.8}{8} = 0.6\ cm$
Hence, $\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

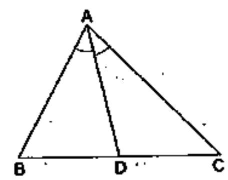
(ii) We have:
AB = 11.7 cm, DB = 6.5 cm
Therefore,
AD = 11.7 -6.5 = 5.2 cm
Similarly,
AC = 11.2 cm, AE = 4.2 cm
Therefore,
EC = 11.2 – 4.2 = 7 cm
Now,
$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{4}{5}$
$\frac{AE}{EC} = \frac{4.2}{7}$
Thus, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

(iii) We have:
AB = 10.8 cm, AD = 6.3 cm
Therefore,

DB = 10.8 – 6.3 = 4.5 cm
Similarly,
AC = 9.6 cm, EC = 4cm
Therefore,
AE = 9.6 – 4 = 5.6 cm
Now,
$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$
$\frac{AE}{EC} = \frac{5.6}{4} = \frac{7}{5}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

(iv) We have :
AD = 7.2 cm, AB = 12 cm
Therefore,
DB = 12 – 7.2 = 4.8 cm
Similarly,
AE = 6.4 cm, AC = 10 cm
Therefore,
EC = 10 – 6.4 = 3.6 cm
Now,
$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{3}{2}$
$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{16}{9}$
This, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

4. In a ΔABC, AD is the bisector of ∠A.
(i) If AB = 6.4cm, AC = 8cm and BD = 5.6cm, find DC.
(ii) If AB = 10cm, AC = 14cm and BC = 6cm, find BD and DC.
(iii) If AB = 5.6cm, BD = 3.2cm and BC = 6cm, find AC.
(iv) If AB = 5.6cm, AC = 4cm and DC = 3cm, find BC.



Sol:
(i) It is give that AD bisects ∠A.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{5.6}{DC} = \frac{6.4}{8}$
$\Rightarrow DC = \frac{8 \times 5.6}{6.4} = 7\ cm$

(ii) It is given that AD bisects $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

Let BD be x cm.

Therefore, DC = (6- x) cm

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

$\Rightarrow 14x = 60\text{-}10x$

$\Rightarrow 24x = 60$

$\Rightarrow x = 2.5$ cm

Thus, BD = 2.5 cm

DC = 6-2.5 = 3.5 cm

(iii) It is given that AD bisector $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

BD = 3.2 cm, BC = 6 cm

Therefore, DC = 6- 3.2 = 2.8 cm

$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2} = 4.9\ cm$

(iv) It is given that AD bisects $\angle A$.

Applying angle – bisector theorem in Δ ABC, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{3} = \frac{5.6}{4}$

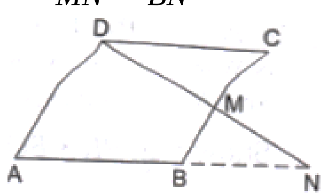
$\Rightarrow BD = \frac{5.6 \times 3}{4}$

$\Rightarrow BD = 4.2$ cm

Hence, BC = 3+ 4.2 = 7.2 cm

5. M is a point on the side BC of a parallelogram ABCD. DM when produced meets AB produced at N. Prove that

(i) $\frac{DM}{MN} = \frac{DC}{BN}$ (ii) $\frac{DN}{DM} = \frac{AN}{DC}$



Sol:

(i) Given: ABCD is a parallelogram

To prove:

(i) $\frac{DM}{MN} = \frac{DC}{BN}$

(ii) $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: In Δ DMC and Δ NMB

∠DMC = ∠NMB (Vertically opposite angle)

∠DCM = ∠NBM (Alternate angles)

By AAA- Similarity

ΔDMC ~ ΔNMB

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

NOW, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we get

$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC}$ [∵ ABCD is a parallelogram]

$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$

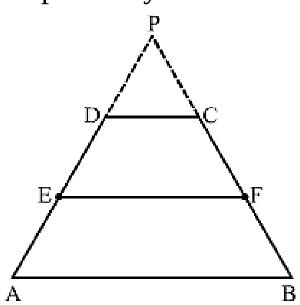
6. Show that the line segment which joins the midpoints of the oblique sides of a trapezium is parallel sides

Sol:

(i)

Let the trapezium be ABCD with E and F as the mid Points of AD and BC,

Respectively Produce AD and BC to Meet at P.



In Δ PAB, DC || AB.

Applying Thales' theorem, we get

$\frac{PD}{DA} = \frac{PC}{CB}$

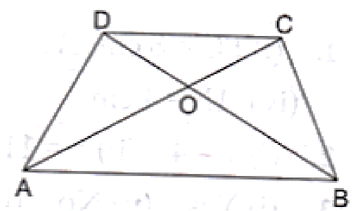
Now, E and F are the midpoints of AD and BC, respectively.

$\Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF}$

$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$

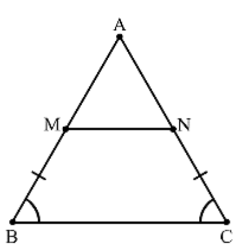
Applying the converse of Thales' theorem in Δ PEF, we get that DC
Hence, EF || AB.
Thus. EF is parallel to both AB and DC.
This completes the proof.

7. In the given figure, ABCD is a trapezium in which AB ∥ DC and its diagonals intersect at O. If AO = (5x – 7), OC = (2x + 1) , BO = (7x – 5) and OD = (7x + 1), find the value of x.



Sol:
In trapezium ABCD, AB ∥ CD and the diagonals AC and BD intersect at O.
Therefore,
$\frac{AO}{OC} = \frac{BO}{OD}$
$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$
$\Rightarrow$ (5x-7) (7x+1) = (7x-5) (2x+1)
$\Rightarrow 35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5$
$\Rightarrow 21x^2 - 41x - 2 = 0$
$\Rightarrow 21x^2 - 42x + x - 2 = 0$
$\Rightarrow 21x(x-2) + 1(x-2) = 0$
$\Rightarrow (x-2)(21x + 1) = 0$
$\Rightarrow x = 2, -\frac{1}{21}$
$\because x \neq -\frac{1}{21}$
$\therefore x = 2$

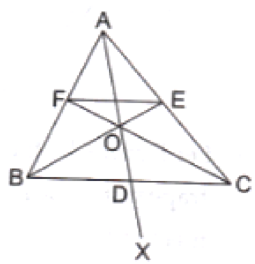
8. In ΔABC, M and N are points on the sides AB and AC respectively such that BM= CN. If $\angle B = \angle C$ then show that MN||BC
Sol:



In ΔABC, $\angle B = \angle C$
∴ AB = AC (Sides opposite to equal angle are equal)
Subtracting BM from both sides, we get

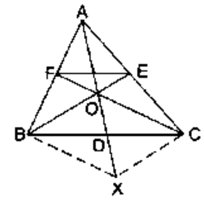
AB – BM = AC – BM
$\Rightarrow$AB – BM = AC – CN (∵BM =CN)
$\Rightarrow$AM =AN
∴∠AMN =∠ ANM (Angles opposite to equal sides are equal)
Now, in ΔABC,
$\angle A + \angle B + \angle C = 180^0$ ----(1)
(Angle Sum Property of triangle)
Again In In ΔAMN,
$\angle A + \angle AMN + \angle ANM = 180^0$ ----(2)
(Angle Sum Property of triangle)
From (1) and (2), we get
$\angle B + \angle C = \angle AMN + \angle ANM$
$\Rightarrow 2\angle B = 2\angle AMN$
$\Rightarrow \angle B = \angle AMN$
Since, ∠B and ∠ AMN are corresponding angles.
∴ MN || BC.

9. ΔABC and ΔDBC lie on the same side of BC, as shown in the figure. From a point P on BC, PQ||AB and PR||BD are drawn, meeting AC at Q and CD at R respectively. Prove that QR||AD.



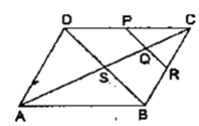
Sol:
In Δ CAB, PQ || AB.
Applying Thales' theorem, we get:
$\frac{CP}{PB} = \frac{CQ}{QA}$ …(1)
Similarly, applying Thales theorem in ΔBDC , Where PR||DM we get:
$\frac{CP}{PB} = \frac{CR}{RD}$ …(2)
Hence, from (1) and (2), we have :
$\frac{CQ}{QA} = \frac{CR}{RD}$
Applying the converse of Thales' theorem, we conclude that QR ∥ AD in Δ ADC.
This completes the proof.

10. In the given figure, side BC of a ΔABC is bisected at D and O is any point on AD. BO and CO produced meet AC and AB at E and F respectively, and AD is produced to X so that D is the midpoint of OX.
Prove that AO : AX = AF : AB and show that EF ∥ BC.



Sol:
It is give that BC is bisected at D.
∴ BD = DC
It is also given that OD =OX
The diagonals OX and BC of quadrilateral BOCX bisect each other.
Therefore, BOCX is a parallelogram.
∴ BO || CX and BX || CO
BX || CF and CX || BE
BX || OF and CX || OE
Applying Thales' theorem in Δ ABX, we get:
$\frac{AO}{AX} = \frac{AF}{AB}$ …(1)
Also, in Δ ACX, CX || OE.
Therefore by Thales' theorem, we get:
$\frac{AO}{AX} = \frac{AE}{AC}$ …(2)
From (1) and (2), we have:
$\frac{AO}{AX} = \frac{AE}{AC}$
Applying the converse of Theorem in Δ ABC, EF || CB.
This completes the proof.

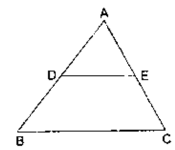
11. ABCD is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that CQ = $\frac{1}{4}$ AC. If PQ produced meets BC at R, prove that R is the midpoint of BC.



Sol:
We know that the diagonals of a parallelogram bisect each other.
Therefore,
CS = $\frac{1}{2}$ AC …(i)
Also, it is given that CQ = $\frac{1}{4}$ AC …(ii)
Dividing equation (ii) by (i), we get:
$\frac{CQ}{CS} = \frac{\frac{1}{4}AC}{\frac{1}{2}AC}$
Or, CQ = $\frac{1}{2}\ CS$
Hence, Q is the midpoint of CS.

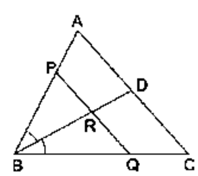
Therefore, according to midpoint theorem in ΔCSD
PQ || DS
If PQ || DS, we can say that QR || SB
In Δ CSB, Q is midpoint of CS and QR ∥ SB.
Applying converse of midpoint theorem , we conclude that R is the midpoint of CB.
This completes the proof.

12. In the adjoining figure, ABC is a triangle in which AB = AC. IF D and E are points on AB and AC respectively such that AD = AE, show that the points B, C, E and D are concyclic.



Sol:
Given:
AD = AE …(i)
AB = AC …(ii)
Subtracting AD from both sides, we get:
⟹ AB – AD = AC – AD
⟹ AB – AD = AC - AE (Since, AD = AE)
⟹ BD = EC …(iii)
Dividing equation (i) by equation (iii), we get:
$\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem, DE∥BC
⟹ ∠DEC + ∠ECB = 180^0 (Sum of interior angles on the same side of a Transversal Line is 0^0 .)
⟹ ∠DEC + ∠CBD = 180^0 (Since, AB = AC ⟹ ∠B = ∠C)
Hence, quadrilateral BCED is cyclic.
Therefore, B,C,E and D are concylic points.

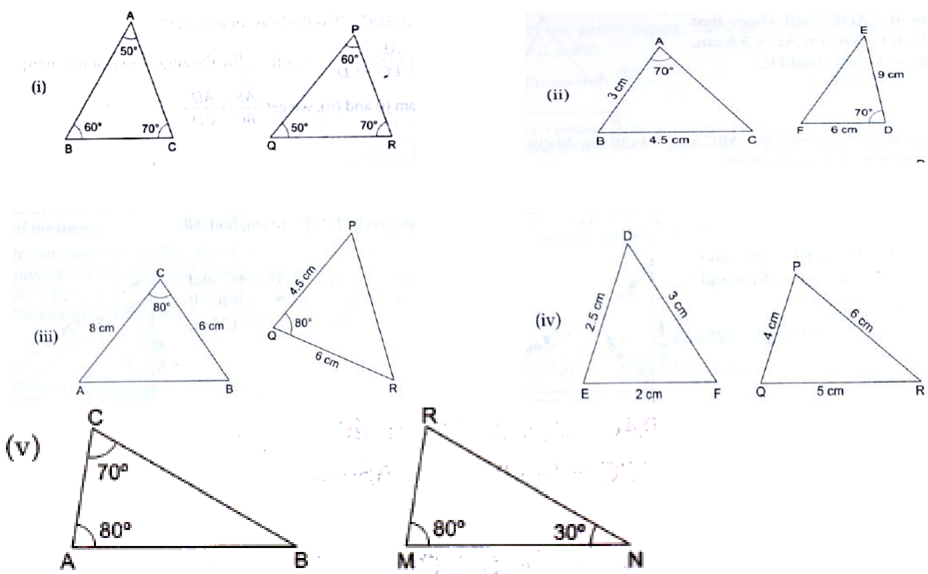
13. In ΔABC, the bisector of ∠B meets AC at D. A line OQ ∥ AC meets AB, BC and BD at O, Q and R respectively. Show that BP × QR = BQ × PR



Sol:
In triangle BQO, BR bisects angle B.
Applying angle bisector theorem, we get:
$\frac{QR}{PR} = \frac{BQ}{BP}$
⟹BP × QR = BQ × PR
This completes the proof.

Exercise – 4B

1. In each of the given pairs of triangles, find which pair of triangles are similar. State the similarity criterion and write the similarity relation in symbolic form:



Sol:

(i)

We have:

$\angle BAC = \angle PQR = 50^0$

$\angle ABC = \angle QPR = 60^0$

$\angle ACB = \angle PRQ = 70^0$

Therefore, by AAA similarity theorem, Δ ABC – QPR

(ii)

We have:

$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2}\ and\ \frac{BC}{DE} = \frac{4.5}{9} = \frac{1}{2}$

But , $\angle ABC \neq \angle EDF$ (Included angles are not equal)

Thus, this triangles are not similar.

(iii)

We have:

$\frac{CA}{QR} = \frac{8}{6} = \frac{4}{3}\ and\ \frac{CB}{PQ} = \frac{6}{4.5} = \frac{4}{3}$

$\Longrightarrow \frac{CA}{QR} = \frac{CB}{PQ}$

Also, $\angle ACB = \angle PQR = 80^0$

Therefore, by SAS similarity theorem, Δ ACB - Δ RQP.

(iv)

We have

$\frac{DE}{QR} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{EF}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{DF}{PR} = \frac{3}{6} = \frac{1}{2}$

$\Longrightarrow \frac{DE}{QR} = \frac{EF}{PQ} = \frac{DF}{PR}$

Therefore, by SSS similarity theorem, Δ FED- Δ PQR

(v)

In Δ ABC

$\angle A + \angle B + \angle C = 180^0$ (Angle Sum Property)

$\Longrightarrow 80^0 + \angle B + 70^0 = 180^0$

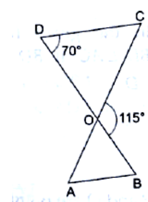
$\Longrightarrow \angle B = 30^0$

$\angle A = \angle M \; and \; \angle B = \angle N$

Therefore, by AA similarity , Δ ABC - Δ MNR

2. In the given figure, ΔODC~ΔOBA, ∠BOC = 115^0 and ∠CDO = 70^0.

Find (i) ∠DCO (ii) ∠DCO (iii) ∠OAB (iv) ∠OBA.



Sol:

(i)

It is given that DB is a straight line.

Therefore,

$\angle DOC + \angle COB = 180^0$

$\angle DOC = 180^0 - 115^0 = 65^0$

(ii)

In Δ DOC, we have:

$\angle ODC + \angle DCO + \angle DOC = 180^0$

Therefore,

$70^0 + \angle DCO + 65^0 = 180^0$

$\Longrightarrow \angle DCO = 180 - 70 - 65 = 45^0$

(iii)

It is given that Δ ODC - Δ OBA

Therefore,

$\angle OAB = \angle OCD = 45^0$

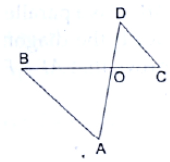
(iv)

Again, Δ ODC- Δ OBA

Therefore,

$\angle OBA = \angle ODC = 70^0$

3. In the given figure, ΔOAB ~ ΔOCD. If AB = 8cm, BO = 6.4cm, OC = 3.5cm and CD = 5cm, find (i) OA (ii) DO.



Sol:

(i) Let OA be X cm.

$\because \Delta$ OAB - Δ OCD

$\therefore \frac{OA}{OC} = \frac{AB}{CD}$

$\Rightarrow \frac{x}{3.5} = \frac{8}{5}$

$\Rightarrow x = \frac{8\times3.5}{5} = 5.6$

Hence, OA = 5.6 cm

(ii) Let OD be Y cm

$\because \Delta$ OAB - Δ OCD

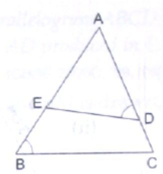
$\therefore \frac{AB}{CD} = \frac{OB}{OD}$

$\Rightarrow \frac{8}{5} = \frac{6.4}{y}$

$\Rightarrow y = \frac{6.4\times5}{8} = 4$

Hence, DO = 4 cm

4. In the given figure, if ∠ADE = ∠B, show that ΔADE ~ ΔABC. If AD = 3.8cm, AE = 3.6cm, BE = 2.1cm and BC = 4.2cm, find DE.



Sol:

Given :

$\angle ADE = \angle ABC \text{ and } \angle A = \angle A$

Let DE be X cm

Therefore, by AA similarity theorem, Δ ADE - Δ ABC

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3.8}{3.6+2.1} = \frac{x}{4.2}$

$\Rightarrow x = \frac{3.8\times4.2}{5.7} = 2.8$

Hence, DE = 2.8 cm

5. The perimeter of two similar triangles ABC and PQR are 32cm and 24cm respectively. If PQ = 12cm, find AB.

Sol:

It is given that triangles ABC and PQR are similar.

Therefore,

$\frac{Perimeter\ (\Delta ABC)}{Perimeter\ (\Delta PQR)} = \frac{AB}{PQ}$

$\Rightarrow \frac{32}{24} = \frac{AB}{12}$

$\Longrightarrow AB = \frac{32\times12}{24} = 16\ cm$

6. The corresponding sides of two similar triangles ABC and DEF are BC = 9.1cm and EF = 6.5cm. If the perimeter of ΔDEF is 25cm, find the perimeter of ΔABC.

Sol:

It is given that Δ ABC - Δ DEF.

Therefore, their corresponding sides will be proportional.

Also, the ratio of the perimeters of similar triangles is same as the ratio of their corresponding sides.

$\Longrightarrow \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF} = \frac{BC}{EF}$

Let the perimeter of ΔABC be X cm

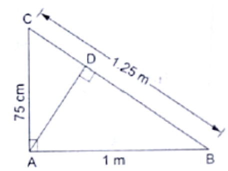
Therefore,

$\frac{x}{25} = \frac{9.1}{6.5}$

$\Longrightarrow x = \frac{9.1\times25}{6.5} = 35$

Thus, the perimeter of ΔABC is 35 cm.

7. In the given figure, ∠CAB = 90^0 and AD⊥BC. Show that ΔBDA ~ ΔBAC. If AC = 75cm, AB = 1m and BC = 1.25m, find AD.



Sol:

In Δ BDA and Δ BAC, we have :

$\angle BDA = \angle BAC = 90^0$

$\angle DBA = \angle CBA \qquad (Common)$

Therefore, by AA similarity theorem, Δ BDA - Δ BAC

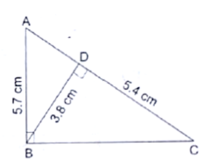
$\Longrightarrow \frac{AD}{AC} = \frac{AB}{BC}$

$\Longrightarrow \frac{AD}{0.75} = \frac{1}{1.25}$

$\Longrightarrow$ AD = $\frac{0.75}{1.25}$

= 0.6 m or 60 cm

8. In the given figure, ∠ABC = 90^0 and BD⊥AC. If AB = 5.7cm, BD = 3.8cm and CD = 5.4cm, find BC.



Sol:

It is given that ABC is a right angled triangle and BD is the altitude drawn from the right angle to the hypotenuse.

In Δ BDC and Δ ABC, we have :

$\angle ABC = \angle BBC = 90^0 \quad (given)$

$\angle C = \angle C \quad (common)$

By AA similarity theorem, we get :

Δ BDC- Δ ABC

$\frac{AB}{BD} = \frac{BC}{DC}$

$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$

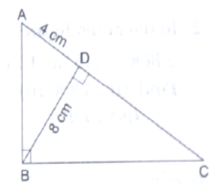
$\Rightarrow BC = \frac{5.7}{3.8} \times 5.4$

$= 8.1$

Hence, BC = 8.1 cm

9. In the given figure, ∠ABC = 90^0 and BD⊥AC.

If BD = 8cm, AD = 4cm, find CD.



Sol:

It is given that ABC is a right angled triangle

and BD is the altitude drawn from the right angle to the hypotenuse.

In Δ DBA and Δ DCB, we have :

$\angle BDA = \angle CDB$

$\angle DBA = \angle DCB = 90^0$

Therefore, by AA similarity theorem, we get :

ΔDBA - Δ DCB

$\Rightarrow \frac{BD}{CD} = \frac{AD}{BD}$

$\Rightarrow CD = \frac{BD^2}{AD}$

CD = $\frac{8\times8}{4} = 16\ cm$

10. P and Q are points on the sides AB and AC respectively of a ΔABC. If AP = 2cm, PB = 4cm, AQ = 3cm and QC = 6cm, show that BC = 3PQ.

Sol:

We have :

$\frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}\ and\ \frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

In Δ APQ and Δ ABC, we have:

$\frac{AP}{AB} = \frac{AQ}{AC}$

$\angle A = \angle A$

Therefore, by AA similarity theorem, we get:

Δ APQ - Δ ABC

Hence, $\frac{PQ}{BC} = \frac{AQ}{AC} = \frac{1}{3}$

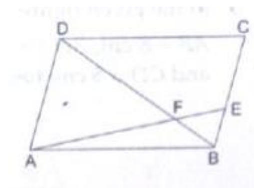
$\Rightarrow \frac{PQ}{BC} = \frac{1}{3}$

⟹ BC = 3PQ

This completes the proof.

11. ABCD is parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that

AF × FB = EF × FD.



Sol:

We have:

$\angle AFD = \angle EFB \quad (Vertically\ Opposite\ angles)$

∵ DA || BC

∴ $\angle DAF = \angle BEF \quad (Alternate\ angles)$

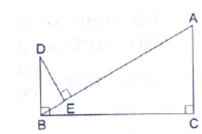
Δ DAF ~ Δ BEF (AA similarity theorem)

$\Rightarrow \frac{AF}{EF} = \frac{FD}{FB}$

Or, AF × FB = FD × EF

This completes the proof.

12. In the given figure, DB⊥BC, DE⊥AB and AC⊥BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Sol:

In ΔBED and ΔACB, we have:

$\angle BED = \angle ACB = 90^0$

∵ $\angle B + \angle C = 180^0$

∴ BD || AC

$\angle EBD = \angle CAB \quad (Alternate\ angles\ )$

Therefore, by AA similarity theorem, we get :

Δ BED ~ Δ ACB

$\Rightarrow \frac{BE}{AC} = \frac{DE}{BC}$

$\Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$

This completes the proof.

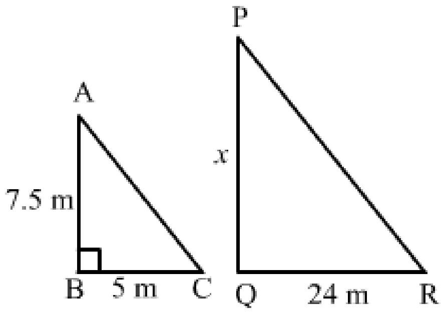
13. A vertical pole of length 7.5cm casts a shadow 5m long on the ground and at the same time a tower casts a shadow 24m long. Find the height of the tower.

Sol:

Let AB be the vertical stick and BC be its shadow.

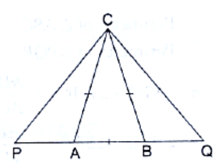
Given:

AB = 7.5 m, BC = 5 m



Let PQ be the tower and QR be its shadow.
Given:
QR = 24 m
Let the length of PQ be x m.
In Δ ABC and Δ PQR, we have:
$\angle ABC = \angle PQR = 90^0$
$\angle ACB = \angle PRQ$ $(Angular\ elevation\ of\ the\ Sun\ at\ the\ same\ time)$
Therefore, by AA similarity theorem , we get :
Δ ABC ~ Δ PQR
$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$
$\Rightarrow \frac{7.5}{5} = \frac{x}{24}$
$x = \frac{7.5}{5} \times 24 = 36cm$
Therefore, PQ = 36 m
Hence, the height of the tower is 36 m.

14. In an isosceles ΔABC, the base AB is produced both ways in P and Q such that AP × BQ = AC^2.
Prove that ΔACP~ΔBCQ.
Sol:
Disclaimer: It should be ΔAPC ~ ΔBCQ instead of ΔACP ~ ΔBCQ



It is given that ΔABC is an isosceles triangle.
Therefore,
CA = CB
$\Rightarrow \angle CAB = \angle CBA$
$\Rightarrow 180^0 - \angle CAB = 180^0 - \angle CBA$
$\Rightarrow \angle CAP = \angle CBQ$
Also,
AP × BQ = AC^2
$\Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ $(\because AC = BC\ )$

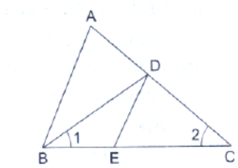
Thus, by SAS similarity theorem, we get

ΔAPC ~ ΔBCQ

This completes the proof.

15. In the given figure, ∠1 = ∠2 and $\frac{AC}{BD} = \frac{CB}{CE}$.

Prove that Δ ACB ~ Δ DCE.



Sol:

We have :

$\frac{AC}{BD} = \frac{CB}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$

$\Rightarrow \frac{AC}{CB} = \frac{CD}{CE}$ $(Since, BD = DC\ as\ \angle 1 = \angle 2\ )$

Also, $\angle 1 = \angle 2$

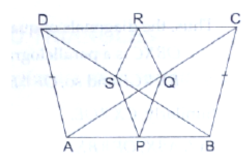
i.e, $\angle DBC = \angle ACB$

Therefore, by SAS similarity theorem, we get :

Δ ACB - Δ DCE

16. ABCD is a quadrilateral in which AD = BC.

If P, Q, R, S be the midpoints of AB, AC, CD and BD respectively, show that PQRS is a rhombus.



Sol:

In Δ ABC, P and Q are mid points of AB and AC respectively.

So, PQ || BC, and PQ = $\frac{1}{2}\ BC$ …(1)

Similarly, in ΔADC, …(2)

Now, in ΔBCD, SR $= \frac{1}{2}\ BC$ …(3)

Similarly, in ΔABD, PS = $\frac{1}{2}\ AD = \frac{1}{2}\ BC$ …(4)

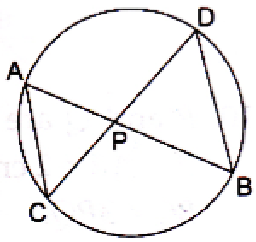
Using (1), (2), (3), and (4).

PQ = QR = SR = PS

Since, all sides are equal

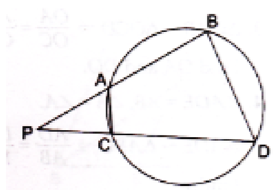
Hence, PQRS is a rhombus.

17. In a circle, two chords AB and CD intersect at a point P inside the circle. Prove that
(a) $\Delta PAC \sim \Delta PDB$ (b) PA. PB= PC.PD



Sol:
Given : AB and CD are two chords
To Prove:
(a) Δ PAC ~ ΔPDB
(b) PA.PB = PC.PD
Proof: In Δ PAC and Δ PDB
$\angle APC = \angle DPB$ $(Vertically\ Opposite\ angles)$
$\angle CAP = \angle BDP$ $(Angles\ in\ the\ same\ segment\ are\ equal)$
$by\ AA\ similarity\ criterion\ \Delta PAC \sim PDB$
When two triangles are similar, then the ratios of lengths of their corresponding sides are proportional.
$\therefore \frac{PA}{PD} = \frac{PC}{PB}$
⟹ PA. PB = PC. PD

18. Two chords AB and CD of a circle intersect at a point P outside the circle.
Prove that: (i) Δ PAC ~ Δ PDB
(ii) PA. PB = PC.PD

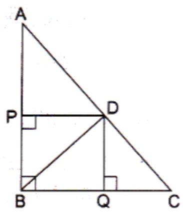


Sol:
Given : AB and CD are two chords
To Prove:
(a) Δ PAC - Δ PDB
(b) PA. PB = PC.PD
Proof: $\angle ABD + \angle ACD = 180^0$ …(1) (Opposite angles of a cyclic quadrilateral are supplementary)
$\angle PCA + \angle ACD = 180^0$ …(2) (Linear Pair Angles)
Using (1) and (2), we get
$\angle ABD = \angle PCA$
$\angle A = \angle A$ (Common)

By AA similarity-criterion Δ PAC - Δ PDB
When two triangles are similar, then the rations of the lengths of their corresponding sides are proportional.
$\therefore \frac{PA}{PD} = \frac{PC}{PB}$
$\Longrightarrow$ PA.PB = PC.PD

19. In a right triangle ABC, right angled at B, D is a point on hypotenuse such that $BD \perp AC$, if $DP \perp AB$ and $DQ \perp BC$ then prove that

(a) $DQ^2 = Dp.QC$ (b) $DP^2 = DQ.AP$ 2



Sol:
We know that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on the both sides of the perpendicular are similar to the whole triangle and also to each other.
(a) Now using the same property in In ΔBDC, we get
ΔCQD ~ ΔDQB
$\frac{CQ}{DQ} = \frac{DQ}{QB}$
$\Longrightarrow DQ^2 = QB.CQ$
Now. Since all the angles in quadrilateral BQDP are right angles.
Hence, BQDP is a rectangle.
So, QB = DP and DQ = PB
$\therefore DQ^2 = DP.CQ$
(b)
Similarly, ΔAPD ~ ΔDPB
$\frac{AP}{DP} = \frac{PD}{PB}$
$\Longrightarrow DP^2 = AP.PB$
$\Longrightarrow DP^2 = AP.DQ$ $[\because DQ = PB]$

Exercise – 4C

1. ΔABC~ΔDEF and their areas are respectively 64 cm^2 and 121cm^2. If EF = 15.4cm, find BC.

Sol:

It is given that Δ ABC ~ Δ DEF.

Therefore, ratio of the areas of these triangles will be equal to the ration of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{BC^2}{EF^2}$

Let BC be X cm.

$\Rightarrow \frac{64}{121} = \frac{x^2}{(15.4)^2}$

$\Rightarrow x^2 = \frac{64 \times 15.4 \times 15.4}{121}$

$\Rightarrow x = \sqrt{\frac{(64 \times 15.4 \times 15.4)}{121}}$

$= \frac{8 \times 15.4}{11}$

= 11.2

Hence, BC = 11.2 cm

2. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If BC = 4.5cm, find the length of QR.

Sol:

It is given that Δ ABC ~ Δ PQR

Therefore, the ration of the areas of triangles will be equal to the ratio of squares of their corresponding sides.

$\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{9}{16} = \frac{4^2}{QR^2}$

$\Rightarrow QR^2 = \frac{4.5 \times 4.5 \times 16}{9}$

$\Rightarrow QR = \sqrt{\frac{(4.5 \times 4.5 \times 16)}{9}}$

$= \frac{4.5 \times 4}{3}$

= 6 cm

Hence, QR = 6 cm

3. ΔABC~ΔPQR and ar(ΔABC) = 4, ar(ΔPQR) . If BC = 12cm, find QR.

Sol:

Given : $ar\ (\ \Delta\ ABC\ ) = 4ar\ (\Delta\ PQR\ )$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$

$\because$ ΔABC ~ ΔPQR

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\therefore \frac{BC^2}{QR^2} = \frac{4}{1}$

$\Rightarrow QR^2 = \frac{12^2}{4}$

$\Rightarrow QR^2 = 36$

$\Rightarrow QR = 6\ cm$

Hence, QR = 6 cm

4. The areas of two similar triangles are 169cm^2 and 121cm^2 respectively. If the longest side of the larger triangle is 26cm, find the longest side of the smaller triangle.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Let the longest side of smaller triangle be X cm.

$\frac{ar\ (Larger\ triangle)}{ar\ (Smaller\ triangle)} = \frac{(Longest\ side\ of\ larger\ traingle)^2}{(Longest\ side\ of\ smaller\ traingle)^2}$

$\Rightarrow \frac{169}{121} = \frac{26^2}{x^2}$

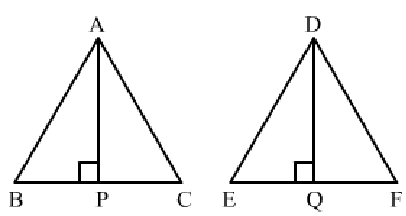
$\Rightarrow x = \sqrt{\frac{26 \times 26 \times 121}{169}}$

= 22

Hence, the longest side of the smaller triangle is 22 cm.

5. ΔABC ~ ΔDEF and their areas are respectively 100cm^2 and 49cm^2. If the altitude of ΔABC is 5cm, find the corresponding altitude of ΔDEF.

Sol:



It is given that ΔABC ~ ΔDEF.

Therefore, the ration of the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the altitude of ΔABC be AP, drawn from A to BC to meet BC at P and the altitude of ΔDEF be DQ, drawn from D to meet EF at Q.

Then,

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{5^2}{DQ^2}$

$\Rightarrow \frac{100}{49} = \frac{25}{DQ^2}$

$\Rightarrow DQ^2 = \frac{49 \times 25}{100}$

$\Rightarrow DQ = \sqrt{\frac{49 \times 25}{100}}$

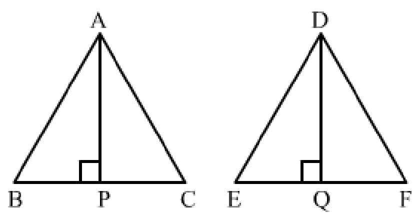
$\Rightarrow DQ = 3.5\ cm$

Hence, the altitude of ΔDEF is 3.5 cm

6. The corresponding altitudes of two similar triangles are 6cm and 9cm respectively. Find the ratio of their areas.

Sol:

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



It is given that Δ ABC ~ Δ DEF.

We know that the ration of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AP)^2}{(DQ)^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(DEF)} = \frac{6^2}{9^2}$

$= \frac{36}{81}$

$= \frac{4}{9}$

Hence, the ratio of their areas is 4 : 9

7. The areas of two similar triangles are 81cm^2 and 49cm^2 respectively. If the altitude of the first triangle is 6.3cm, find the corresponding altitude of the other.

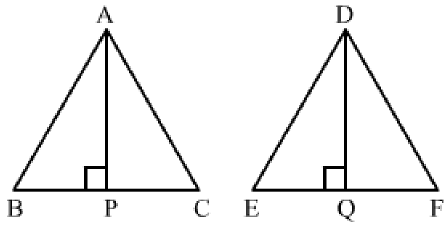
Sol:

It is given that the triangles are similar.

Therefore, the areas of these triangles will be equal to the ratio of squares of their corresponding sides.

Also, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Let the two triangles be ABC and DEF with altitudes AP and DQ, respectively.



$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AP^2}{DQ^2}$

$\Rightarrow \frac{81}{49} = \frac{6.3^2}{DQ}$

$\Rightarrow DQ^2 = \frac{49}{81} \times 6.3^2$

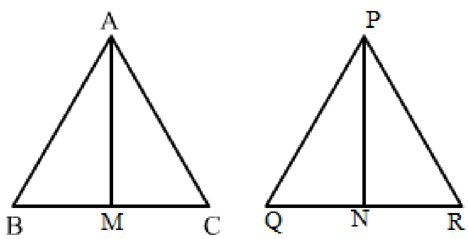
$\Rightarrow DQ^2 = \sqrt{\frac{49}{81} \times 6.3 \times 6.3}$

Hence, the altitude of the other triangle is 4.9 cm.

8. The areas of two similar triangles are 64cm^2 and 100cm^2 respectively. If a median of the smaller triangle is 5.6cm, find the corresponding median of the other.

Sol:

Let the two triangles be ABC and PQR with medians AM and PN, respectively.



Therefore, the ratio of areas of two similar triangles will be equal to the ratio of squares of their corresponding medians.

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AM^2}{PN^2}$

$\Rightarrow \frac{64}{100} = \frac{5.6^2}{PN^2}$

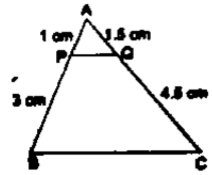
$\Rightarrow PN^2 = \frac{64}{100} \times 5.6^2$

$\Rightarrow PN^2 = \sqrt{\frac{100}{64} \times 5.6 \times 5.6}$

= 7 cm

Hence, the median of the larger triangle is 7 cm.

9. In the given figure, ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1cm, PB = 3cm, AQ = 1.5cm, QC = 4.5cm, prove that area of ΔAPQ is $\frac{1}{16}$ of the area of ΔABC.



Sol:

We have :

$\frac{AP}{AB} = \frac{1}{1+3} = \frac{1}{4}$ and $\frac{AQ}{AC} = \frac{1.5}{1.5+4.5} = \frac{1.5}{6} = \frac{1}{4}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Also, $\angle A = \angle A$

By SAS similarity, we can conclude that ΔAPQ- ΔABC.

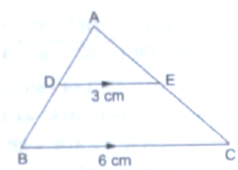
$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16}$

$\Rightarrow ar(\Delta APQ) = \frac{1}{16} \times ar\ (\Delta ABC)$

Hence proved.

10. In the given figure, DE ∥ BC. If DE = 3cm, BC = 6cm and ar(ΔADE) = 15cm^2, find the area of ΔABC.



Sol:

It is given that DE || BC

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

By AA similarity, we can conclude that Δ ADE ~ Δ ABC

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

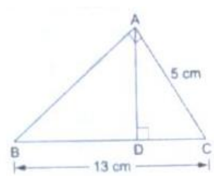
$\Rightarrow \frac{15}{ar(\Delta ABC)} = \frac{3^2}{6^2}$

$\Rightarrow ar\ (\Delta ABC) = \frac{15 \times 36}{9}$

= 60 cm^2

Hence, area of triangle ABC is 60 cm^2

11. ΔABC is right angled at A and AD⊥BC. If BC = 13cm and AC = 5cm, find the ratio of the areas of ΔABC and ΔADC.



Sol:

In ΔABC and ΔADC, we have:

$\angle BAC = \angle ADC = 90^0$

$\angle ACB = \angle ACD\ (common)$

By AA similarity, we can conclude that Δ BAC~ Δ ADC.

Hence, the ratio of the areas of these triangles is equal to the ratio of squares of their corresponding sides.

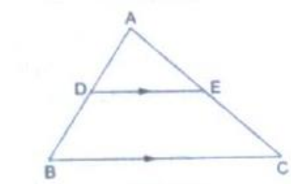
$\therefore \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2}$

$\Longrightarrow \frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{13^2}{5^2}$

$= \frac{169}{25}$

Hence, the ratio of areas of both the triangles is 169**:**25

12. In the given figure, DE ∥ BC and DE: BC = 3:5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.



Sol:

It is given that DE || BC.

$\therefore \angle ADE = \angle ABC\ (Corresponding\ angles)$

$\angle AED = \angle ACB\ (Corresponding\ angles)$

Applying AA similarity theorem, we can conclude that Δ ADE ~ ΔABC.

$\therefore \frac{ar(\Delta ABC)}{ar(ADE)} = \frac{BC^2}{DE^2}$

Subtracting 1 from both sides, we get:

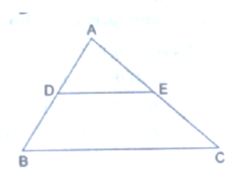
$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = \frac{5^2}{3^2} - 1$

$\Longrightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = \frac{25-9}{9}$

$\Longrightarrow \frac{ar(BCED)}{ar(\Delta ADE)} = \frac{16}{9}$

Or, $\frac{ar(\Delta ADE)}{ar(BCED)} = \frac{9}{16}$

13. In ΔABC, D and E are the midpoints of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.



Sol:

It is given that D and E are midpoints of AB and AC.

Applying midpoint theorem, we can conclude that DE ∥ BC.

Hence, by B.P.T., we get :

$\frac{AD}{AB} = \frac{AE}{AC}$

Also, $\angle A = \angle A$

Applying SAS similarity theorem, we can conclude that Δ ADE~ Δ ABC.

Therefore, the ration of areas of these triangles will be equal to the ratio of squares of their corresponding sides.

$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2}$

$= \frac{1}{4}$

Exercise – 4D

1. The sides of certain triangles are given below. Determine which of them right triangles are.

(i) 9cm, 16cm, 18cm (ii) 7cm, 24cm, 25cm

(iii) 1.4cm, 4.8cm, 5cm (iv) 1.6cm, 3.8cm, 4cm

(v) (a – 1) cm, $2\sqrt{a}$ cm, (a + 1) cm

Sol:

For the given triangle to be right-angled, the sum of the two sides must be equal to the square of the third side.

Here, let the three sides of the triangle be a, b and c.

(i)

a = 9 cm, b = 16 cm and c = 18 cm

Then,

$a^2 + b^2 = 9^2 + 16^2$

$= 81 + 256$

$= 337$

$c^2 = 19^2$

$= 361$

$a^2 + b^2 \neq c^2$

Thus, the given triangle is not right-angled.

(ii)

A=7 cm, b = 24 cm and c = 25 cm

Then,

$a^2 + b^2 = 7^2 + 24^2$

= 49 + 576

= 625

$c^2 = 25^2$

= 625

$a^2 + b^2 = c^2$

Thus, the given triangle is a right-angled.

(iii)

A= 1.4 cm, b= 4.8 cm and c= 5 cm

Then,

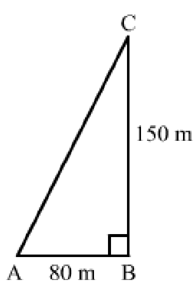
$a^2 + b^2 = (1.4)^2 + (4.8)^2$

= 1.96 + 23.04

= 25

$c^2 = 5^2$
= 25
$a^2 + b^2 = c^2$
Thus, the given triangle is right-angled.
(iv) A = 1.6 cm, b= 3.8 cm and c= 4 cm
Then
$a^2 + b^2 = (1.6)^2 + (3.8)^2$
= 2.56 + 14.44
= 16
$a^2 + b^2 \neq c^2$
Thus, the given triangle is not right-angled.
(v)
P = (a-1) cm, q = 2 $\sqrt{a}\ cm\ and\ r = (a+1)cm$
Then,
$p^2 + q^2 = (a-1)^2 + \left(2\sqrt{a}\right)^2$
= $a^2 + 1 - 2a + 4a$
= $a^2 + 1 + 2a$
= $(a+1)^2$
$r^2 = (a+1)^2$
$p^2 + q^2 = r^2$
Thus, the given triangle is right-angled.

2. A man goes 80m due east and then 150m due north. How far is he from the starting point?
Sol:
Let the man starts from point A and goes 80 m due east to B.
Then, from B, he goes 150 m due north to c.



We need to find AC.
In right- angled triangle ABC, we have:
$AC^2 = AB^2 + BC^2$
$AC = \sqrt{80^2 + 150^2}$
= $\sqrt{6400 + 22500}$
= $\sqrt{28900}$
= 170 m

Hence, the man is 170 m away from the starting point.

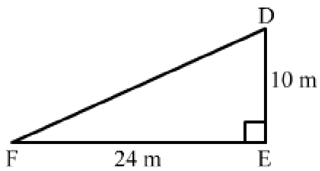
3. A man goes 10m due south and then 24m due west. How far is he from the starting point?

Sol:

Let the man starts from point D and goes 10 m due south at E. He then goes 24 m due west at F.

In right ΔDEF, we have:

DE = 10 m, EF = 24 m



$DF^2 = EF^2 + DE^2$

$DF = \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576}$

$= \sqrt{676}$

= 26 m

Hence, the man is 26 m away from the starting point.

4. A 13m long ladder reaches a window of a building 12m above the ground. Determine the distance of the foot of the ladder from the building.

Sol:

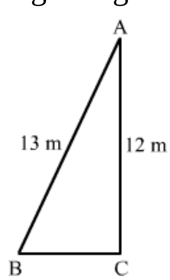
Let AB and AC be the ladder and height of the building.

It is given that :

AB = 13 m and AC = 12 m

We need to find distance of the foot of the ladder from the building, i.e, BC.

In right-angled triangle ABC, we have:



$AB^2 = AC^2 + BC^2$

$\Rightarrow BC = \sqrt{13^2 - 12^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25}$

= 5 m

Hence, the distance of the foot ladder from the building is 5 m

5. A ladder is placed in such a way that its foot is at a distance of 15m from a wall and its top reaches a window 20m above the ground. Find the length of the ladder.

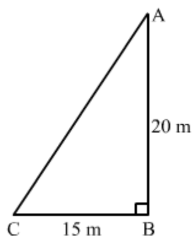
Sol:

Let the height of the window from the ground and the distance of the foot of the ladder from the wall be AB and BC, respectively.

We have :

AB = 20 m and BC = 15 m

Applying Pythagoras theorem in right-angled ABC, we get:



$AC^2 = AB^2 + BC^2$

$\Longrightarrow AC = \sqrt{20^2 + 15^2}$

$= \sqrt{400 + 225}$

$= \sqrt{625}$

= 25 m

Hence, the length of the ladder is 25 m.

6. Two vertical poles of height 9m and 14m stand on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

Sol:

Let the two poles be DE and AB and the distance between their bases be BE.

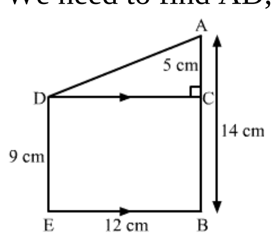
We have:

DE = 9 m, AB = 14 m and BE = 12 m

Draw a line parallel to BE from D, meeting AB at C.

Then, DC = 12 m and AC = 5 m

We need to find AD, the distance between their tops.



Applying Pythagoras theorem in right-angled ACD, we have:

$AD^2 = AC^2 + DC^2$

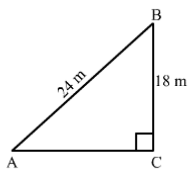
$AD^2 = 5^2 + 12^2 = 25 + 144 = 169$

$AD = \sqrt{169} = 13\ m$

Hence, the distance between the tops to the two poles is 13 m.

7. A guy wire attached to a vertical pole of height 18 m is 24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol:



Let AB be a guy wire attached to a pole BC of height 18 m. Now, to keep the wire taut let it to be fixed at A.

Now, In right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + CA^2$

$\Rightarrow 24^2 = 18^2 + CA^2$

$\Rightarrow CA^2 = 576 - 324$

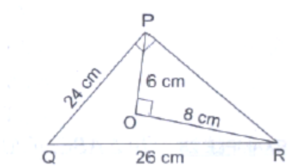
$\Rightarrow CA^2 = 252$

$\Rightarrow CA = 6\sqrt{7}\, m$

Hence, the stake should be driven 6 $\sqrt{7}m$ far from the base of the pole.

8. In the given figure, O is a point inside a ΔPQR such that ∠PQR such that ∠POR = 90^0, OP = 6cm and OR = 8cm. If PQ = 24cm and QR = 26cm, prove that ΔPQR is right-angled.

Sol:



Applying Pythagoras theorem in right-angled triangle POR, we have:

$PR^2 = PO^2 + OR^2$

$\Rightarrow PR^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow PR = \sqrt{100} = 10\ cm$

IN Δ PQR,

$PQ^2 + PR^2 = 24^2 + 10^2 = 576 + 100 = 676$

And $QR^2 = 26^2 = 676$

$\therefore PQ^2 + PR^2 = QR^2$

Therefore, by applying Pythagoras theorem, we can say that ΔPQR is right-angled at P.

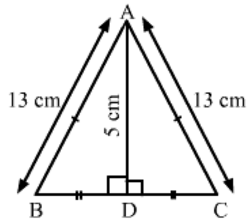
9. ΔABC is an isosceles triangle with AB = AC = 13cm. The length of altitude from A on BC is 5cm. Find BC.

Sol:

It is given that Δ ABC is an isosceles triangle.

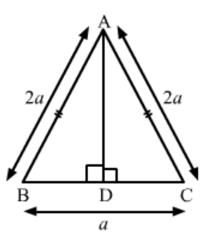
Also, AB = AC = 13 cm

Suppose the altitude from A on BC meets BC at D. Therefore, D is the midpoint of BC.
AD =5 cm
$\Delta\ ADB\ and\ \Delta\ ADC$ are right-angled triangles.
Applying Pythagoras theorem, we have;



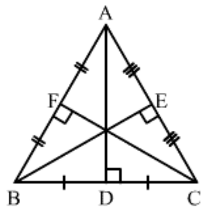
$AB^2 = AD^2 + BD^2$
$BD^2 = AB^2 - AD^2 = 13^2 - 5^2$
$BD^2 = 169 - 25 = 144$
$BD = \sqrt{144} = 12$
Hence,
BC =2(BD) = 2 × 12 = 24 cm

10. Find the length of altitude AD of an isosceles ΔABC in which AB = AC = 2a units and BC = a units.
Sol:
In isosceles Δ ABC, we have:
AB = AC = 2a units and BC = a units
Let AD be the altitude drawn from A that meets BC at D.
Then, D is the midpoint of BC.
BD = BC = $\frac{a}{2}\ units$
Applying Pythagoras theorem in right-angled ΔABD, we have:



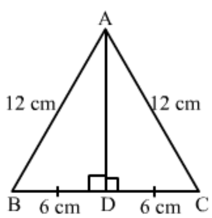
$AB^2 = AD^2 + BD^2$
$AD^2 = AB^2 - BD^2 = (2a)^2 - \left(\frac{a}{2}\right)^2$
$AD^2 = 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4}$
$AD = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2}\ units.$

11. ΔABC is am equilateral triangle of side 2a units. Find each of its altitudes.
Sol:



Let AD, BE and CF be the altitudes of ΔABC meeting BC, AC and AB at D, E and F, respectively.
Then, D, E and F are the midpoint of BC, AC and AB, respectively.
In right-angled ΔABD, we have:
AB = 2a and BD = a
Applying Pythagoras theorem, we get:
$AB^2 = AD^2 + BD^2$
$AD^2 = AB^2 - BD^2 = (2a)^2 - a^2$
$AD^2 = 4a^2 - a^2 = 3a^2$
$AD = \sqrt{3}a\ units$
Similarly,
BE = $a\sqrt{3}\ units\ and\ CF = a\sqrt{3}\ units$

12. Find the height of an equilateral triangle of side 12cm.
Sol:
Let ABC be the equilateral triangle with AD as an altitude from A meeting BC at D. Then, D will be the midpoint of BC.
Applying Pythagoras theorem in right-angled triangle ABD, we get:



$AB^2 = AD^2 + BD^2$
$\Rightarrow AD^2 = 12^2 - 6^2$ ($\because BD = \frac{1}{2} BC = 6$)
$\Rightarrow AD^2 = 144 - 36 = 108$
$\Rightarrow AD = \sqrt{108} = 6\sqrt{3}\ cm.$
Hence, the height of the given triangle is $6\sqrt{3}\ cm$.

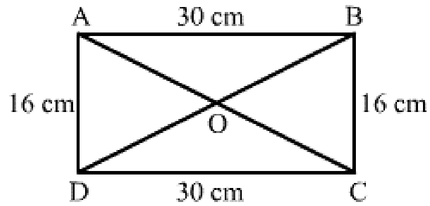
13. Find the length of a diagonal of a rectangle whose adjacent sides are 30cm and 16cm.

Sol:

Let ABCD be the rectangle with diagonals AC and BD meeting at O.

According to the question:

AB = CD = 30 cm and BC = AD = 16 cm



Applying Pythagoras theorem in right-angled triangle ABC, we get:

$AC^2 = AB^2 + BC^2 = 30^2 + 16^2 = 900 + 256 = 1156$

$AC = \sqrt{1156} = 34\ cm$

Diagonals of a rectangle are equal.

Therefore, AC = BD = 34 cm

14. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

Sol:

Let ABCD be the rhombus with diagonals (AC = 24 cm and BD = 10 cm) meeting at O.

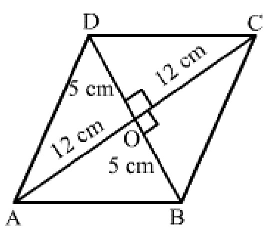
We know that the diagonals of a rhombus bisect each other at angles.

Applying Pythagoras theorem in right-angled AOB, we get:

$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$

$AB^2 = 144 + 25 = 169$

$AB = \sqrt{169} = 13\ cm$



Hence, the length of each side of the rhombus is 13 cm.

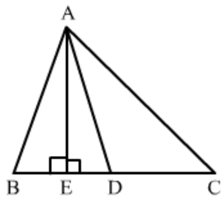
15. In ΔABC, D is the midpoint of BC and AE⊥BC. If AC>AB, show that $AB^2 = AD^2 + \frac{1}{4} BC^2 - BC.DE$

Sol:

In right-angled triangle AED, applying Pythagoras theorem, we have:

$AB^2 = AE^2 + ED^2 \quad \dots (i)$

In right-angled triangle AED, applying Pythagoras theorem, we have:



$AD^2 = AE^2 + ED^2$

$\Longrightarrow AE^2 = AD^2 - ED^2 \quad ...(ii)$

Therefore,

$AB^2 = AD^2 - ED^2 + EB^2 \; (from\ (i) and\ (ii))$

$AB^2 = AD^2 - ED^2 + (BD - DE)^2$

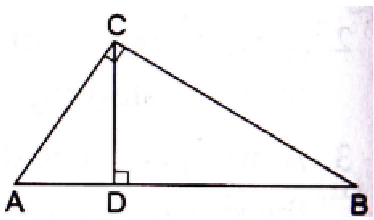
$= AD^2 - ED^2 + \left(\frac{1}{2} BC - DE\right)^2$

$= AD^2 - DE^2 + \frac{1}{4} BC^2 + DE^2 - BC.DE$

$= AD^2 + \frac{1}{4} BC^2 - BC.DE$

This completes the proof.

16. In the given figure, $\angle ACB = 90^\circ$ $CD \perp AB$ Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$



Sol:

Given: $\angle ACB = 90^0$ $and\ CD \perp AB$

To Prove; $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Proof: In Δ ACB and Δ CDB

$\angle ACB = \angle CDB = 90^0 \; (Given)$

$\angle ABC = \angle CBD \; (Common)$

By AA similarity-criterion Δ ACB ~ ΔCDB

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$\therefore \frac{BC}{BD} = \frac{AB}{BC}$

$\Longrightarrow BC^2 = BD.AB \quad ...(1)$

In Δ ACB and Δ ADC

$\angle ACB = \angle ADC = 90^0 \; (Given)$

$\angle CAB = \angle DAC \; (Common)$

By AA similarity-criterion Δ ACB ~ ΔADC

When two triangles are similar, then the ratios of their corresponding sides are proportional.

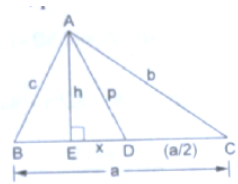
$\therefore \frac{AC}{AD} = \frac{AB}{AC}$

$\Rightarrow AC^2$ = AD. AB ….(2)

Dividing (2) by (1), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$

17. In the given figure, D is the midpoint of side BC and AE⊥BC. If BC = a, AC = b, AB = c, AD = p and AE = h, prove that



(i) $b^2 = p^2 + ax + \frac{a^2}{x}$

(ii) $c^2 = p^2 - ax + \frac{a^2}{x}$

(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

(iv) $b^2 - c^2 = 2ax$

Sol:

(i)

In right-angled triangle AEC, applying Pythagoras theorem, we have:

$AC^2 = AE^2 + EC^2$

$\Rightarrow b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = h^2 + x^2 + \frac{a^2}{4} + ax \ldots (i)$

In right – angled triangle AED, we have:

$AD^2 = AE^2 + ED^2$

$\Rightarrow p^2 = h^2 + x^2 \ldots (ii)$

Therefore,

from (i) and (ii),

$b^2 = p^2 + ax + \frac{a^2}{x}$

(ii)

In right-angled triangle AEB, applying Pythagoras, we have:

$AB^2 = AE^2 + EB^2$

$\Rightarrow c^2 = h^2 + \left(\frac{a}{2} - x\right)^2 \ (\because BD = \frac{a}{2} \ and \ BE = BD - x)$

$\Rightarrow c^2 = h^2 + x^2 - \frac{a^2}{4} \ (\because h^2 + x^2 = p^2)$

$\Rightarrow c^2 = p^2 - ax + \frac{a^2}{x}$

(iii)

Adding (i) and (ii), we get:

$\Rightarrow b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$

$= 2p^2 + ax - ax + \frac{a^2+a^2}{4}$

$= 2p^2 + \frac{a^2}{2}$

(iv)

Subtracting (ii) from (i), we get:

$b^2 - c^2 = p^2 + ax + \frac{a^2}{4} - \ (p^2 - ax + \frac{a^2}{4})$

$= p^2 - p^2 + ax + \ ax + \frac{a^2}{4} - \frac{a^2}{4}$

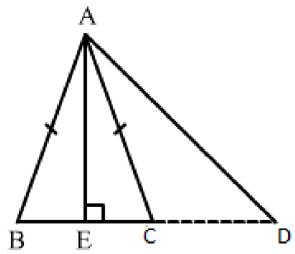
= 2ax

18. In ΔABC, AB = AC. Side BC is produced to D. Prove that $AD^2 - AC^2 = \text{BD.CD}$

Sol:

Draw AE⊥BC, meeting BC at D.

Applying Pythagoras theorem in right-angled triangle AED, we get:



Since, ABC is an isosceles triangle and AE is the altitude and we know that the altitude is also the median of the isosceles triangle.

So, BE = CE

And DE+CE=DE+BE=BD

$AD^2 = AE^2 + DE^2$

$\Rightarrow AE^2 = AD^2 - DE^2$ …(i)

In ΔACE,

$AC^2 = AE^2 + EC^2$

$\Rightarrow AE^2 = AC^2 - EC^2$ ... (ii)

Using (i) and (ii),

$\Rightarrow AD^2 - DE^2 = AC^2 - EC^2$

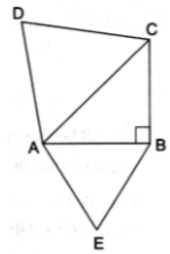
$\Rightarrow AD^2 - AC^2 = DE^2 - EC^2$

= (DE + CE) (DE – CE)

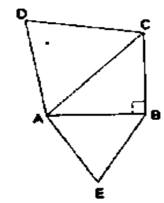
= (DE + BE) CD

= BD.CD

19. ABC is an isosceles triangle, right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



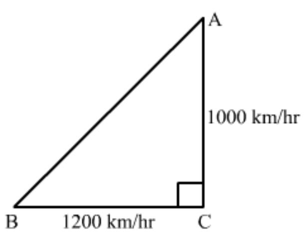
Sol:



We have, ABC as an isosceles triangle, right angled at B.
Now, AB = BC
Applying Pythagoras theorem in right-angled triangle ABC, we get:
$AC^2 = AB^2 + BC^2 = 2AB^2\ (\because AB = AC) \ldots (i)$
$\because$ ΔACD ~ Δ ABE
We know that ratio of areas of 2 similar triangles is equal to squares of the ratio of their corresponding sides.
$\therefore \frac{ar(\Delta ABE)}{ar(\Delta ACD)} = \frac{AB^2}{AC^2} = \frac{AB^2}{2AB^2}\ [from\ (i)]$
$= \frac{1}{2} = 1 : 2$

20. An aeroplane leaves an airport and flies due north at a speed of 1000km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Sol:

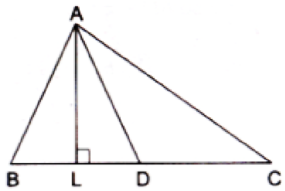


Let A be the first aeroplane flied due north at a speed of 1000 km/hr and B be the second aeroplane flied due west at a speed of 1200 km/hr
Distance covered by plane A in $1\frac{1}{2}\ hours = 1000 \times \frac{3}{2} = 1500\ km$
Distance covered by plane B in $1\frac{1}{2}\ hours = 1200 \times \frac{3}{2} = 1800\ km$
Now, In right triangle ABC
By using Pythagoras theorem, we have
$AB^2 = BC^2 + CA^2$
= $(1800)^2 + (1500)^2$
= 3240000 + 2250000
= 5490000
$\therefore AB^2 = 5490000$

$\Longrightarrow$ AB = 300 $\sqrt{61}\,m$

Hence, the distance between two planes after $1\frac{1}{2}$ $hours\ is$ 300 $\sqrt{61}m$

21. In a ΔABC, AD is a median and $AL \perp BC$.

Prove that 

(a) $AC^2 = AD^2 + BC.DL + \left(\frac{BC}{2}\right)^2$

(b) $AB^2 = AD^2 - BC.DL + \left(\frac{BC}{2}\right)^2$

(c) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Sol:

(a) In right triangle ALD

Using Pythagoras theorem, we have

$AC^2 = AL^2 + LC^2$

$= AD^2 - DL^2 + \ (DL + DC)^2$ [Using (1)]

$= AD^2 - DL^2 + \ \left(DL + \frac{BC}{2}\right)^2$ [$\because$ AD is a median]

$= AD^2 - DL^2 + DL^2 + \ \left(\frac{BC}{2}\right)^2 + BC.DL$

$\therefore AC^2 = AD^2 + BC.DL + \ \left(\frac{BC}{2}\right)^2$ …. (2)

(b) In right triangle ALD

Using Pythagoras theorem, we have

$AL^2 = AD^2 - DL^2$ ... (3)

Again, In right triangle ABL

Using Pythagoras theorem, we have

$AB^2 = AL^2 + LB^2$

$= AD^2 - DL^2 + LB^2$ $[Using\ (3)]$

$= AD^2 - DL^2 + \ (BD - DL)^2$

$= AD^2\ DL^2 + \ \left(\frac{1}{2}\ BC - DL\ \right)^2$

$= AD^2 - DL^2 + \ \left(\frac{BC}{2}\right)^2 - BC.DL + DL^2$

$\therefore AB^2 = AD^2 - BC.DL + \ \left(\frac{BC}{2}\right)^2$ ….. (4)

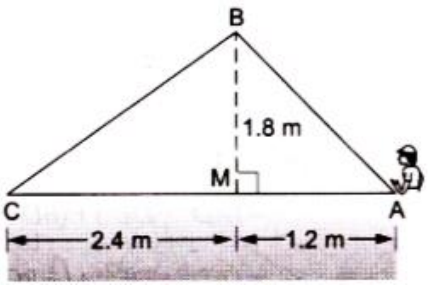
(c) Adding (2) and (4), we get,

$= AC^2 + AB^2 = AD^2 + BC.DL + \left(\frac{BC}{2}\right)^2 + AD^2 - BC.DL + \left(\frac{BC}{2}\right)^2$

$= 2\ AD^2 + \frac{BC^2}{4} + \frac{BC^2}{4}$

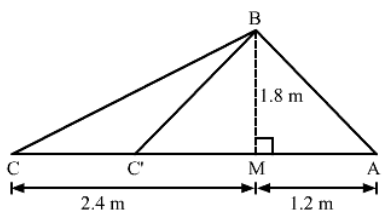
$= 2\ AD^2 + \frac{1}{2}\ BC^2$

22. Naman is doing fly-fishing in a stream. The trip fishing rod is 1.8m above the surface of the water and the fly at the end of the string rests on the water 3.6m away from him and 2.4m from the point directly under the tip of the rod. Assuming that the string(from the tip of his rod to the fly) is taut, how much string does he have out (see the adjoining figure) if he pulls in the string at the rate of 5cm per second, what will be the horizontal distance of the fly from him after 12 seconds?



23.

Sol:



Naman pulls in the string at the rate of 5 cm per second.

Hence, after 12 seconds the length of the string he will pulled is given by

12 × 5 = 60 cm or 0.6 m

Now, in ΔBMC

By using Pythagoras theorem, we have

$BC^2 = CM^2 + MB^2$

$= (2.4)^2 + (1.8)^2$

= 9

∴ BC = 3 m

Now, BC' = BC – 0.6

= 3 – 0.6

= 2.4 m

Now, In ΔBC'M

By using Pythagoras theorem, we have

$C'M^2 = BC'^2 - MB^2$

= $(2.4)^2 - (1.8)^2$
= 2.52
∴ C'M = 1.6 m
The horizontal distance of the fly from him after 12 seconds is given by
C'A = C'M + MA
= 1.6 + 1.2
= 2.8 m

Exercise – 4E

1. State the two properties which are necessary for given two triangles to be similar.
Sol:
The two triangles are similar if and only if
1. The corresponding sides are in proportion.
2. The corresponding angles are equal.

2. State the basic proportionality theorem.
Sol:
If a line is draw parallel to one side of a triangle intersect the other two sides, then it divides the other two sides in the same ratio.

3. State and converse of Thale's theorem.
Sol:
If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

4. State the midpoint theorem
Sol:
The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is equal to one half of the third side.

5. State the AAA-similarity criterion
Sol:
If the corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.

6. State the AA-similarity criterion
Sol:
If two angles are correspondingly equal to the two angles of another triangle, then the two triangles are similar.

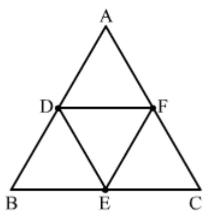
7. State the SSS-similarity criterion for similarity of triangles
Sol:
If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar.

8. State the SAS-similarity criterion
Sol:
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.

9. State Pythagoras theorem
Sol:
The square of the hypotenuse is equal to the sum of the squares of the other two sides.
Here, the hypotenuse is the longest side and it's always opposite the right angle.

10. State the converse of Pythagoras theorem.
Sol:
If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

11. If D, E, F are the respectively the midpoints of sides BC, CA and AB of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.
Sol:



By using mid theorem i.e., the segment joining two sides of a triangle at the midpoints of those sides is parallel to the third side and is half the length of the third side.
∴ DF || BC
And $DF = \frac{1}{2} BC$
⟹ DF = BE
Since, the opposite sides of the quadrilateral are parallel and equal.
Hence, BDFE is a parallelogram
Similarly, DFCE is a parallelogram.
Now, in ΔABC and ΔEFD
$\angle ABC = \angle EFD \qquad (Opposite\ angles\ of\ a\ parallelogram)$
$\angle BCA = \angle EDF \qquad (Opposite\ angles\ of\ a\ parallelogram)$

By AA similarity criterion, ΔABC ~ ΔEFD
If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

$\therefore \frac{area(\Delta DEF)}{area(\Delta ABC)} = \left(\frac{DF}{BC}\right)^2 = \left(\frac{DF}{2\,DF}\right)^2 = \frac{1}{4}$

Hence, the ratio of the areas of ΔDEF and ΔABC is 1 : 4.

12. Two triangles ABC and PQR are such that AB = 3 cm, AC = 6cm, $\angle A = 70°$, PR = 9cm $\angle P = 70°$ and PQ = 4.5 cm. Show that $\Delta ABC \sim \Delta PQR$ and state that similarity criterion.

Sol:
Now, In ΔABC and ΔPQR

$\angle A = \angle P = 70^0 \quad (Given)$

$\frac{AB}{PQ} = \frac{AC}{PR} \left[\because \frac{3}{4.5} = \frac{6}{9} \Rightarrow \frac{1}{1.5} = \frac{1}{1.5}\right]$

By SAS similarity criterion, ΔABC~ ΔPQR

13. In ΔABC~ΔDEF such that 2AB = DE and BC = 6cm, find EF.

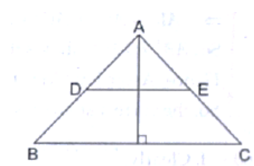
Sol:
When two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.
Here, ΔABC ~ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{AB}{2\,AB} = \frac{6}{EF}$

$\Rightarrow$ EF = 12 cm

14. In the given figure, DE∥BC such that AD = x cm, DB = (3x + 4) cm, AE = (x + 3) cm and EC = (3x + 19) cm. Find the value of x.



Sol:
In ΔADE and ΔABC

$\angle ADE = \angle ABC \quad (Corresponding\ angles\ in\ DE \parallel BC)$

$\angle AED = \angle ACB \quad (Corresponding\ angles\ in\ DE \parallel BC$

By AA similarity criterion, ΔADE ~ ΔABC
If two triangles are similar, then the ratio of their corresponding sides are proportional

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{AD}{AD+DB} = \frac{AE}{AE+EC}$

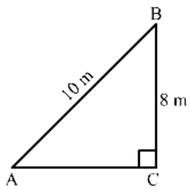
$\Rightarrow \frac{x}{x+3x+4} = \frac{x+3}{x+3+3x+19}$

$\Rightarrow \frac{x}{4x+4} = \frac{x+3}{x+3+3x+19}$

$\Rightarrow \frac{x}{2x+2} = \frac{x+3}{2x+11}$

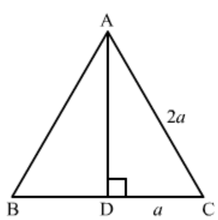
$\Rightarrow 2x^2 + 11x = 2x^2 + 2x + 6x + 6$
$\Rightarrow$ 3x = 6
$\Rightarrow$ x = 2
Hence, the value of x is 2.

15. A ladder 10m long reaches the window of a house 8m above the ground. Find the distance of the foot of the ladder from the base of the wall.
Sol:



Let AB be A ladder and B is the window at 8 m above the ground C.
Now, In right triangle ABC
By using Pythagoras theorem, we have
$AB^2 = BC^2 + CA^2$
$\Rightarrow 10^2 = 8^2 + CA^2$
$\Rightarrow CA^2 = 100 - 64$
$\Rightarrow CA^2 = 36$
$\Rightarrow$ CA = 6m
Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

16. Find the length of the altitude of an equilateral triangle of side 2a cm.
Sol:



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.
Suppose ABC is an equilateral triangle having AB = BC = CA = 2a.
Suppose AD is the altitude drawn from the vertex A to the side BC.
So, it will bisects the side BC
$\therefore$ DC = a
Now, In right triangle ADC
By using Pythagoras theorem, we have
$AC^2 = CD^2 + DA^2$
$\Rightarrow (2a)^2 = a^2 + DA^2$

$\Rightarrow DA^2 = 4a^2 - a^2$
$\Rightarrow DA^2 = 3a^2$
$\Rightarrow$ DA = $\sqrt{3}a$
Hence, the length of the altitude of an equilateral triangle of side 2a cm is $\sqrt{3}a\ cm$

17. ΔABC~ΔDEF such that ar(ΔABC) = 64 cm^2 and ar(ΔDEF) = 169cm^2. If BC = 4cm, find EF.
Sol:
We have Δ ABC ~ Δ DEF
If two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta ABC)}{area(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$

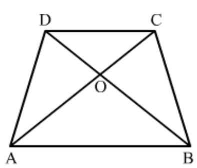
$\Rightarrow \frac{64}{169} = \left(\frac{BC}{EF}\right)^2$

$\Rightarrow \left(\frac{8}{13}\right)^2 = \left(\frac{4}{EF}\right)^2$

$\Rightarrow \frac{8}{13} = \frac{4}{EF}$

$\Rightarrow$ EF = 6.5 cm

18. In a trapezium ABCD, it is given that AB$\parallel$CD and AB = 2CD. Its diagonals AC and BD intersect at the point O such that ar(ΔAOB) = 84cm^2. Find ar(ΔCOD).
Sol:



In Δ AOB and COD
$\angle ABO = \angle CDO$ $\quad$ $(Alternte\ angles\ in\ AB \parallel CD)$
$\angle AOB = \angle COD$ $\quad$ $(Vertically\ opposite\ angles)$
By AA similarity criterion, ΔAOB ~ ΔCOD
If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta AOB)}{area(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

$\Rightarrow \frac{84}{area(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2$

$\Rightarrow area\ (\Delta\ COD\ ) = 12\ cm^2$

19. The corresponding sides of two similar triangles are in the ratio 2: 3. If the area of the smaller triangle is 48cm^2, find the area of the larger triangle.
Sol:

If two triangles are similar, then the ratio of their areas is equal to the squares of their corresponding sides.

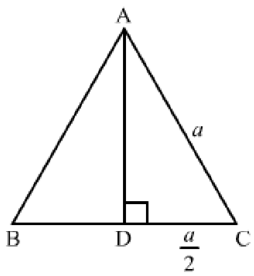
$\therefore \frac{area\ of\ smaller\ triangle}{area\ of\ lager\ triangle} = \left(\frac{Side\ of\ smaller\ triangle}{Side\ of\ larger\ triangle}\right)^2$

$\Rightarrow \frac{48}{area\ of\ larger\ triangle} = \left(\frac{2}{3}\right)^2$

$\Rightarrow area\ of\ larger\ triangle = 108\ cm^2$

20. In an equilateral triangle with side a, prove that area = $\frac{\sqrt{3}}{4}\ a^2$.

Sol:



We know that the altitude of an equilateral triangle bisects the side on which it stands and forms right angled triangles with the remaining sides.

Suppose ABC is an equilateral triangle having AB =BC = CA = a.

Suppose AD is the altitude drawn from the vertex A to the side BC.

So, It will bisects the side BC

$\therefore DC = \frac{1}{2}\ a$

Now, In right triangle ADC

By using Pythagoras theorem, we have

$AC^2 = CD^2 + DA^2$

$\Rightarrow a^2 - \left(\frac{1}{2}a\right)^2 + DA^2$

$\Rightarrow DA^2 = a^2 - \frac{1}{4}a^2$

$\Rightarrow DA^2 = \frac{3}{4}a^2$

$\Rightarrow DA = \frac{\sqrt{3}}{2}\ a$

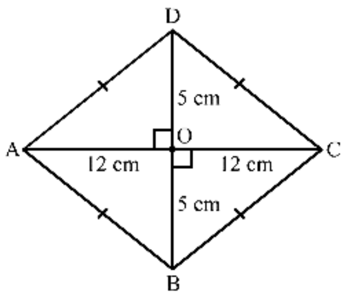
$Now, area\ (\Delta ABC) = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}\ a$

$= \frac{\sqrt{3}}{4}\ a^2$

21. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

Sol:

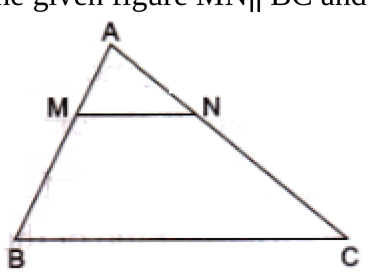


Suppose ABCD is a rhombus.
We know that the diagonals of a rhombus perpendicularly bisect each other.
$\therefore \angle AOB = 90^0 , AO = 12\ cm\ and\ BO = 5\ cm$
Now, In right triangle AOB
By using Pythagoras theorem we have
$AB^2 = AO^2 + BO^2$
$= 12^2 + 5^2$
$= 144 + 25$
$= 169$
$\therefore AB^2 = 169$
$\Longrightarrow$ AB = 13 cm
Since, all the sides of a rhombus are equal.
Hence, AB = BC = CD = DA = 13 cm

22. Two triangles DEF an GHK are such that $\angle D = 48°$ and $\angle H = 57°$. If $\Delta DEF \sim \Delta GHK$ then find the measures of $\angle F$

Sol:
If two triangles are similar then the corresponding angles of the two triangles are equal.
Here, ΔDEF ~ ΔGHK
$\therefore \angle E = \angle H = 57^0$
Now, In Δ DEF
$\angle D + \angle E + \angle F = 180^0 \quad (Angle\ sum\ property\ of\ triangle)$
$\Longrightarrow \angle F = 180^0 - 48^0 - 57^0 = 75^0$

23. In the given figure MN|| BC and AM: MB= 1: 2



Find $\frac{area(\Delta AMN)}{area(\Delta ABC)}$

Sol:

We have

AM : MB = 1 : 2

$\Rightarrow \frac{MB}{AM} = \frac{2}{1}$

Adding 1 to both sides, we get

$\Rightarrow \frac{MB}{AM} + 1 = \frac{2}{1} + 1$

$\Rightarrow \frac{MB+AM}{AM} = \frac{2+1}{1}$

$\Rightarrow \frac{AB}{AM} = \frac{3}{1}$

Now, In ΔAMN and ΔABC

$\angle AMN = \angle ABC \quad (Corresponding\ angles\ in\ MN \parallel BC)$

$\angle ANM = \angle ACB \quad (Corresponding\ angles\ in\ MN \parallel BC)$

By AA similarity criterion, ΔAMN ~ Δ ABC

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta AMN)}{area(\Delta ABC)} = \left(\frac{AM}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

24. In triangle BMP and CNR it is given that PB= 5 cm, MP = 6cm BM = 9 cm and NR = 9cm. If $\Delta BMP \sim \Delta CNR$ then find the perimeter of ΔCNR

Sol:

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

Here, ΔBMP ~ ΔCNR

$\therefore \frac{BM}{CN} = \frac{BP}{CR} = \frac{MP}{NR} \quad ..(1)$

$Now, \frac{BM}{CN} = \frac{MP}{NR} \quad [Using\ (1)]$

$\Rightarrow CN = \frac{BM \times NR}{MP} = \frac{9\times 9}{6} = 13.5\ cm$

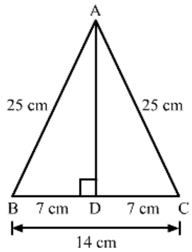
$Again, \frac{BM}{CN} = \frac{BP}{CR} = [Using\ (1)]$

$\Rightarrow CR = \frac{BP \times CN}{BM} = \frac{5\times 13.5}{9} = 7.5\ cm$

Perimeter of ΔCNR = CN + NR + CR = 13.5 + 9 + 7.5 = 30 cm

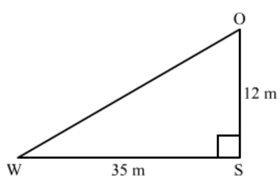
25. Each of the equal sides of an isosceles triangle is 25 cm. Find the length of its altitude if the base is 14 cm.

Sol:



We know that the altitude drawn from the vertex opposite to the non-equal side bisects the non-equal side.
Suppose ABC is an isosceles triangle having equal sides AB and BC.
So, the altitude drawn from the vertex will bisect the opposite side.
Now, In right triangle ABD
By using Pythagoras theorem, we have
$AB^2 = BD^2 + DA^2$
$\Longrightarrow 25^2 = 7^2 + DA^2$
$\Longrightarrow DA^2 = 625 - 49$
$\Longrightarrow DA^2 = 576$
$\Longrightarrow DA = 24\ cm$

26. A man goes 12m due south and then 35m due west. How far is he from the starting point.
Sol:



In right triangle SOW
By using Pythagoras theorem, we have
$OW^2 = WS^2 + SO^2$
$= 35^2 + 12^2$
$= 1225 + 144$
$= 1369$
$\therefore OW^2 = 1369$
$\Longrightarrow OW = 37\ m$
Hence, the man is 37 m away from the starting point.

27. If the lengths of the sides BC, CA and AB of a ΔABC are a, b and c respectively and AD is the bisector $\angle A$ then find the lengths of BD and DC
Sol:
Let DC = X
$\therefore$ BD = a-X

By using angle bisector there in ΔABC, we have

$\frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \frac{c}{b} = \frac{a-x}{x}$

$\Rightarrow cx = ab - bx$

$\Rightarrow x\,(b + c) = ab$

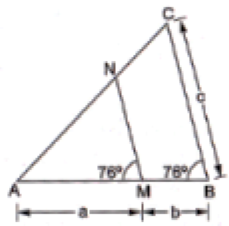
$\Rightarrow x = \frac{ab}{(b+c)}$

Now, $a - x = a - \frac{ab}{b+c}$

$= \frac{ab+ac-ab}{b+c}$

$= \frac{ac}{a+b}$

28. In the given figure, $\angle AMN = \angle MBC = 76°$. If p, q and r are the lengths of AM, MB and BC respectively then express the length of MN of terms of P, q and r.



Sol:

In ΔAMN and ΔABC

$\angle AMN = \angle ABC = 76^0 \quad (Given)$

$\angle A = \angle A \quad (Common)$

By AA similarity criterion, ΔAMN ~ ΔABC

If two triangles are similar, then the ratio of their corresponding sides are proportional

$\therefore \frac{AM}{AB} = \frac{MN}{BC}$

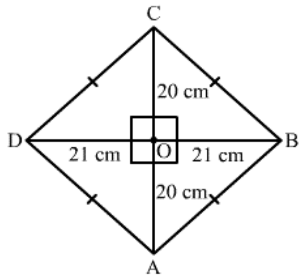
$\Rightarrow \frac{AM}{AM+MB} = \frac{MN}{BC}$

$\Rightarrow \frac{a}{a+b} = \frac{MN}{c}$

$\Rightarrow MN = \frac{ac}{a+b}$

29. Find the length of each side of a rhombus are 40 cm and 42 cm. find the length of each side of the rhombus.

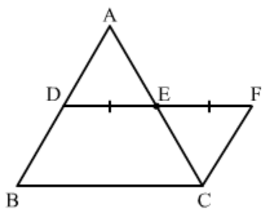
Sol:



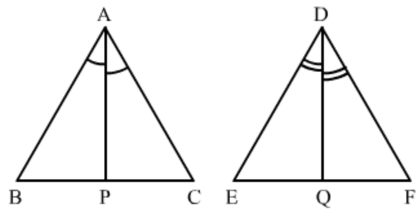
Suppose ABCD is a rhombus.
We know that the diagonals of a rhombus perpendicularly bisect each other.
$\therefore \angle AOB = 90^0, AO = 20\ cm\ and\ BO = 21\ cm$
Now, In right triangle AOB
By using Pythagoras theorem we have
$AB^2 = AO^2 + OB^2$
$= 20^2 + 21^2$
$= 400 + 441$
$= 841$
$\therefore AB^2 = 841$
$\Longrightarrow$ AB = 29 cm
Since, all the sides of a rhombus are equal.
Hence, AB = BC = CD = DA = 29 cm

30. For each of the following statements state whether true(T) or false (F)
(i) Two circles with different radii are similar.
(ii) any two rectangles are similar
(iii) if two triangles are similar then their corresponding angles are equal and their corresponding sides are equal
(iv) The length of the line segment joining the midpoints of any two sides of a triangles is equal to half the length of the third side.
(v) In a ΔABC, AB = 6 cm, $\angle A = 45°$ and AC = 8 cm and in a ΔDEF, DF = 9 cm $\angle D = 45°$ and DE= 12 cm then $\Delta ABC \sim \Delta DEF$.
(vi) the polygon formed by joining the midpoints of the sides of a quadrilateral is a rhombus.
(vii) the ratio of the perimeter of two similar triangles is the same as the ratio of the their corresponding medians.
(ix) if O is any point inside a rectangle ABCD then $OA^2 + OC^2 = OB^2 + OD^2$
(x) The sum of the squares on the sides of a rhombus is equal to the sum of the squares on its diagonals.
Sol:
(i)
Two rectangles are similar if their corresponding sides are proportional.
(ii) True

Two circles of any radii are similar to each other.
(iii)false
If two triangles are similar, their corresponding angles are equal and their corresponding sides are proportional.
(iv) True
Suppose ABC is a triangle and M, N are



Construction: DE is expanded to F such that EF = DE
To proof = DE = $\frac{1}{2}BC$
Proof: In ΔADE and ΔCEF
AE = EC (E is the mid point of AC)
DE = EF (By construction)
AED = CEF (Vertically Opposite angle)
By SAS criterion, ΔADE ~= ΔCEF
CF = AD (CPCT)
⟹ BD = CF
$\angle ADE = \angle EFC \quad (CPCT)$
Since, $\angle ADE$ and $\angle EFC$ are $alternate$ $angle$
Hence, AD ∥ CF and BD ∥ CF
When two sides of a quadrilateral are parallel, then it is a parallelogram
∴ DF = BC and BD ∥ CF
∴BDFC is a parallelogram
Hence, DF = BC
⟹ DE + EF = BC
⟹ $DE = \frac{1}{2}\ BC$
(v) False
In ΔABC, AB = 6 cm, $\angle A = 45^0$ and $AC = 8$ cm and in $\Delta DEF, DF = 9$ $cm, \angle D = 45^0$ and $DE = 12$ $cm,$ $then$ $\Delta ABC \sim \Delta DEF.$
In ΔABC and ΔDEF
(vi) False
The polygon formed by joining the mid points of the sides of a quadrilateral is a parallelogram.
(vii) True



Given: ΔABC ~ ΔDEF

To prove = $\frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2$

Proof: in ΔABP and ΔDEQ

$\angle BAP = \angle EDQ \quad (As\ \angle A = \angle D, so\ their\ Half\ is\ also\ equal)$

$\angle B = \angle E \qquad (\angle ABC \sim \Delta DEF)$

By AA criterion, ΔABP and ΔDEQ

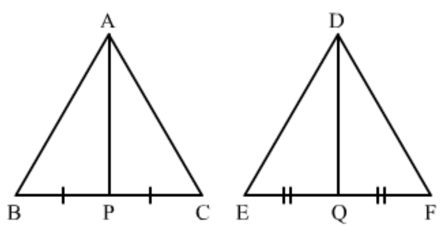
$\frac{AB}{DE} = \frac{AP}{DQ}$ ….(1)

Since, ΔABC ~ ΔDEF

$\therefore \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$

$\Longrightarrow \frac{Ar(\Delta ABC)}{Ar(\Delta DEF)} = \left(\frac{AP}{DQ}\right)^2 \quad [Using\ (1)]$

(viii)



Given: ΔABC ~ ΔDEF

To Prove = $\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AP}{DQ}$

Proof: In ΔABP and ΔDEQ

$\angle B = \angle E \qquad (\therefore \Delta ABC \sim \Delta DEF\ )$

∴ ΔABC ~ ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Longrightarrow \frac{AB}{DE} = \frac{2\ BP}{2\ EQ}$

$\Longrightarrow \frac{AB}{DE} = \frac{BP}{EQ}$

By SAS criterion, ΔABP ~ ΔDEQ

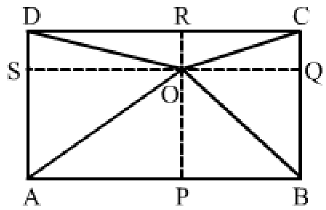
$\frac{AB}{DE} = \frac{AP}{DQ}$ …. (1)

Since, ΔABC ~ ΔDEF

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Longrightarrow \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AP}{DQ}$ $\quad [Using\ (1)]$

(ix) True



Suppose ABCD is a rectangle with O is any point inside it.

Construction: $OA^2 + OC^2 = OB^2 + OD^2$

Proof:

$OA^2 + OC^2 = (AS^2 + OS^2) + (OQ^2 + QC^2)$ [Using Pythagoras theorem in right triangle AOP and COQ]

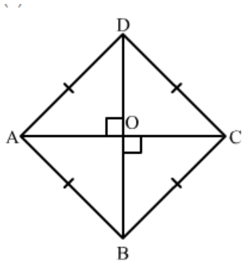
= $(BQ^2 + OS^2) + (OQ^2 + DS^2)$

= $(BQ^2 + OQ^2) + (OS^2 + DS^2)$ [Using Pythagoras theorem in right triangle BOQ and DOS]

= $OB^2 + OD^2$

Hence, LHS = RHS

(x) True



Suppose ABCD is a rhombus having AC and BD its diagonals.

Since, the diagonals of a rhombus perpendicular bisect each other.

Hence, AOC is a right angle triangle

In right triangle AOC

By using Pythagoras theorem, we have

$AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

[∴ Diagonals of a rhombus perpendicularly bisect each other]

$\Longrightarrow AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$

$\Longrightarrow 4\ AB^2 = AC^2 + BD^2$

$\Longrightarrow AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$

$\Longrightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ $[\because All\ sides\ of\ a\ rhombus\ are\ equal]$

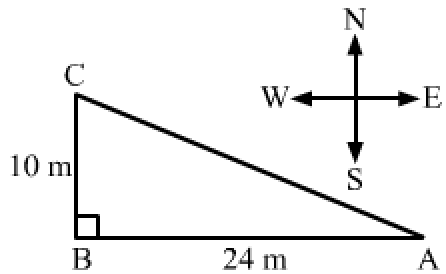
Exercise – MCQ

1. A man goes 24m due west and then 10m due both. How far is he from the starting point?

(a) 34m (b) 17m (c) 26m (d) 28m

Sol:

(c) 26 m



Suppose, the man starts from point A and goes 24 m due west to point B. From here, he goes 10 m due north and stops at C.

In right triangle ABC, we have:

AB = 24 m, BC = 10 m

Applying Pythagoras theorem, we get:

$AC^2 = AB^2 + BC^2 = 24^2 + 10^2$

$AC^2 = 576 + 100 = 676$

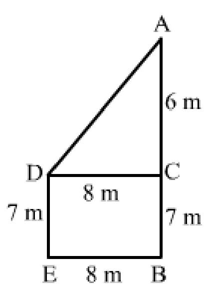
$AC = \sqrt{676} = 26$

2. Two poles of height 13m and 7m respectively stand vertically on a plane ground at a distance of 8m from each other. The distance between their tops is

(a) 9m (b) 10m (c) 11m (d) 12m

Sol:

(b) 10 m



Let AB and DE be the two poles.

According to the question:

AB = 13 m

DE = 7 m

Distance between their bottoms = BE = 8 m

Draw a perpendicular DC to AB from D, meeting AB at C. We get:

DC = 8m, AC = 6 m

Applying, Pythagoras theorem in right-angled triangle ACD, we have

$AD^2 = DC^2 + AC^2$

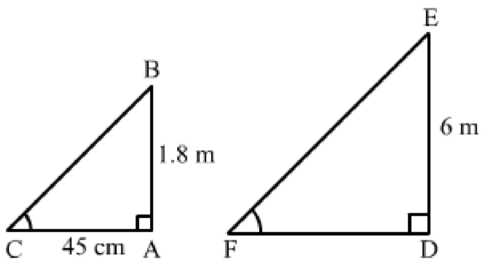
$= 8^2 + 6^2 = 64 + 36 = 100$

$AD = \sqrt{100} = 10\ M$

3. A vertical stick 1.8m long casts a shadow 45cm long on the ground. At the same time, what is the length of the shadow of a pole 6m high?

(a) 2.4m (b) 1.35m (c) 1.5m (d) 13.5m

Sol:



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

AB = 1.8 m

AC = 45 cm = 0.45 m

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

DE = 6 m

DF =?

Now, in right-angled triangles ABC and DEF, we have:

$\angle BAC = \angle EDF = 90^0$

$\angle ACB = \angle DFE \qquad (Angular\ elevation\ of\ the\ \ Sun\ \ at\ the\ same\ time)$

Therefore, by AA similarity theorem, we get:

Δ ABC ~ ΔDFE

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$

$\Rightarrow \frac{1.8}{0.45} = \frac{6}{DF}$

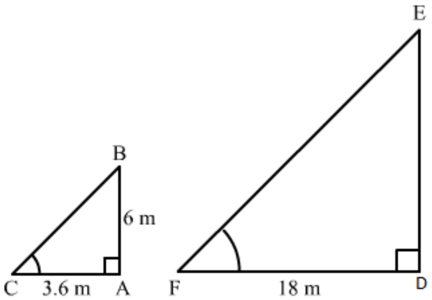
$\Rightarrow\ DF = \frac{6\times0.45}{1.8} = 1.5\ m$

4. A vertical pole 6m long casts a shadow of length 3.6m on the ground. What is the height of a tower which casts a shadow of length 18m at the same time?

(a) 10.8m (b) 28.8m (c) 32.4m (d) 30m

Sol:

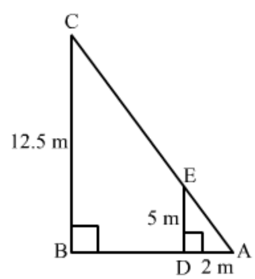
(d)



Let AB and AC be the vertical pole and its shadow, respectively.
According to the question:
AB = 6 m
AC = 3.6 m
Again, let DE and DF be the tower and its shadow.
According to the question:
DF = 18 m
DE =?
Now, in right -angled triangles ABC and DEF, we have:
$\angle BAC = \angle EDF = 90^0$
$\angle ABC = \angle DFE = \quad (Angular\ elevation\ of\ the\ sun\ at\ the\ same\ time)$
Therefore, by AA similarity theorem, we get:
Δ ABC - Δ DEF
$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$
$\Rightarrow \frac{6}{3.6} = \frac{DE}{18}$
$\Rightarrow DE = \frac{6\times18}{3.6} = 30\ m$

5. The shadow of a 5m long stick is 2m long. At the same time the length of the shadow of a 12.5m high tree(in m) is
(a) 3.0 (b) 3.5 (c) 4.5 (d) 5.0

Sol:



Suppose DE is a 5 m long stick and BC is a 12.5 m high tree.
Suppose DA and BA are the shadows of DE and BC respectively.
Now, In ΔABC and ΔADE
$\angle ABC = \angle ADE = 90^0$

$\angle A = \angle A \quad (Common)$

By AA- similarity criterion

$\Delta ABC \sim \Delta ADE$

If two triangles are similar, then the ratio of their corresponding sides are equal.

$\therefore \frac{AB}{AD} = \frac{BC}{DE}$

$\Rightarrow \frac{AB}{2} = \frac{12.5}{5}$

$\Rightarrow$ AB = 5 cm

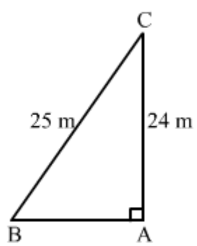
Hence, the correct answer is option (d).

6. A ladder 25m long just reaches the top of a building 24m high from the ground. What is the distance of the foot of the ladder from the building?

(a) 7m (b) 14m (c) 21m (d) 24.5m

Sol:

(a) 7 m



Let the ladder BC reaches the building at C.

Let the height of building where the ladder reaches be AC.

According to the question:

BC = 25 m

AC = 24 m

In right-angled triangle CAB, we apply Pythagoras theorem to find the value of AB.

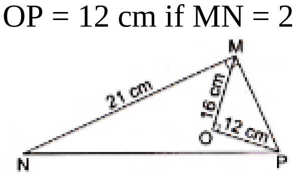
$BC^2 = AC^2 + AB^2$

$\Rightarrow AB^2 = BC^2 - AC^2 = 25^2 - 24^2$

$\Rightarrow AB^2 = 625 - 576 = 49$

$\Rightarrow AB = \sqrt{49} = 7\ m$

7. In the given figure, O is the point inside a ΔMNP such that $\angle MOP = 90°$ OM = 16 cm and OP = 12 cm if MN = 21cm and $\angle NMP = 90°$ then NP= ?



Sol:

Now, In right triangle MOP

By using Pythagoras theorem, we have

$MP^2 = PO^2 + OM^2$
= $12^2 + 16^2$
= 144 + 256
= 400
$\therefore MP^2 = 400$
$\Rightarrow$ MO = 20 cm
Now, In right triangle MPN
By using Pythagoras theorem, we have
$PN^2 = NM^2 + MP^2$
= $21^2 + 20^2$
= $441 + 400$
= 841
$\therefore MP^2 = 841$
$\Rightarrow$ MP = 29 cm
Hence, the correct answer is option (b).

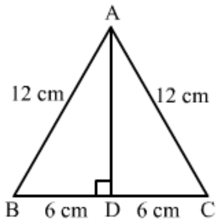
8. The hypotenuse of a right triangle is 25cm. The other two sides are such that one is 5cm longer than the other. The lengths of these sides are
(a) 10cm, 15cm (b) 15cm, 20cm
(c) 12cm, 17cm (d) 13cm, 18cm
Sol:
(b) 15 cm, 20 cm
It is given that length of hypotenuse is 25 cm.
Let the other two sides be x cm and (x−5) cm.
Applying Pythagoras theorem, we get:
$25^2 = x^2 + (x - 5\,)^2$
$\Rightarrow 625 = x^2 + x^2 + 25 - 10x$
$\Rightarrow 2x^2 - 10x - 600 = 0$
$\Rightarrow x^2 - 5x - 300 = 0$
$\Rightarrow x^2 - 20x + 15x - 300 = 0$
$\Rightarrow x(x - 20) + 15(x - 20) = 0$
$\Rightarrow$ (x – 20) (x + 15) =0
$\Rightarrow x - 20 = 0 \; or \; x + 15 = 0$
$\Rightarrow x = 20 \; or \; x = \; -15$
Side of a triangle cannot be negative.
Therefore, x = 20 cm
Now,
x – 5 = 20 – 5 = 15 cm

9. The height of an equilateral triangle having each side 12cm, is

(a) $6\sqrt{2}$ cm (b) $6\sqrt{3}$m (c) $3\sqrt{6}$m (d) $6\sqrt{6}$m

Sol:

(b) $6\sqrt{3}cm$



Let ABC be the equilateral triangle with AD as its altitude from A.

In right-angled triangle ABD, we have

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2$

$= 12^2 - 6^2$

$= 144 – 36 = 108$

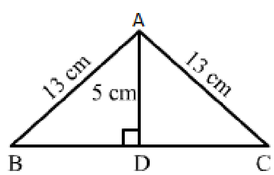
AD = $\sqrt{108} = 6\sqrt{3}cm$

10. ΔABC is an isosceles triangle with AB = AC = 13cm and the length of altitude from A on BC is 5cm. Then, BC = ?

(a) 12cm (b) 16cm (c) 18cm (d) 24cm

Sol:

(d) 24 cm



In triangle ABC, let the altitude from A on BC meets BC at D.

We have:

AD = 5 cm, AB = 13 cm and D is the midpoint of BC.

Applying Pythagoras theorem in right-angled triangle ABD, we get:

$AB^2 = AD^2 + BD^2$

$\Rightarrow BD^2 = AB^2 - AD^2$

$\Rightarrow BD^2 = 13^2 - 5^2$

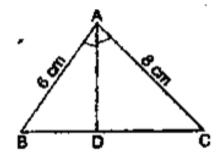
$\Rightarrow BD^2 = 169 - 25$

$\Rightarrow BD^2 = 144$

$\Rightarrow BD = \sqrt{144} = 12\ cm$

Therefore, BC = 2BD = 24 cm

11. In a ΔABC, it is given that AB = 6cm, AC = 8cm and AD is the bisector of $\angle A$. Then, BD : DC = ?



(a) 3 : 4 (b) 9 : 16 (c) 4 : 3 (d) $\sqrt{3}$: 2

Sol:

(a) 3 : 4

In Δ ABD and ΔACD, we have:

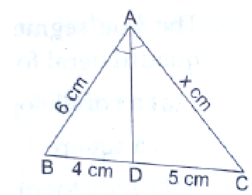
$\angle BAD = \angle CAD$

Now,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$

BD : DC = 3 : 4

12. In ΔABC, it is given that AD is the internal bisector of $\angle A$. If BD = 4cm, DC = 5cm and AB = 6cm, then AC = ?



(a) 4.5cm (b) 8cm (c) 9cm (d) 7.5cm

Sol:

(d) 7.5 cm

It is given that AD bisects angle A

Therefore, applying angle bisector theorem, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$

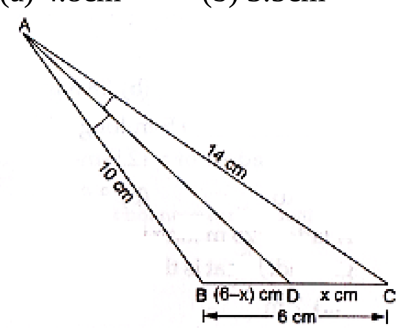
$\Rightarrow \frac{4}{5} = \frac{6}{x}$

$\Rightarrow x = \frac{5 \times 6}{4} = 7.5$

Hence, AC = 7.5 cm

13. In a ΔABC, it is given that AD is the internal bisector of $\angle A$. If AB = 10cm, AC = 14cm and BC = 6cm, then CD = ?

(a) 4.8cm (b) 3.5cm (c) 7cm (d) 10.5cm



Sol:

By using angle bisector in ΔABC, we have

$\frac{AB}{AC} = \frac{BD}{DC}$

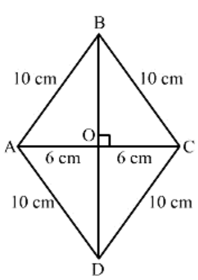
$\Rightarrow \frac{10}{14} = \frac{6-x}{x}$

$\Rightarrow 10x = 84 - 14x$
$\Rightarrow 24x = 84$
$\Rightarrow x = 3.5$
Hence, the correct answer is option (b).

14. In a triangle, the perpendicular from the vertex to the base bisects the base. The triangle is
(a) right-angled (b) isosceles
(c) scalene (d) obtuse-angled
Sol:
(b) Isosceles
In an isosceles triangle, the perpendicular from the vertex to the base bisects the base.

15. In an equilateral triangle ABC, if AD ⊥ BC, then which of the following is true?
(a) $2AB^2 = 3AD^2$ (b) $4AB^2 = 3AD^2$
(c) $3AB^2 = 4AD^2$ (d) $3AB^2 = 2AD^2$
Sol:
(c) $3AB^2 = 4AD^2$
Applying Pythagoras theorem in right-angled triangles ABD and ADC, we get:
$AB^2 = AD^2 + BD^2$
$\Rightarrow AB^2 = \left(\frac{1}{2}AB\right)^2 + AD^2 \quad \left(\because \Delta ABC \text{ is equilateral and } AD = \frac{1}{2}AB\right)$
$\Rightarrow AB^2 = \frac{1}{4}AB^2 + AD^2$
$\Rightarrow AB^2 - \frac{1}{4}AB^2 = AD^2$
$\Rightarrow \frac{3}{4}AB^2 = AD^2$
$\Rightarrow 3AB^2 = 4AD^2$

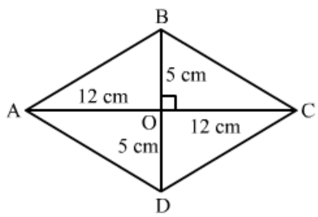
16. In a rhombus of side 10cm, one of the diagonals is 12cm long. The length of the second diagonal is
(a) 20cm (b) 18cm (c) 16cm (d) 22cm
Sol:
(c) 16 cm



Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.
Also, diagonals of a rhombus bisect each other at right angles.

If AC = 12 cm, AO = 6 cm
Applying Pythagoras theorem in right-angled triangle AOB. We get:
$AB^2 = AO^2 + BO^2$
$\Rightarrow BO^2 = AB^2 - AO^2$
$\Rightarrow BO^2 = 10^2 - 6^2 = 100 - 36 = 64$
$\Rightarrow BO = \sqrt{64} = 8$
$\Rightarrow BD = 2 \times BO = 2 \times 8 = 16\ cm$
Hence, the length of the second diagonal BD is 16 cm.

17. The lengths of the diagonals of a rhombus are 24cm and 10cm. The length of each side of the rhombus is
(a) 12cm (b) 13cm (c) 14cm (d) 17cm
Sol:
(b) 13 cm



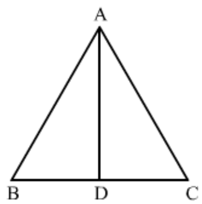
Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.
We have:
AC = 24 cm and BD = 10 cm
We know that diagonals of a rhombus bisect each other at right angles.
Therefore applying Pythagoras theorem in right-angled triangle AOB, we get:
$AB^2 = AO^2 + BO^2 = 12^2 + 5^2$
= 144 + 25 = 169
$AB = \sqrt{169} = 13$
Hence, the length of each side of the rhombus is 13 cm.

18. If the diagonals of a quadrilateral divide each other proportionally, then it is a
(a) parallelogram (b) trapezium
(c) rectangle (d) square
Sol:
(b) trapezium
Diagonals of a trapezium divide each other proportionally.

19. The line segments joining the midpoints of the adjacent sides of a quadrilateral form
(a) parallelogram (b) trapezium
(c) rectangle (d) square
Sol:

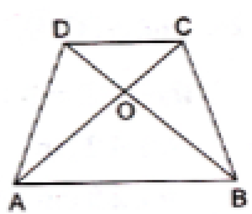
(a) A parallelogram
The line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

20. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is
(a) scalene (b) equilateral
(c) isosceles (d) right-angled
Sol:
(c) isosceles



Let AD be the angle bisector of angle A in triangle ABC.
Applying angle bisector theorem, we get:
$\frac{AB}{AC} = \frac{BD}{DC}$
It is given that AD bisects BC.
Therefore, BD = DC
$\Rightarrow \frac{AB}{AC} = 1$
$\Rightarrow$ AB = AC
Therefore, the triangle is isosceles.

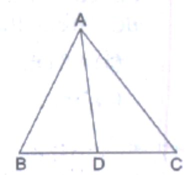
21. In the given figure, ABCD is a trapezium whose diagonals AC and BD intersect at O such that OA = (3x – 1) cm, OB = (2x + 1)cm, OC = (5x – 3)cm and OD = (6x – 5)cm. Then, x = ?
(a) 2 (b) 3 (c) 2.5 (d) 4



Sol:
(a) 2
We know that the diagonals of a trapezium are proportional.
Therefore $\frac{OA}{OC} = \frac{OB}{OD}$
$\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$
$\Rightarrow$ (3X – 1) (6X – 5) = (2X + 1) (5X -3)

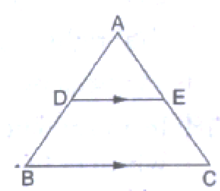
$\Rightarrow 18X^2 - 15X - 6X + 5 = 10X^2 - 6X + 5X - 3$
$\Rightarrow 18X^2 - 21X + 5 = 10X^2 - X - 3$
$\Rightarrow 18X^2 - 21X + 5 - 10X^2 + X + 3 = 0$
$\Rightarrow 8X^2 - 20X + 8 = 0$
$\Rightarrow 4\,(2X^2 - 5X + 2) = 0$
$\Rightarrow 2X^2 - 5X + 2 = 0$
$\Rightarrow 2X^2 - 4X - X + 2 = 0$
$\Rightarrow 2X(X - 2) - 1\,(X - 2) = 0$
$\Rightarrow (X - 2)(2X - 1) = 0$
$\Rightarrow Either\ x - 2 = 0\ or\ 2x - 1 = 0$
$\Rightarrow Either\ x = 2\ or\ x = \frac{1}{2}$
$When\ x = \frac{1}{2}, 6x - 5 = -2 < 0, which\ is\ not\ possible.$
Therefore, x = 2

22. In ΔABC, it is given that $\frac{AB}{AC} = \frac{BD}{DC}$. If ∠B = 70^0 and ∠C = 50^0, then ∠BAD = ?
(a) 30^0 (b) 40^0 (c) 45^0 (d) 50^0



Sol:
(a) 30^0
We have:
$\frac{AB}{AC} = \frac{BD}{DC}$
Applying angle bisector theorem, we can conclude that AD bisects $\angle A$.
In ΔABC,
$\angle A + \angle B + \angle C = 180^0$
$\Rightarrow \angle A = 180 - \angle B - \angle C$
$\Rightarrow \angle A = 180 - 70 - 50 = 60^0$
$\because \angle BAD = \angle CAD = \frac{1}{2}\angle BAC$
$\therefore \angle BAD = \frac{1}{2} \times 60 = 30^0$

23. In ΔABC, DE‖BC so that AD = 2.4cm, AE = 3.2cm and EC = 4.8cm. Then, AB = ?
(a) 3.6cm (b) 6cm (c) 6.4cm (d) 7.2cm



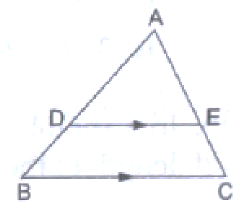
Sol:
(b) 6 cm
It is given that DE || BC.
Applying basic proportionality theorem, we have:
$\frac{AD}{BD} = \frac{AE}{EC}$
$\Rightarrow \frac{2.4}{BD} = \frac{3.2}{4.8}$

$\Rightarrow BD = \frac{2.4 \times 4.8}{3.2} = 3.6\ cm$

Therefore, AB = AD + BD = 2.4 + 3.6 = 6 cm

24. In a ΔABC, if DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that AB = 7.2cm, AC = 6.4cm and AD = 4.5cm. Then, AE = ?

(a) 5.4cm (b) 4cm (c) 3.6cm (d) 3.2cm



Sol:

(b) 4cm

It is given that DE || BC.

Applying basic proportionality theorem, we get:

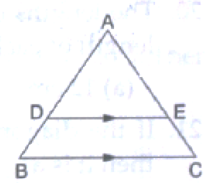
$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{4.5}{7.2} = \frac{AE}{6.4}$

$\Rightarrow AE = \frac{4.5 \times 6.4}{7.2} = 4\ cm$

25. In ΔABC, DE ∥ BC so that AD = (7x – 4)cm, AE = (5x – 2)cm, DB = (3x + 4)cm and EC = 3x cm. Then, we have:

(a) x = 3 (b) x = 5 (c) x = 4 (d) x = 2.5



Sol:

(c) x = 4

It is given DE || BC.

Applying Thales' theorem. We get:

$\frac{AD}{BD} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

$\Rightarrow 3x(7x-4) = (5x-2)(3x+4)$

$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$

$\Rightarrow 6x^2 - 26x + 8 = 0$

$\Rightarrow 2\ (3x^2 - 13x + 4) = 0$

$\Rightarrow 3x^2 - 13x + 4 = 0$

$\Rightarrow 3x^2 - 12x - x + 4 = 0$

$\Rightarrow 3x\ (x-4) - 1\ (x-4) = 0$

$\Rightarrow (x-4)(3x-1) = 0$

$\Rightarrow x - 4 = 0\ or\ 3x - 1 = 0$

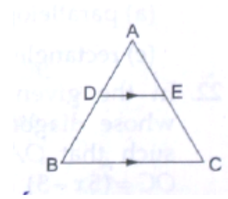
$\Rightarrow x - 4\ or\ x = \frac{1}{3}$

If $x = \frac{1}{3}, 7x - 4 = -\frac{5}{3} < 0\ ; it\ is\ not\ possible.$

Therefore, x = 4

26. In ΔABC, DE ∥ BC such that $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6cm, then AE = ?

(a) 4.2cm (b) 3.1cm (c) 2.8cm (d) 2.1cm



Sol:

(d) 2.1 cm

It is given that DE || BC.

Applying Thales' theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

Let AE be x cm.

Therefore, EC = (5.6 – x) cm

$\Rightarrow \frac{3}{5} = \frac{x}{5.6 - x}$

$\Rightarrow 3(5.6 - x) = 5x$

$\Rightarrow 16.8 - 3x = 5x$

$\Rightarrow 8x = 16.8$

$\Rightarrow$ x = 2.1 cm

27. $\Delta ABC \sim \Delta DEF$ and the perimeters of ΔABC and ΔDEF are 30cm and 18cm respectively. If BC = 9cm, then EF = ?

(a) 6.3cm (b) 5.4cm (c) 7.2cm (d) 4.5cm

Sol:

(b) 5.4 cm

$\Delta ABC \sim \Delta DEF$

Therefore,

$\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{BC}{EF}$

$\Rightarrow \frac{30}{18} = \frac{9}{EF}$

$\Rightarrow EF = \frac{9 \times 18}{30} = 5.4\ cm$

28. $\Delta ABC \sim \Delta DEF$ such that AB = 9.1cm and DE = 6.5cm. If the perimeter of ΔDEF is 25cm, what is the perimeter of ΔABC?

(a) 35cm (b) 28cm (c) 42cm (d) 40cm

Sol:

(a) 35 cm

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Rightarrow \frac{Perimeter(\Delta ABC)}{25} = \frac{9.1}{6.5}$

$\Rightarrow Perimeter\ (\Delta ABC) = \frac{9.1 \times 25}{6.5} = 35\ cm$

29. In ΔABC, it is given that AB = 9cm, BC = 6cm and CA = 7.5cm. Also, ΔDEF is given such that EF = 8cm and ΔDEF~ΔABC. Then, perimeter of ΔDEF is

(a) 22.5cm (b) 25cm (c) 27cm (d) 30cm

Sol:

(d) 30 cm

Perimeter of ΔABC = AB + BC + CA = 9 + 6 + 7.5 = 22.5 cm

∵ ΔDEF ~ ΔABC

$\therefore \frac{Perimeter\Delta(ABC)}{Perimeter(\Delta DEF)} = \frac{BC}{EF}$

$\Longrightarrow \frac{22.5}{Perimeter(\Delta DEF)} = \frac{6}{8}$

Perimeter(ΔDEF) = $\frac{22.5\times8}{6} = 30\ cm$

30. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. Ratio of these area of triangles ABC and BDE is

(a) 2 : 1 (b) 1 : 4 (c) 1 : 2 (d) 4 : 1

Sol:

Give: ABC and BDE are two equilateral triangles

Since, D is the midpoint of BC and BDE is also an equilateral triangle.

Hence, E is also the midpoint of AB.

Now, D and E are the midpoint of BC and AB.

In a triangle, the line segment that joins midpoint of the two sides of a triangle is parallel to the third side and is half of it.

$DE \parallel CA \text{ and } DE = \frac{1}{2}\ CA$

Now, in ΔABC and ΔEBD

$\angle BED = \angle BAC \quad (Corresponding\ angles)$

$\angle B = \angle B \quad (Common)$

By AA-similarity criterion

ΔABC ~ ΔEBD

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area(\Delta ABC)}{area(\Delta DBE)} = \left(\frac{AC}{ED}\right)^2 = \left(\frac{2ED}{ED}\right)^2 = \frac{4}{1}$

Hence, the correct answer is option (d).

31. It is given that ΔABC~ΔDEF. If ∠A = 30^0, ∠C = 50^0, AB = 5cm, AC = 8cm and DF = 7.5cm, then which of the following is true?

(a) DE = 12cm, ∠F = 50^0 (b) DE = 12cm, ∠F = 100^0

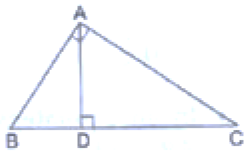
(c) DE = 12cm, ∠D = 100^0 (d) EF = 12cm, ∠D = 30^0

Sol:

(b) DE = 12 cm, $\angle F = 100^0$

Disclaimer: In the question, it should be ΔABC ~ ΔDFE instead of ΔABC ~ ΔDEF.
In triangle ABC,
$\angle A + \angle B + \angle C = 180^0$
$\therefore \angle B = 180 - 30 - 50 = 100^0$
∵ ΔABC ~ ΔDFE
$\therefore \angle D = \angle A = 30^0$
$\angle F = \angle B = 100^0$
And $\angle E = \angle C = 50^0$
Also,
$\frac{AB}{DF} = \frac{AC}{DE} \quad \Rightarrow \frac{5}{7.5} = \frac{8}{DE}$
$\Rightarrow DE = \frac{8 \times 7.5}{5} = 12\ cm$

32. In the given figure, ∠BAC = 90^0 and AD⊥BC. Then,



(a) BC.CD = BC^2
(b) AB.AC = BC^2
(c) BD.CD = AD^2
(d) AB.AC = AD^2

Sol:
(c) BD . CD = AD^2
In Δ BDA and ΔADC, we have:
$\angle BDA = \angle ADC = 90^0$
$\angle ABD = 90^0 - \angle DAB$
$= 90^0 - (90^0 - \angle DAC)$
$= 90^0 - 90^0 + \angle DAC$
$= \angle DAC$
Applying AA similarity, we conclude that $\Delta BDA - \Delta ADC$.
$\Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$
$\Rightarrow AD^2 = BD.CD$

33. In ΔABC, $AB = 6\sqrt{3}$, AC = 12 cm and BC = 6cm. Then $\angle B$ is

Sol:
$AB = 6\sqrt{3}cm$
$\Rightarrow AB^2 = 108\ cm^2$
AC = 12 cm
$\Rightarrow AC^2 = 144\ cm^2$

BC = 6 cm
$\Rightarrow BC^2 = 36\ cm$
$\therefore AC^2 = AB^2 + BC^2$
Since, the square of the longest side is equal to the sum of two sides, so ΔABC is a right angled triangle.
∴ The angle opposite to ∠90°
Hence, the correct answer is option (c)

34. In ΔABC and ΔDEF, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$, then
(a) ∠B = ∠E (b) ∠A = ∠D (c) ∠B = ∠D (d) ∠A = ∠F
Sol:
(c)$\angle B = \angle D$
Disclaimer: In the question, the ratio should be $\frac{AB}{DE} = \frac{BC}{FD} = \frac{AC}{EF}$.
We can write it as:
$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{FE}$
Therefore, Δ ABC − EDF
Hence, the corresponding angles, i.e.,$\angle B\ and\ \angle D, will\ be\ equal.$
$i.e., \angle B = \angle D$

35. In ΔDEF and ΔPQR, it is given that ∠D = ∠Q and ∠R = ∠E, then which of the following is not true?
(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$
Sol:
(b) $\frac{DE}{PQ} = \frac{EF}{RP}$
In ΔDEF and ΔPQR, we have:
$\angle D = \angle Q\ and\ \angle R = \angle E$
Applying AA similarity theorem, we conclude that ΔDEF ~ ΔQRP.
$Hence, \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{PR}$

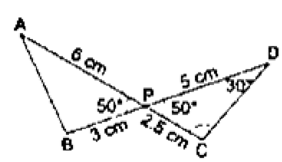
36. If ΔABC~ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?
(a) BC.EF = AC.FD
(b) AB.EF = AC.DE
(c) BC.DE = AB.EF
(d) BC.DE = AB.FD
Sol:
(c) BC. DE = AB. EF
ΔABC ~ ΔEDF

Therefore,
$\frac{AB}{DE} = \frac{AC}{EF} = \frac{BC}{DF}$
$\Longrightarrow$ BC. DE $\neq$ AB. EF

37. In ΔABC and ΔDEF, it is given that ∠B = ∠E, ∠F = ∠C and AB = 3DE, then the two triangles are
(a) congruent but not similar (b) similar but not congruent
(c) neither congruent nor similar (d) similar as well as congruent
Sol:
(b) similar but not congruent
In ΔABC and ΔDEF, we have:
$\angle B = \angle E \text{ and } \angle F = \angle C$
Applying AA similarity theorem, we conclude that ΔABC - ΔDEF.
Also,
AB = 3DE
$\Longrightarrow$ AB $\neq$ DE
Therefore, ΔABC and ΔDEF are not congruent.

38. If in ΔABC and ΔPQR, we have: $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then
(a) ΔPQR ~ ΔCAB (b) ΔPQR ~ ΔABC
(c) ΔCBA ~ ΔPQR (d) ΔBCA ~ ΔPQR
Sol:
(a) ΔPQR ~ ΔCAB
In ΔABC and ΔPQR, we have:
$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$
$\Longrightarrow$ ΔABC ~ ΔQRP
We can also write it as ΔPQR ~ ΔCAB.

39. In the given figure, two line segment AC and BD intersect each other at the point P such that PA = 6cm, PB = 3cm, PC = 2.5cm, PD = 5cm, ∠APB = 50^0 and ∠CDP = 30^0, then ∠PBA = ?
(a) 50^0 (b) 30^0
(c) 60^0 (b) 100^0



Sol:
(d)100^0
In Δ APB and Δ DPC, we have:
$\angle APB = \angle DPC = 50^0$
$\frac{AP}{BP} = \frac{6}{3} = 2$

$\frac{DP}{CP} = \frac{5}{2.5} = 2$

Hence, $\frac{AP}{BP} = \frac{DP}{CP}$

Applying SAS theorem, we conclude that Δ APB- Δ DPC.

$\therefore \angle PBA = \angle PCD$

In Δ DPC, we have:

$\angle CDP + \angle CPD + \angle PCD = 180^0$

$\Rightarrow \angle PCD = 180^0 - \angle CDP - \angle CPD$

$\Rightarrow \angle PCD = 180^0 - 30^0 - 50^0$

$\Rightarrow \angle PCD = 100^0$

Therefore, $\angle PBA = 100^0$

40. Corresponding sides of two similar triangles are in the ratio 4:9 Areas of these triangles are in the ration

(a) 2:3 (b) 4:9 (c) 9:4 (d) 16:81

Sol:

If two triangles are similar, then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$\therefore \frac{area\ of\ first\ triangle}{area\ of\ second\ triangle} = \left(\frac{Side\ of\ first\ triangle}{Side\ of\ second\ triangle}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is option (d).

41. It is given that ΔABC~ΔPQR and $\frac{BC}{QR} = \frac{2}{3}$, then $\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = ?$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

Sol:

(d) 9:4

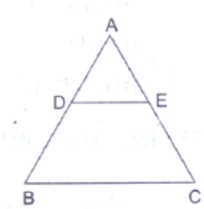
It is given that ΔABC ~ ΔPQR and $\frac{BC}{QR} = \frac{2}{3}$

Therefore,

$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{QR^2}{BC^2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

42. In an equilateral ΔABC, D is the midpoint of AB and E is the midpoint of AC. Then, ar(ΔABC) : ar(ΔADE) = ?

(a) 2 : 1 (b) 4 : 1 (c) 1 : 2 (d) 1 : 4



Sol:

(b) 4:1

In ΔABC, D is the midpoint of AB and E is the midpoint of AC.

Therefore, by midpoint theorem,

Also, by Basic Proportionality Theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

Also,

AB = AC = BC ($\because$ΔABC is an equilateral triangle)

So, $\frac{AD}{DB} = \frac{AE}{EC} = 1$

In ΔABC and ΔADE, we have:

$\angle A = \angle A$

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$

$\therefore$ Δ ABC - ΔADE (SAS criterion)

$\therefore ar\ (\Delta ABC) : ar\ (\Delta ADE) = (AB)^2 : (AD)^2$

$\Longrightarrow ar\ (\Delta ABC) : ar\ (\Delta ADE) = 2^2 : 1^2$

$\Longrightarrow ar(\Delta ABC): ar\ (\Delta ADE) = 4:1$

43. In ΔABC and ΔDEF, we have: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then ar(ΔABC) : Δ(DEF) = ?

(a) 5 : 7 (b) 25 : 49 (c) 49 : 25 (d) 125 : 343

Sol:

(b) 25 :49

In ΔABC and ΔDEF, we have :

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} = \frac{5}{7}$

Therefore, by SSS criterion, we conclude that ΔABC ~ ΔDEF.

$\Longrightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \left(\frac{5}{7}\right)^2 = \frac{25}{49} = 25 : 49$

44. ΔABC~ΔDEF such that ar(ΔABC) = 36cm^2 and ar(ΔDEF) = 49cm^2. Then, the ratio of their corresponding sides is

(a) 36 : 49 (b) 6 : 7 (c) 7 : 6 (d) $\sqrt{6} : \sqrt{7}$

Sol:

(b) 6:7

$\because$ ΔABC ~ ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ... (i)

Also,

$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Longrightarrow \frac{36}{49} = \frac{AB^2}{DE^2}$

$\Longrightarrow \frac{6}{7} = \frac{AB}{DE}$

$\Longrightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{6}{7}$ $\left(from\ (i)\right)$

Thus, the ratio of corresponding sides is 6 : 7.

45. Two isosceles triangles have their corresponding angles equal and their areas are in the ratio 25: 36. The ratio of their corresponding heights is

(a) 25 : 36 (b) 36 : 25 (c) 5 : 6 (d) 6: 5

Sol:

(c) 5:6

Let x and y be the corresponding heights of the two triangles.

It is given that the corresponding angles of the triangles are equal.

Therefore, the triangles are similar. (By AA criterion)

Hence,

$\frac{ar(\Delta_1)}{ar(\Delta_2)} = \frac{25}{36} = \frac{x^2}{y^2}$

$\Rightarrow \frac{x^2}{y^2} = \frac{25}{36}$

$\Longrightarrow \frac{x^2}{y^2} = \sqrt{\frac{25}{36}} = \frac{5}{6} = 5 : 6$

46. The line segments joining the midpoints of the sides of a triangle form four triangles, each of which is

(a) congruent to the original triangle

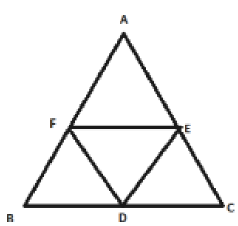
(b) similar to the original triangle

(c) an isosceles triangle

(d) an equilateral triangle

Sol:

(b) similar to the original triangle



The line segments joining the midpoint of the sides of a triangle form four triangles, each of which is similar to the original triangle.

47. If ΔABC~ΔQRP, $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$, AB = 18cm and BC = 15cm, then PR = ?

(a) 18cm (b) 10cm

(c) 12 cm (d) $\frac{20}{3}$ cm

Sol:

(b) 10 cm

∵ ΔABC ~ ΔQRP

$\therefore \frac{AB}{QR} = \frac{BC}{PR}$

Now,

$\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$

$\Rightarrow \left(\frac{AB}{QR}\right)^2 = \frac{9}{4}$

$\Rightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{3}{2}$

Hence, 3PR = 2BC = 2 × 15 = 30

PR = 10 cm

48. In the given figure, O is the point of intersection of two chords AB and CD such that OB = OD and ∠AOC = 45^0. Then, ΔOAC and ΔODB are

(a) equilateral and similar

(b) equilateral but not similar

(c) isosceles and similar

(d) isosceles but not similar

Sol:

(c) isosceles and similar

In ΔAOC and ΔODB, we have:

$\angle AOC = \angle DOB$ $(Vertically\ opposite\ angles)$

$and\ \angle OAC = \angle ODB$ $(Angles\ in\ the\ same\ segment\ )$

$Therefore, by\ AA\ similarity\ theorem, we\ conclude\ that\ \Delta AOC - \Delta DOB.$

$\Rightarrow \frac{OC}{OB} = \frac{OA}{OD} = \frac{AC}{BD}$

Now, OB = OD

$\Rightarrow \frac{OC}{OA} = \frac{OB}{OD} = 1$

⟹ OC = OA

Hence, ΔOAC and ΔODB are isosceles and similar.

49. In an isosceles ΔABC, if AC = BC and $AB^2 = 2AC^2$, then ∠C = ?

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

Sol:

(d) 90^0

Given:

AC = BC

$AB^2 = 2AC^2 = AC^2 + AC^2 = AC^2 + BC^2$

Applying Pythagoras theorem, we conclude that ΔABC is right angled at C.

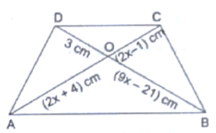
Or, $\angle C = 90^0$

50. In ΔABC, if AB = 16cm, BC = 12cm and AC = 20cm, then ΔABC is
(a) acute-angled (b) right-angled (c) obtuse-angled
Sol:
(b) right-angled
We have:
$AB^2 + BC^2 = 16^2 + 12^2 = 256 + 144 = 400$
$and, AC^2 = 20^2 = 400$
$\therefore AB^2 + BC^2 = AC^2$
Hence, ΔABC is a right-angled triangle.

51. Which of the following is a true statement?
(a) Two similar triangles are always congruent
(b) Two figures are similar if they have the same shape and size.
(c)Two triangles are similar if their corresponding sides are proportional.
(d) Two polygons are similar if their corresponding sides are proportional.
Sol:
(c)Two triangles are similar if their corresponding sides are proportional.
According to the statement:
ΔABC~ ΔDEF
$if\ \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

52. Which of the following is a false statement?
(a) If the areas of two similar triangles are equal, then the triangles are congruent.
(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.
(c) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding.
(d) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.
Sol:
(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.
Because the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

53. Match the following columns:

Column I	Column II
(a) In a given ΔABC, DE ∥ BC and $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6cm, then AE = ……..cm.	(p) 6
(b) If ΔABC~ΔDEF such that 2AB = 3DE and BC = 6cm, then EF = …….cm.	(q) 4
(c) If ΔABC~ΔPQR such that ar(ΔABC) : ar(ΔPQR) = 9 : 16 and BC = 4.5cm, then QR = ……..cm.	(r) 3
(d) In the given figure, AB ∥ CD and OA = (2x + 4)cm, OB = (9x – 21)cm, OC = (2x – 1)cm and OD = 3cm. Then x = ? 	(s) 2.1

The correct answer is:

(a) - ……, (b)-……, (c)-……, (d)-…..,

Sol:

(a) −(s)

Let AE be X.

Therefore, EC = 5.6 – X

It is given that DE || BC.

Therefore, by B.P.T.,we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Longrightarrow \frac{3}{5} = \frac{x}{5.6-x}$

$\Longrightarrow 3(5.6 - x) = 5x$

$\Longrightarrow 16.8 - 3x = 5x$

$\Longrightarrow 8x = 16.8$

$\Longrightarrow x = 2.1\ cm$

(b) −(q)

∵ ΔABC-ΔDEF

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\Longrightarrow \frac{3}{2} = \frac{6}{EF}$

$EF = \frac{6\times2}{3} = 4\ cm$

(c) −(p)

∵ ΔABC ~ ΔPQR

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{9}{16} = \frac{4.5^2}{QR^2} \Rightarrow QR = \sqrt{\frac{4.5 \times 4.5 \times 16}{9}} = \frac{4.5 \times 4}{3} = 6\ cm$

(d) −(r)

∵ AB || CD

$\therefore \frac{OA}{OB} = \frac{OC}{OD}\ (Thales' theorem)$

$\Rightarrow \frac{2x+4}{9x-21} = \frac{2x-1}{3}$

$3(2x+4) = (2x-1(9x-21))$

$\Rightarrow 6x + 12 = 18x^2 - 42x - 9x + 21$

$\Rightarrow 18x^2 - 57x + 9 = 0$

$\Rightarrow 6x^2 - 19x + 3 = 0$

$\Rightarrow 6x^2 - 18x - x + 3 = 0$

$\Rightarrow$ (6x – 1) (x -3) = 0

$\Rightarrow x = 3\ or\ x = -\frac{1}{6}$

$But\ x = -\frac{1}{6} makes\ (2x - 1) < 0, which\ is\ not\ possible.$

Therefore, x= 3

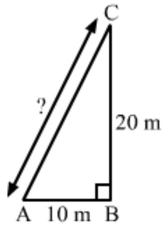
54. Match the following columns:

Column I	Column II
(a) A man goes 10m due east and then 20m due north. His distance from the starting point is ……m.	(p) $25\sqrt{3}$
(b) In an equilateral triangle with each side 10cm, the altitude is …..cm.	(q) $5\sqrt{3}$
(c) The area of an equilateral triangle having each side 10cm is …..cm^2.	(r) $10\sqrt{5}$
(d) The length of a diagonal of a rectangle having length 8m and breadth 6m is ….m.	(s) 10

The correct answer is:

(a) - ……, (b)-……, (c)-……, (d)-…..,

Sol:



(a) −(r)

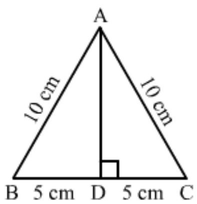
Let the man starts from A and goes 10 m due east at B and then 20 m due north at C.

Then, in right-angled triangle ABC, we have:

$AB^2 + BC^2 = AC^2$

$\Rightarrow AC = \sqrt{10^2 + 20^2} = \sqrt{100 + 200} = 10\sqrt{3}$

Hence, the man is 10 $\sqrt{3} m\ away\ from\ the\ staring\ point$.



(b) −(q)

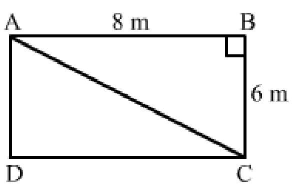
Let the triangle be ABC with altitude AD.

In right-angled triangle ABC, we have:

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = 10^2 - 5^2 \left(\because BD = \frac{1}{2} BC\right)$

$\Rightarrow AD = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm$



(c) – (p)

Area of an equilateral triangle with side a = $\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 10^2 = \sqrt{3} \times 5 \times 5$

$= 25\sqrt{3}\ cm^2$

(d) – (s)

Let the rectangle be ABCD with diagonals AC and BD.

In right-angled triangle ABC, we have:

$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36$

$\Rightarrow AC = \sqrt{100} = 10\ m$

Exercise – Formative Assessment

1. $\Delta ABC \sim \Delta DEF$ and the perimeters of ΔABC and $\sim \Delta DEF$ are 32cm and 24cm respectively. If AB = 10cm, then DE = ?

(a) 8cm (b) 7.5cm (c) 15cm (d) $5\sqrt{3}$cm

Sol:

(b) 7.5 cm

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$

$\Longrightarrow \frac{32}{24} = \frac{10}{DE}$

$\Longrightarrow DE = \frac{10\times 24}{32} = 7.5\ cm$

2. In ΔABC, DE ‖ BC. If DE = 5cm, BC = 8cm and AD = 3.5cm, then AB = ?

(a) 5.6cm (b) 4.8cm (c) 5.2cm (d) 6.4cm

Sol:

(a) 5.6 cm

$\because$ DE ∥ BC

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad (Thales' theorem)$

$\Longrightarrow \frac{3.5}{AB} = \frac{5}{8}$

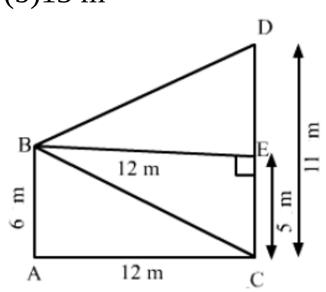
$\Longrightarrow AB = \frac{3.5\times 8}{5} = 5.6\ cm$

3. Two poles of heights 6m and 11m stand vertically on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

(a) 12m (b) 13m (c) 14m (d) 15m

Sol:

(b)13 m



Let the poles be and CD

It is given that:

AB = 6 m and CD = 11 m

Let AC be 12 m

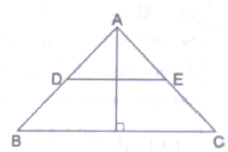
Draw a perpendicular farm Bon CD, meeting CD at E

Then,
BE = 12 m
We have to find BD.
Applying Pythagoras theorem in right-angled triangle BED, we have:
$BD^2 = BE^2 + ED^2$
$= 12^2 + 5^2$ $(\because ED = CD - CE = 11 - 6)$
= 144 + 25 = 169
BD = 13 m

4. The areas of two similar triangles are 25cm^2 and 36cm^2 respectively. If the altitude of the first triangle is 3.5cm, then the corresponding altitude of the other triangle.
(a) 5.6cm (b) 6.3cm (c) 4.2cm (d) 7cm
Sol:
(c)
We know that the ratio of areas of similar triangles is equal to the ratio of squares of their corresponding altitudes.
Let h be the altitude of the other triangle.
Therefore,
$\frac{25}{36} = \frac{(3.5)^2}{h^2}$
$\Rightarrow h^2 = \frac{(3.5)^2 \times 36}{25}$
$\Rightarrow h^2 = 17.64$
$\Rightarrow h = 4.2\ cm$

5. If ΔABC~ΔDEF such that 2AB = DE and BC = 6cm, find EF.
Sol:
∵ ΔABC ~ Δ DEF
$\therefore \frac{AB}{DE} = \frac{BC}{EF}$
$\Rightarrow \frac{1}{2} = \frac{6}{EF}$
$\Rightarrow EF = 12\ cm$

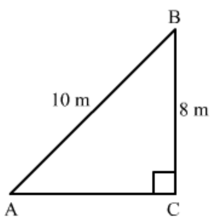
6. In the given figure, DE║BC such that AD = x cm, DB = (3x + 4) cm, AE = (x + 3) cm and EC = (3x + 19) cm. Find the value of x.



Sol:
∵ DE || BC
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $(Basic\ proportinality\ theorem)$
$\frac{x}{3x+4} = \frac{x+3}{3x+19}$
$\Rightarrow x\,(3x + 19) = (x + 3)(3x + 4)$
$\Rightarrow 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

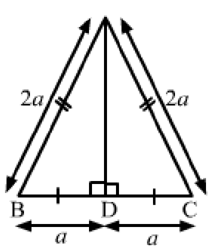
$\Rightarrow 19x - 13x = 12$
$\Rightarrow 6x = 12$
$\Rightarrow x = 2$

7. A ladder 10m long reaches the window of a house 8m above the ground. Find the distance of the foot of the ladder from the base of the wall.
Sol:
Let the ladder be AB and BC be the height of the window from the ground.



We have:
AB 10 m and BC = 8 m
Applying theorem in right-angled triangle ACB, we have:
$AB^2 = AC^2 + BC^2$
$\Rightarrow AC^2 = AB^2 - BC^2 = 10^2 - 8^2 = 100 - 64 = 36$
$\Rightarrow AC = 6\ m$
Hence, the ladder is 6 m away from the base of the wall.

8. Find the length of the altitude of an equilateral triangle of side 2a cm.
Sol:
Let the triangle be ABC with AD as its altitude. Then, D is the midpoint of BC.
In right-angled triangle ABD, we have:



$AB^2 = AD^2 + DB^2$
$\Rightarrow AD^2 = AB^2 - DB^2 = 4a^2 - a^2 \qquad \left(\because BD = \frac{1}{2} BC\right)$
$= 3a^2$
$AD = \sqrt{3}a$
Hence, the length of the altitude of an equilateral triangle of side 2a cm is $\sqrt{3a}\ cm$.

9. ΔABC~ΔDEF such that ar(ΔABC) = 64cm^2 and ar(ΔDEF) = 169cm^2. If BC = 4cm, find EF.
Sol:

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{64}{169} = \frac{4^2}{EF^2}$

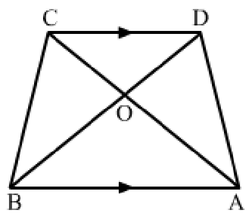
$\Rightarrow EF^2 = \frac{16\times169}{64}$

$\Rightarrow EF = \frac{4\times13}{8} = 6.5\ cm$

10. In a trapezium ABCD, it is given that AB $\parallel$ CD and AB = 2CD. Its diagonals AC and BD intersect at the point O such that ar(ΔAOB) = 84cm^2. Find ar(ΔCOD).

Sol:

In ΔAOB and ΔCOD, we have:



$\angle AOB = \angle COD\ (Vertically\ opposite\ angles)$

$\angle OAB = \angle OCD\ (Alternate\ angles\ as\ AB \parallel CD\ )$

Applying AA similarity criterion, we get :

Δ AOB - ΔCOD

$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$

$\Rightarrow \frac{84}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

$\Rightarrow \frac{84}{ar(\Delta COD)} = \left(\frac{2CD}{CD}\right)^2$

$\Rightarrow ar(\Delta COD) = \frac{84}{4} = 21\ cm^2$

11. The corresponding sides of two similar triangles are in the ratio 2: 3. If the area of the smaller triangle is 48cm^2, find the area of the larger triangle.

Sol:

It is given that the triangles are similar.

Therefore, the ratio of areas of similar triangles will be equal to the ratio of squares of their corresponding sides.

$\therefore \frac{48}{Area\ of\ larger\ triangle} = \frac{2^2}{3^2}$

$\Rightarrow \frac{48}{Area\ of\ larger\ triangle} = \frac{4}{9}$

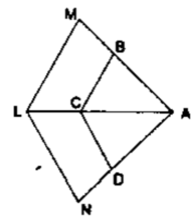
$\Rightarrow Area\ of\ larger\ triangle = \frac{48\times9}{4} = 108\ cm^2$

12. In the given figure, LM ∥ CB and LN ∥ CD. Prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

Sol:

LM || CB and LN || CD

Therefore, applying Thales' theorem, we have:



$\frac{AB}{AM} = \frac{AC}{AL}$ and $\frac{AD}{AN} = \frac{AC}{AL}$

$\Longrightarrow \frac{AB}{AM} = \frac{AD}{AN}$

$\therefore \frac{AM}{AB} = \frac{AN}{AD}$

This completes the proof.

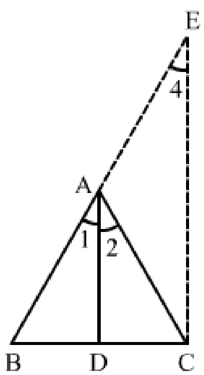
13. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Sol:

Let the triangle be ABC with AD as the bisector of ∠A which meets BC at D.

We have to prove:

$\frac{BD}{DC} = \frac{AB}{AC}$



Draw CE || DA, meeting BA produced at E.

CE || DA

Therefore,

$\angle 2 = \angle 3 \quad (Alternate\ angles)$

$and\ \angle 1 = \angle 4 \qquad (Corresponding\ angles)$

But,

$\angle 1 = \angle 2$

Therefore,

$\angle 3 = \angle 4$

$\Longrightarrow$ AE = AC

In ΔBCE, DA || CE.

Applying Thales' theorem, we gave:

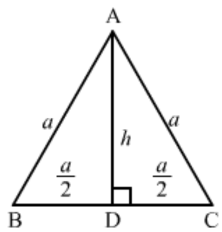
$\frac{BD}{DC} = \frac{AB}{AE}$

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$

This completes the proof.

14. In an equilateral triangle with side a, prove that area = $\frac{\sqrt{3}}{4}\ a^2$.

Sol:



Let ABC be the equilateral triangle with each side equal to a.
Let AD be the altitude from A, meeting BC at D.
Therefore, D is the midpoint of BC.
Let AD be h.
Applying Pythagoras theorem in right-angled ABD, we have:

$AB^2 \ = AD^2 \ + BD^2$

$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$

$\Rightarrow h = \frac{\sqrt{3}}{2}\ a$

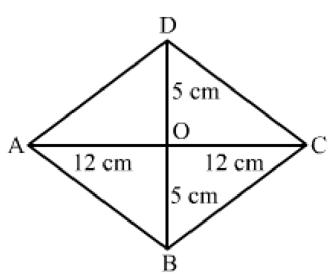
Therefore,

$Area\ of\ triangle\ ABC = \frac{1}{2} \times base \times height = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}\ a$

This completes the proof.

15. Find the length of each side of a rhombus whose diagonals are 24cm and 10cm long.

Sol:

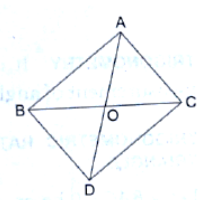


Let ABCD be the rhombus with diagonals AC and BD intersecting each other at O.
We know that the diagonals of a rhombus bisect each other at right angles.
∴ If AC – 24 cm and BD = 10 cm, AO = 12 cm and BO = 5 cm
Applying Pythagoras theorem in right-angled triangle AOB, we get:

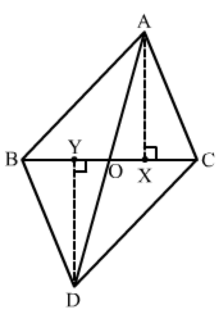
$AB^2 = AO^2 + BO^2 = 12^2 + 5^2 = 144 + 25 = 169$
AB = 13 cm
Hence, the length of each side of the given rhombus is 13 cm.

16. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.
Sol:
Let the two triangles be ABC and PQR.
We have:
ΔABC ~ ΔPQR,
Here,
BC = a, AC = b and AB = c
PQ = r, PR = q and QR = p
We have to prove:
$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$
ΔABC ~ ΔPQR; therefore, their corresponding sides will be proportional.
$\Longrightarrow \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \quad (say) \quad ...(i)$
$\Longrightarrow a = kp, b = kq \; and \; c = kr$
$\therefore \frac{Premieter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r} = k \quad ...(ii)$
$From\ (i) and\ (ii), we\ get$:
$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR}$
This completes the proof.

17. In the given figure, ΔABC and ΔDBC have the same base BC. If AD and BC intersect at O, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Sol:



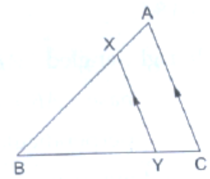
$Construction : Draw\ AX \perp CO\ and\ DY \perp BO.$
As,
$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times AX \times BC}{\frac{1}{2} \times DY \times BC}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AX}{DY} \dots (i)$

$In\ \Delta\ ABC\ and\ \Delta DBC, \angle AXY = \angle DYO = 90^0\ (BY\ constructin) \angle AOX =$
$\angle DOY\ (Vertically\ opposite\ anges) \therefore \Delta AXO \sim \Delta DYO (BY\ AA\ criterion) \therefore \frac{AX}{DY}$
$= \frac{AO}{DO}\ (Thales' stheorem) \dots\ (ii) From\ (i) and\ (ii), we\ have: \frac{ar(\Delta ABC)}{ar(\Delta DBC)}$
$= \frac{AX}{DY} = \frac{AO}{DO}\ or, \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

This completes the proof.

18. In the given figure, XY ∥ AC and XY divides ΔABC into two regions, equal in area. Show that $\frac{AX}{AB} = \frac{(2-\sqrt{2})}{2}$.



Sol:

In Δ ABC and ΔBXY, we have:

$\angle B = \angle B$

$\angle BXY = \angle BAC \quad (Corresponding\ angles)$

$Thus, \Delta ABC - \Delta BXY \quad (AA\ criterion\ )$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{AB^2}{BX^2} = \frac{AB^2}{(AB-AX)^2} \quad \dots (i)$

$Also, \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{2}{1} \{\therefore ar\ (\Delta BXY) = ar(trapezium\ AXYV)\} \dots (ii)$

$From\ (i) and\ (ii), we\ have:$

$\frac{AB^2}{(AB-AX)^2} = \frac{2}{1}$

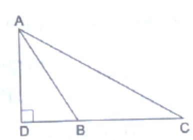
$\Rightarrow \frac{AB}{(AB-AX)} = \sqrt{2}$

$\Rightarrow \frac{(AB-AX)}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{AX}{AB} = \frac{1}{\sqrt{2}}$

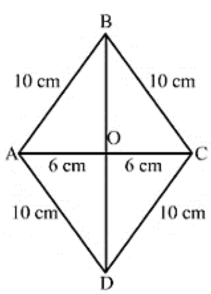
$\Rightarrow \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$

19. In the given figure, ΔABC is an obtuse triangle, obtuse-angled at B. If AD⊥CB, prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



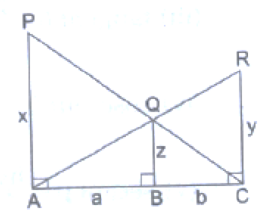
Sol:

Applying Pythagoras theorem in right-angled triangle ADC, we get:



$AC^2 = AD^2 + DC^2$
$\Rightarrow AC^2 - DC^2 = AD^2$
$\Rightarrow AD^2 = AC^2 - DB^2$ …(1)
Applying Pythagoras theorem in right-angled triangle ADB, we get:
$AB^2 = AD^2 + DB^2$
$\Rightarrow AB^2 - DB^2 = AD^2$
$\Rightarrow AD^2 = AB^2 - DB^2$ …(2)
$From\ equation\ (1) and\ (2), we\ have:$
$AC^2 - DC^2 = AB^2 - DB^2$
$\Rightarrow AC^2 = AB^2 + DC^2 - DB^2$
$\Rightarrow AC^2 = AB^2 + (DB + BC)^2 - DB^2 \quad (\because DB + BC = DC)$
$\Rightarrow AC^2 = AB^2 + DB^2 + BC^2 + 2DB.BC - DB^2$
$\Rightarrow AC^2 = AB^2 + BC^2 + 2BC.BD$
This completes the proof.

20. In the given figure, each one of PA, QB and RC is perpendicular to AC. If AP = x, QB = z, RC = y, AB = a and BC = b, show that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



Sol:
In Δ PAC and ΔQBC, we have:
$\angle A = \angle B \quad (Both\ angles\ are\ 90^0)$
$\angle P = \angle Q \quad (Corresponding\ angles)$
And
$\angle C = \angle C \quad (common\ angles)$
$Therefore, \Delta PAC \sim \Delta QBC$
$\frac{AP}{BQ} = \frac{AC}{BC}$
$\Rightarrow \frac{x}{2} = \frac{a+b}{b}$
$\Rightarrow a + b = \frac{ay}{z}$ ... (1)
In Δ RCA and ΔQBA, we have:
$\angle C = \angle B \quad (Both\ angles\ are\ 90^0)$
$\angle R = \angle Q \quad (Corresponding\ angles)$
And
$\angle A = \angle A \quad (common\ angles)$
$Therefore, \Delta RCA \sim \Delta QBA$
$\frac{RC}{BQ} = \frac{AC}{AB}$
$\Rightarrow \frac{y}{z} = \frac{a+b}{a}$
$\Rightarrow a + b = \frac{ay}{z}$ ... (2)
From equation (1) and (2), we have:

$\frac{bx}{z} = \frac{ay}{z}$

$\Longrightarrow \mathrm{bx} = \mathrm{ay}$

$\Longrightarrow \frac{a}{b} = \frac{x}{y}$ ... (3)

Also,

$\frac{x}{z} = \frac{a+b}{b}$

$\Longrightarrow \frac{x}{z} = \frac{a}{b} + 1$

$Using\ the\ value\ of\ \frac{a}{b}\ from\ equation\ (3), we\ have:$

$\Longrightarrow \frac{x}{z} = \frac{x}{y} + 1$

$Dividing\ both\ sides\ by\ x, we\ get:$

$\frac{1}{z} = \frac{1}{y} + \frac{1}{x}$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

$This\ completes\ the\ proof.$