8. Linear Equations

Exercise 8A

1. Question

Solve:

$8x + 3 = 27 + 2x$

Answer

$8x + 3 = 27 + 2x$

By transposition,

$\Rightarrow 8x - 2x = 27 - 3$

$\Rightarrow 6x = 24$

$\Rightarrow x = 4$

2. Question

Solve:

$5x + 7 = 2x - 8$

Answer

$5x + 7 = 2x - 8$

By transposition,

$\Rightarrow 5x - 2x = -8 - 7$

$\Rightarrow 3x = -15$

$\Rightarrow x = -5$

3. Question

Solve:

$2z - 1 = 14 - z$

Answer

$2z - 1 = 14 - z$

By transposition,

$\Rightarrow 2z + z = 14 + 1$

⇒ 3z = 15

Dividing by 3, on both the sides we get,

$$\Rightarrow \frac{3z}{3} = \frac{15}{3}$$

⇒ z = 5

4. Question

Solve:

9x + 5 = 4(x - 2) + 8

Answer

9x + 5 = 4(x - 2) + 8

By transposition,

⇒ 9x + 5 = 4x - 8 + 8

⇒ 9x - 4x = - 5 + 0

⇒ 5x = - 5

⇒ x = - 1

5. Question

Solve:

$$\frac{7y}{5} = y - 4$$

Answer

$$\frac{7y}{5} = y - 4$$

By cross multiplication

$$\Rightarrow \frac{7y}{5} - y = -4$$

Taking LCM of 5 and 1 = 5 on LHS

$$\Rightarrow \frac{7y - 5y}{5} = -4$$

⇒ 2y = - 5 × 4

⇒ y = - 5 × 2 = - 10

6. Question

Solve:

$3x + \frac{2}{3} = 2x + 1$

Answer

$3x + \frac{2}{3} = 2x + 1$

By cross multiplication

$\Rightarrow 3x - 2x = -\frac{2}{3} + 1$

Taking LCM of 3 and 1 = 3 on RHS

$\Rightarrow x = \frac{-2+3}{3} = \frac{1}{3}$

7. Question

Solve:

15(y - 4) - 2(y - 9) + 5(y + 6) = 0

Answer

15(y - 4) - 2(y - 9) + 5(y + 6) = 0

Opening the brackets and multiplying, we get,

⇒ 15y - 60 - 2y + 18 + 5y + 30 = 0

⇒ 15y - 2y + 5y - 60 + 18 + 30 = 0

⇒ 18y = 12

$\Rightarrow y = \frac{12}{18} = \frac{2}{3}$

8. Question

Solve:

3(5x - 7) - 2(9x - 11) = 4(8x - 13) - 17

Answer

3(5x - 7) - 2(9x - 11) = 4(8x - 13) - 17

Multiplying we get,

⇒ 15x - 21 - 18x + 22 = 32x - 52 – 17

Solving, we get

(15x - 18x) + (22 - 21) = 32x - (52 + 17)

⇒ - 3x + 1 = 32x - 69

⇒ 35x = 70

⇒ x = 2

9. Question

Solve:

$$\frac{x-5}{2}-\frac{x-3}{5}=\frac{1}{2}$$

Answer

$$\frac{x-5}{2}-\frac{x-3}{5}=\frac{1}{2}$$

Taking LCM of 2 and 5 = 10 on LHS

$$\Rightarrow \frac{5(x-5)-2(x-3)}{10}=\frac{1}{2}$$

By cross multiplication

⇒ 5x - 25 - 2x + 6 = 10/2

⇒ 3x = 5 + 19

$$\Rightarrow x=\frac{24}{3}=8$$

10. Question

Solve:

$$\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$$

Answer

$$\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$$

Taking LCM of 3 and 4 = 12 on LHS and LCM of 3 and 1 = 3 on RHS

$$\Rightarrow \frac{3(3t-2)-4(2t+3)}{12}=\frac{2-3t}{3}$$

By cross multiplication

$$\Rightarrow \frac{3(3t-2)-4(2t+3)}{4}=2-3t$$

⇒ 9t - 6 - 8t - 12 = 4(2 - 3t)

⇒ 9t - 6 - 8t - 12 = 8 - 12t

⇒ t - 18 = 8 - 12t

⇒ t + 12t = 8 + 18

$\Rightarrow \ t = \frac{26}{13} = 2$

11. Question

Solve:

$$\frac{2x+7}{5} - \frac{3x+11}{2} = \frac{2x+8}{3} - 5$$

Answer

$$\frac{2x+7}{5} - \frac{3x+11}{2} = \frac{2x+8}{3} - 5$$

Taking LCM of 5 and 2 = 10 on LHS and LCM of 3 and 1 = 3 on RHS

$$\frac{2(2x\ +\ 7) - 5(3x\ +\ 11)}{10} = \frac{2x\ +\ 8 - 15}{3}$$

By cross multiplication

⇒ 3(4x + 14 - 15x - 55) = 10(2x - 7)

⇒ 3(- 11x - 41) = 20x - 70

⇒ - 33x - 20x = 123 - 70

$\Rightarrow \ x = -\frac{53}{53} = -1$

12. Question

Solve:

$$\frac{5x-4}{6} = 4x+1-\frac{3x+10}{2}$$

Answer

$$\frac{5x-4}{6} = 4x+1-\frac{3x+10}{2}$$

Taking LCM of 1 and 2 = 2 on RHS

$$\frac{5x - 4 - 6(4x + 1) + 3(3x + 10)}{6} = 0$$

By cross multiplication

⇒ 5x - 4 - 24x - 6 + 9x + 30 = 0

⇒ - 10x = - 20

$\Rightarrow x = \frac{20}{10} = 2$

13. Question

Solve:

$$5x - \frac{1}{3}(x+1) = 6\left(x + \frac{1}{30}\right)$$

Answer

$$5x - \frac{1}{3}(x+1) = 6\left(x + \frac{1}{30}\right)$$

Taking LCM on both the sides

$\frac{15x-(x+1)}{3} = \frac{6(30x+1)}{30}$

By cross multiplication

⇒ 10(14x - 1) = 6(30x + 1)

⇒ 140x - 180x = 6 + 10

⇒ - 40x = 16

$\Rightarrow x = -\frac{2}{5}$

14. Question

Solve:

$$4 - \frac{2(z-4)}{3} = \frac{1}{2}(2z+5)$$

Answer

$$4 - \frac{2(z-4)}{3} = \frac{1}{2}(2z+5)$$

Taking LCM of 1 and 3 on LHS = 3

$\frac{12-2(z-4)}{3} = \frac{2z+5}{2}$

By cross multiplication

⇒ 2(12 - 2z + 8) = 3(2z + 5)

⇒ 40 - 4z = 6z + 15

⇒ - 10z = - 25

$\Rightarrow z = \frac{25}{10} = \frac{5}{2}$

15. Question

Solve:

$$\frac{3(y-5)}{4} - 4y = 3 - \frac{(y-3)}{2}$$

Answer

$$\frac{3(y-5)}{4} - 4y = 3 - \frac{(y-3)}{2}$$

Taking LCM of 4 and 1 on LHS = 4 and 1 and 2 on RHS = 2

$\frac{3(y-5)-16y}{4} = \frac{6-y+3}{2}$

By cross multiplication

⇒ 3y - 15 - 16y = 2(9 - y)

⇒ - 13y + 2y = 18 + 15

⇒ - 11y = 33

⇒ y = - 3

16. Question

Solve:

$$\frac{8x-3}{3x} = 2$$

Answer

$$\frac{8x-3}{3x} = 2$$

By cross multiplication

8x - 3 = 6x

⇒ 2x = 3

$\Rightarrow x = \frac{3}{2}$

17. Question

Solve:

$$\frac{9x}{7-6x} = 15$$

Answer

$$\frac{9x}{7-6x} = 15$$

By cross multiplication

9x = 15(7 - 6x)

⇒ 9x + 90x = 105

⇒ 99x = 105

$\Rightarrow x = \frac{35}{33}$

18. Question

Solve:

$$\frac{3x}{5x+2} = -4$$

Answer

$$\frac{3x}{5x+2} = -4$$

By cross multiplication

3x = - 4 (5x + 2)

⇒ 3x = -20x - 8

⇒ 3x + 20x = -8

⇒ 23x = -8

$\Rightarrow x = \frac{-8}{23}$

19. Question

Solve:

$$\frac{6y-5}{2y}=\frac{7}{9}$$

Answer

$$\frac{6y-5}{2y}=\frac{7}{9}$$

By cross multiplication

9 (6y – 5) = 7 × 2y

⇒ 54y – 45 = 14y

⇒ 54y – 14y = 45

⇒ 40y = 45

$\Rightarrow y = \frac{45}{40}$

Or $y = \frac{9}{8}$

20. Question

Solve:

$$\frac{2-9z}{17-4z}=\frac{4}{5}$$

Answer

$$\frac{2-9z}{17-4z}=\frac{4}{5}$$

By cross multiplication

5(2 - 9z) = 4(17 - 4z)

⇒ 10 - 45z = 68 - 16z

⇒ - 45z + 16z = 68 - 10

$\Rightarrow x = -\frac{58}{29} = -2$

21. Question

Solve:

$$\frac{4x+7}{9-3x}=\frac{1}{4}$$

Answer

$$\frac{4x+7}{9-3x}=\frac{1}{4}$$

By cross multiplication

4(4x + 7) = (9 - 3x)

⇒ 16x + 28 = 9 - 3x

⇒ 19x = - 19

⇒ x = - 1

22. Question

Solve:

$$\frac{7y+4}{y+2}=-\frac{4}{3}$$

Answer

$$\frac{7y+4}{y+2}=-\frac{4}{3}$$

By cross multiplication

3(7y + 4) = - 4(y + 2)

⇒ 21y + 12 = - 4y - 8

⇒ 25y = - 20

$\Rightarrow x=-\frac{4}{5}$

23. Question

Solve:

$$\frac{15(2-y)-5(y+6)=10}{1-3y}$$

Answer

$$\frac{15(2 - y) - 5(y + 6) = 10}{1 - 3y}$$

By cross multiplication

30 - 15y - 5y - 30 = 10 - 30y

⇒ - 20y + 30y = 10

⇒ 10y = 10

⇒ x = 1

24. Question

Solve:

$$\frac{2x-(7-5x)}{9x-(3+4x)}=\frac{7}{6}$$

Answer

$$\frac{2x-(7-5x)}{9x-(3+4x)}=\frac{7}{6}$$

By cross multiplication

6(2x - 7 + 5x) = 7(9x - 3 - 4x)

⇒ 42x - 42 = 35x - 21

⇒ 7x = 21

⇒ x = 3

25. Question

Solve:

$$m-\frac{(m-1)}{2}=1-\frac{(m-2)}{3}$$

Answer

$$m-\frac{(m-1)}{2}=1-\frac{(m-2)}{3}$$

Taking LCM of 1 and 2 on LHS = 2 and 1 and 3 on RHS = 3

$$\frac{2m-m+1}{2} = \frac{3-m+2}{3}$$

Taking transposition

⇒ 3(m + 1) = 2(5 - m)

⇒ 3m + 3 = 10 - 2m

⇒ 5m = 7

⇒ m = 7/5

26. Question

Solve:

$$\frac{3x+5}{4x+2}=\frac{3x+4}{4x+7}$$

Answer

$$\frac{3x+5}{4x+2}=\frac{3x+4}{4x+7}$$

Taking transposition

(4x + 7)(3x + 5) = (3x + 4)(4x + 2)

⇒ $12x^2$ + 20x + 21x + 35 = $12x^2$ + 6x + 16x + 8

⇒ $12x^2$ - $12x^2$ + 41x - 22x = 8 - 35

⇒ 19x = - 27

$$\Rightarrow x=-\frac{27}{19}$$

27. Question

Solve:

$$\frac{9x-7}{3x+5}=\frac{3x-4}{x+6}$$

Answer

$$\frac{9x-7}{3x+5}=\frac{3x-4}{x+6}$$

By cross multiplication

(9x - 7)(x + 6) = (3x - 4)(3x + 5)

⇒ $9x^2$ + 54x - 7x - 42 = $9x^2$ + 15x - 12x - 20

⇒ $9x^2$ - $9x^2$ + 47x - 3x = - 20 + 42

⇒ 44x = 22

$\Rightarrow x = \frac{1}{2}$

28. Question

Solve:

$$\frac{2-7x}{1-5x} = \frac{3+7x}{4+5x}$$

Answer

$$\frac{2-7x}{1-5x} = \frac{3+7x}{4+5x}$$

By cross multiplication

$(2 - 7x)(4 + 5x) = (3 + 7x)(1 - 5x)$

$\Rightarrow 8 + 10x - 28x - 35x^2 = 3 - 15x + 7x - 35x^2$

$\Rightarrow - 35x^2 - 35x^2 - 18x + 8x = 3 - 8$

$\Rightarrow - 10x = - 5$

$\Rightarrow x = \frac{1}{2}$

Exercise 8B

1. Question

Two numbers are in the ratio 8:3. If the sum of the numbers is 143, find the numbers.

Answer

Since the numbers are in the ratio 8:3 so Let the numbers be 8x and 3x

According to the question

$8x + 3x = 143$

$\Rightarrow 11x = 143$

$\Rightarrow x = 13$

So the numbers are $8x = 8 \times 13 = 104$ and $3x = 3 \times 13 = 39$

2. Question

$\frac{2}{3}$ of a number is 20 less than the original number. Find the number.

Answer

Let the numbers be x

According to the question

$$\frac{2}{3}x + 20 = x$$

By cross multiplication

$$\Rightarrow x - \frac{2}{3}x = 20$$

Taking LCM of 1 and 3 on LHS = 3

$$\Rightarrow \frac{3x-2x}{3} = 20$$

⇒ x = 60

So the number 60

3. Question

Four - fifths of a number is 10 more than two - thirds of the number. Find the number.

Answer

Let the numbers be x

According to the question

$$\frac{4}{5}x - 10 = \frac{2}{3}x$$

$$\Rightarrow \frac{4}{5}x - \frac{2}{3}x = 10$$

$$\Rightarrow \frac{12x-10x}{15} = 10$$

⇒ 2x = 10 × 15 = 150⇒ x = 75

So the number is 75.

4. Question

Twenty - four is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. Find each part.

Answer

Let the two parts be x and (24 - x)

According to the question

7x + 5(24 - x) = 146

By cross multiplication

⇒ 2x = 146 - 120

⇒ 2x = 26

⇒ x = 13

So the parts are 13 and (24 - 13) = 11

5. Question

Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.

Answer

Let the numbers be x

According to the question

$$\frac{1}{5}x + 5 = \frac{1}{4}x - 5$$

Taking LCM of 5 and 1 on LHS = 5 and 4 and 1 on RHS = 1

$$\Rightarrow \frac{1}{5}x - \frac{1}{4}x = -10$$

$$\Rightarrow \frac{4x-5x}{20} = -10$$

$$\Rightarrow x = 200$$

So the number 200

6. Question

Three numbers are in the ratio of 4 : 5 : 6. If the sum of the largest and the smallest equals the sum of the third and 55, find the numbers.

Answer

Let the numbers be 4x,5x and 6x

According to the question

6x + 4x = 5x + 55

By cross multiplication

⇒ 10x - 5x = 55

⇒ 5x = 55

⇒ x = 11

So the numbers are 4x = 4 × 11 = 44, 5x = 5 × 11 = 55 and 6x = 6 × 11 = 66

7. Question

If 10 be added to four times a certain number, the result is 5 less than five times the number. Find the number.

Answer

Let the number be x

According to the question

10 + 4x = 5x - 5 [10 is added to 4 times the number, 5 less than 5 times the number]

By transposing

⇒ 5x - 4x = 10 + 5

⇒ x = 15

So the number is 15

8. Question

Two numbers are such that the ratio between them is 3 : 5. If each is increased by 10, the ratio between the new numbers so formed is 5:7. Find the original numbers.

Answer

Let the numbers be 3x and 5x

According to the question

$$\frac{3x + 10}{5x + 10} = \frac{5}{7}$$

By cross multiplication

⇒ 7(3x + 10) = 5(5x + 10)

⇒ 21x + 70 = 25x + 50

⇒ 4x = 20

⇒ x = 5

So the numbers are 3x = 3 × 5 = 15 and 5x = 5 × 5 = 25

9. Question

Find three consecutive odd numbers whose sum is 147. Hint. Let the required numbers be (2x + 1), (2x + 3) and (2x + 5).

Answer

Let the numbers be (2x + 1), (2x + 3) and (2x + 5)

According to the question

2x + 1 + 2x + 3 + 2x + 5 = 147

By cross multiplication

⇒ 6x + 9 = 147

⇒ 6x = 147 - 9

$\Rightarrow x = \frac{138}{6} = 23$

So the numbers are (2x + 1) = 47, (2x + 3) = 49 and (2x + 5) = 51

10. Question

Find three consecutive even numbers whose sum is 234.

Hint. Let the required numbers be 2x, (2x + 2) and (2x + 4).

Answer

Let the numbers be 2x, (2x + 2) and (2x + 4)

According to the question

By cross multiplication

2x + 2x + 2 + 2x + 4 = 234

⇒ 6x + 6 = 234

⇒ 6x = 228

$\Rightarrow x = \frac{228}{6} = 38$

So the numbers are 2x = 76, (2x + 2) = 78 and (2x + 4) = 80

11. Question

The sum of the digits of a two - digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number. Check your solution.

Answer

Let the digits be x and y so the number = (10x + y), on reversing the digits number = (10y + x)

According to the question

x + y = 12 .....(A)

And 10y + x - 10x - y = 54

⇒ 9y - 9x = 54

⇒ y - x = 54/9 = 6

⇒ y = 6 + x

Putting in (A) we get

x + 6 + x = 12

⇒ 2x = 6

⇒ x = 3

⇒ y = 6 + x = 9

So the number is 39

Checking the answer:

Digit sum = 3 + 9 = 12

Reversing the digits number becomes = 93

93 - 39 = 54

Hence, verified.

12. Question

The digit in the tens place of a two - digit number is three times that in the units place. If the digits are reversed, the new number will be 36 less than the original number. Find the original number. Check your solution.

Answer

Let the unit digit be y and tens digit is x so numbers = (10x + y), on reversing the digits number = (10y + x)

According to the question

x = 3y - (A)

And 10y + x + 36 = 10x + y

⇒ 10y - y + 36 = 10x - x

⇒ 9y - 9x = - 36

Putting (A) we get

9y – 27y = - 36

⇒ - 18y = - 36

⇒ y = 2

⇒ x = 3y = 6

So the number is 62

Checking the answer:

Digit at tens place = 6 = 3 × digit at unit place 6

Reversing the digits number becomes = 26

26 + 36 = 62

Hence, verified.

13. Question

The denominator of a rational number is greater than its numerator by 7. If the numerator is increased by 17 and the denominator decreased by 6, the new number becomes 2. Find the original number.

Answer

Let the rational numbers be $\frac{x}{y}$

According to the question

y = x + 7x = y - 7....(1)

And $\frac{x + 17}{y - 6} = 2$

Putting (1), we get,

$\frac{y - 7 + 17}{y - 6} = 2$

By cross multiplication

⇒ y - 7 + 17 = 2(y - 6)

⇒ y + 10 = 2y - 12

⇒ 2y - y = 10 + 12

⇒ y = 22

⇒ x = y - 7 = 22 - 7 = 15

So the number is $\frac{15}{22}$

14. Question

In a fraction, twice the numerator is 2 more than the denominator. If 3 is added to the numerator and to the denominator, the new fraction is $\frac{2}{3}$. Find the original fraction.

Answer

Let the numerator is x.

Now, according to question twice the numerator (2x) is 2 more than denominator. Then denominator = 2x - 2The fraction $= \frac{x}{2x - 2}$

Now, the numerator is increased by 3, numerator becomes x + 3

The denominator is increased by 3, denominator becomes (2x - 2 + 3) = 2x + 1Therefore, the new fraction $= \frac{x + 3}{2x + 1}$

According to question,

$$\frac{x + 3}{2x + 1} = \frac{2}{3}$$

Cross-multiplying we get,

3(x + 3) = 2(2x + 1)3x + 9 = 4x + 23x - 4x = 2 - 9-x = -7x = 7

Now, putting the value of x, we get that Original fraction

$$= \frac{x}{2x-2} = \frac{7}{2\times 7-2} = \frac{7}{12}$$

Hence, the original fraction is 7/12.

15. Question

The length of a rectangle exceeds its breadth by 7 cm. If the length is decreased by 4 cm and the breadth is increased by 3 cm, the area of the new rectangle is the same as the area of the original rectangle. Find the length and the breadth of the original rectangle.

Answer

To Find: Length and Breadth of the original rectangle'Let the length and breadth of a rectangle be l cm and b cm

According to the question

Breadth of rectangle is 7 less than the length of the rectangle,

l - 7 = b .....(1)

Area of a rectangle = (l × b)

Now length of the rectangle is decrease by 4, and breadth increased by 3,

Area of new rectangle = (l - 4)(b + 3)

Area of new rectangle = Area of Old rectangle(l - 4)(b + 3) = lb

Now

Putting the value of b from equation 1, we get,

(l - 4)(l - 7 + 3) = l(l - 7)

(l - 4)(l - 4) = l(l - 7)Opening the brackets, we get,

⇒ l^2 - 4l - 4l + 16 = l^2 - 7l

⇒ l^2 - 8l + 16 = l^2 - 7l

⇒ - l = - 16

⇒ l = 16 cm

b = l - 7 = 16 - 7 = 9 cm

Hence, length and breadth of original rectangle are 16 cm and 9 cm.

16. Question

The width of a rectangle is two - thirds its length. If the perimeter is 180 metres, find the dimensions of the rectangle.

Answer

Let the length and breadth of a rectangle be l m and b m

According to the question

$b = \frac{2}{3}l$ (A)

Perimeter of a rectangle = 2(l + b)

And 2(l + b) = 180

Putting (A) we get

$2\left(l + \frac{2}{3}l\right) = 180$

$\Rightarrow \frac{3l+2l}{3} = 90$

⇒ 5l = 90 × 3

⇒ l = 54 m

⇒ b = 2/3 (54) = 36m

17. Question

An altitude of a triangle is five - thirds the length of its corresponding base. If the altitude be increased by 4 cm and the base decreased by 2 cm, the area of the triangle remains the same. Find the base and the altitude of the triangle.

Answer

Let the length of the altitude and base of a triangle be l cm and b cm

According to the question

$l = \frac{5}{3}b$ (A)

Area of a triangle $= \frac{1}{2}(\text{base} \times \text{length of the altitude})$

And $\frac{1}{2}(l + 4)(b-2) = \frac{1}{2}l \times b$

Putting (A) we get

$\Rightarrow \left(\frac{5}{3}b + 4\right)(b-2) = \frac{5}{3}b \times b$

Taking LCM of 3 and 1 = 3 on LHS

$\Rightarrow \frac{5}{3}b^2 + 4b - \frac{10}{3}b - 8 = \frac{5}{3}b^2$

$\Rightarrow \frac{12b-10b}{3} = 8$

⇒ 2b = 24 cm

$$b = 12\,cm \text{ and } l = \frac{5}{3}b = 20cm$$

18. Question

Two angles of a triangle are in the ratio 4: 5. If the sum of these angles is equal to the third angle, find the angles of the triangle.

Answer

Let the given two angles of a triangle be 4x and 5x

According to the question

3^{rd} angle = 4x + 5x = 9x

Using angle sum property of a triangle

4x + 5x + 9x = 180°

⇒ 18x = 180°

⇒ x = 10

So, the angles of the given triangle are:

4x = 40°, 5x = 50° and 9x = 90°

19. Question

A steamer goes downstream from one port to another in 9 hours. It covers the same distance upstream in 10 hours. If the speed of the stream be 1 km/h, find the speed of the steamer in still water and the distance between the ports.

Answer

Let the speed of the steamer in still water be x km/h

Speed in downstream = x + 1, Speed in upstream = x - 1

Distance = speed × time

According to the question

9(x + 1) = 10(x - 1)

By cross multiplication

⇒ 9x + 9 = 10x - 10

⇒ x = 19 km/h

Distance between the ports = 9(19 + 1) = 180 km

20. Question

The distance between two stations is 300 km. Two motorcyclists start simultaneously from these stations and move towards each other. The speed of one of them is 7 km/h more than that of the other. If the distance between them after 2 hours of their start is 34 km, find the speed of each motorcyclist. Check your solution.

Answer

Let the speed of motorcyclists be x km/h and y km/h

According to the question

x + 7 = y (A)

And 2y + 2x + 34 = 300

Putting (A) we get

⇒ 2(x + 7) + 2x + 34 = 300

⇒ 2x + 14 + 2x = 300 - 34

⇒ 4x = 266 - 14

$$\Rightarrow x = \frac{252}{4} = 63\ \text{km/h}$$

$$\Rightarrow y = x + 7 = 63\frac{\text{km}}{\text{h}} + 7 = 70\ \text{km/h}$$

Checking the answer:

2(70) + 2(63) + 34 = 140 + 126 + 34 = 300 = Distance between them

Hence, verified .

21. Question

Divide 150 into three parts such that the second number is five - sixths the first and the third number is four - fifths the second.

Answer

Let the first part be x of 150

According to the question second part is $\frac{5}{6}x$

And the third part is $\frac{4}{5}\left(\frac{5}{6}x\right)$

Adding all of them

$$x + \frac{5}{6}x + \frac{20}{30}x = 150$$

Taking LCM of 6 and 30 = 30

$\Rightarrow \frac{30x + 25x + 20x}{30} = 150$

$\Rightarrow 75x = 150 \times 30$

$\Rightarrow x = \frac{4500}{75} = 60$

Second part = $\frac{5}{6}x = 50$

Third part = $\frac{4}{5}\left(\frac{5}{6}x\right) = 40$

22. Question

Divide 4500 into two parts such that 5% of the first part is equal to 10% of the second part.

Answer

Let the first part and second part be x and y respectively

According to the question

$\frac{5}{100}x = \frac{10}{100}y$

$\Rightarrow y = \frac{5}{10}x = \frac{1}{2}x$

Adding them

$x + \frac{1}{2}x = 4500$

$\Rightarrow \frac{3x}{2} = 4500$

$\Rightarrow 3x = 4500 \times 2$

$\Rightarrow x = \frac{9000}{3} = 3000$

Second part = $\frac{1}{2}x = 1500$

23. Question

Rakhi's mother is four times as old as Rakhi. After 5 years, her mother will be three times as old as she will be then. Find their present ages.

Answer

Let the age of Rakhi and Rakhi’s mother be x and 4x respectively

According to the question

(4x + 5) = 3(x + 5)

⇒ 4x - 3x = 15 - 5

So, Rakhi' age = x = 10 Years

and Rakhi' s mother is 4x = 40 years

24. Question

Monu's father is 26 years younger than Monu's grandfather and 29 years older than Monu. The sum of the ages of all the three is 135 years. What is the age of each one of them?

Answer

Let the age of Monu’s father be x years

According to the question

Age of Monu = x – 29 years

And age of Monu’s grandfather = x + 26

Adding all of these,

x + x - 29 + x + 26 = 135

⇒ 3x = 135 + 3

$$\Rightarrow x = \frac{138}{3} = 46$$

So, Monu' s father is 46 Years

and Monu is 46 - 29 = 17 years

Monu’s grandfather is 46 + 26 = 72 years

25. Question

A man is 10 times older than his grandson. He is also 54 years older than him. Find their present ages.

Answer

Let the age of man be x years

According to the question

Age of his grandson = $\frac{1}{10}x$

Also,

$$x - \frac{1}{10}x = 54$$

$$\text{taking LCM of } 1 \text{ and } 10 = 10$$

⇒ 9x = 540

$$\Rightarrow x = \frac{540}{9} = 60$$

So, Man is 60 Years

$$\text{and Grandson is } \frac{1}{10}x = 6 \text{ years}$$

26. Question

The difference between the ages of two cousins is 10 years. 15 years ago, if the elder one was twice as old as the younger one, find their present ages.

Answer

Let the ages of cousins be x years and x - 10 years

According to the question

x - 15 = 2(x - 10 - 15)

By cross multiplication

⇒ x - 15 = 2x - 50

⇒ x = 35

So, cousins are 35 Years and 25 years in age

27. Question

Half of a herd of deer are grazing in the field and three - fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

Answer

Let the number of deer in the herd be x.

Number of those who are grazing = $\frac{x}{2}$

Remaining = $\frac{x}{2}$

Number of those who are playing =

$$\left(\frac{3}{4}\times\frac{x}{2}\right)=\frac{3x}{8}$$

$$\therefore \frac{x}{2}+\frac{3x}{8}+9=x.$$

$$\Rightarrow \frac{4x + 3x + 72}{8} = x$$

⇒ 7x + 72 = 8xx = 72

Exercise 8C

1. Question

If 2x - 3 = x + 2, then x = ?

A. 1

B. 3

C. 5

D. 7

Answer

2x - 3 = x + 2

By transposing x and 3

⇒ 2x - x = 3 + 2

⇒ x = 5

2. Question

If $5x+\frac{7}{2}=\frac{3}{2}x-14$, then x = ?

A. 5

B. - 5

C. 6

D. - 6

Answer

$$5x+\frac{7}{2}=\frac{3}{2}x-14$$

By cross multiplication

$$\Rightarrow 5x-\frac{3}{2}x = -14-\frac{7}{2}$$

Taking LCM of 1and2 = 2

$\Rightarrow \frac{10x - 3x}{2} = \frac{-28 - 7}{2}$

$\Rightarrow 7x = -35$

$\Rightarrow x = -5$

3. Question

If $z = \frac{4}{5}(z + 10)$, then z = ?

A. 40

B. 20

C. 10

D. 60

Answer

$z = \frac{4}{5}(z + 10)$

By cross multiplication, $z - \frac{4}{5}z = \frac{40}{5}$

Taking LCM of 1 and 5 = 5

$\Rightarrow \frac{z}{5} = \frac{40}{5}$

$\Rightarrow x = 40$

4. Question

If $3m = 5m - \frac{8}{5}$, then m = ?

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{4}{5}$

D. $\frac{1}{5}$

Answer

$$3m = 5m - \frac{8}{5}$$

By cross multiplication, 5m-3m = 8/5

$$\Rightarrow 2m = \frac{8}{5}$$

$$\Rightarrow m = \frac{4}{5}$$

5. Question

If 5t - 3 = 3t - 5, then t = ?

A. 1

B. - 1

C. 2

D. – 2

Answer

5t - 3 = 3t - 5 .... (1)

By transposition of -3 on RHS we get,

5t = 3t - 5 + 35t = 3t - 2By transposition of 3t on LHS we get,5t - 3t = - 2

⇒ 2t = - 2

⇒ t = - 1

Check:

Put the value of t in (1),LHS5(-1) - 3 = -5 -3

= -8RHS3t - 5 = 3(-1) - 5

= -3 - 5

= -8

As LHS =RHS

The value t=-1 is correct.

6. Question

If $2y+\frac{5}{3}=\frac{26}{3}-y,$ then y = ?

A. 1

B. $\frac{2}{3}$

C. $\frac{6}{5}$

D. $\frac{7}{3}$

Answer

$$2y+\frac{5}{3}=\frac{26}{3}-y,$$

By cross multiplication,

$2y + y = \frac{26-5}{3}$

⇒ 3y = 7

$\Rightarrow y = \frac{7}{3}$

7. Question

If $\frac{6x+1}{3}+1=\frac{x-3}{6}$ then x = ?

A. 1

B. - 1

C. 3

D. - 3

Answer

$$\frac{6x+1}{3}+1=\frac{x-3}{6}$$

Taking LCM of 1 and 3 = 3,

$\frac{6x+1+3}{3} = \frac{x-3}{6}$

⇒ 2(6x + 4) = (x - 3)

⇒ 12x - x = - 3 - 8

⇒ x = - 1

8. Question

If $\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$, then n = ?

A. 30

B. 42

C. 36

D. 28

Answer

$$\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$$

Taking LCM of 2, 4, 6 = 12

$$\frac{6n-9n\ +\ 10n}{12}=21$$

⇒ 7n = 21 × 12

⇒ n = 36

9. Question

if $\frac{x+1}{2x+3}=\frac{3}{8}$, then x = ?

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

Answer

$$\frac{x+1}{2x+3}=\frac{3}{8}$$

By cross multiplication, 8(x + 1) = 3(2x + 3)

⇒ 8x - 6x = 9 - 8

⇒ 2x = 1

$$\Rightarrow x=\frac{1}{2}$$

10. Question

If $\frac{4x+8}{5x+8}=\frac{5}{6}$ then x = ?

A. 4

B. 6

C. 8

D. 12

Answer

$$\frac{4x+8}{5x+8}=\frac{5}{6}$$

By cross multiplication,

6(4x + 8) = 5(5x + 8)

⇒ 24x - 25x = 40 - 48

⇒ - x = - 8

⇒ x = 8

11. Question

If $\frac{n}{n+15}=\frac{4}{9}$, then n = ?

A. 4

B. 6

C. 8

D. 12

Answer

$$\frac{n}{n+15}=\frac{4}{9}$$

By cross multiplication,

9n = 4(n + 15)

⇒ 5n = 60

⇒ n = 12

12. Question

If 3(t - 3) = 5(2t + 1), then t = ?

A. - 2

B. 2

C. - 3

D. 3

Answer

3(t - 3) = 5(2t + 1)

Opening the brackets,

3t - 9 = 10t + 5⇒ 3t - 10t = 5 + 9⇒ -7t = 14⇒ 7t = - 14

⇒ t = - 2

13. Question

Four - fifths of a number is greater than three - fourths of the number by 4. The number is

A. 12

B. 64

C. 80

D. 102

Answer

$$\frac{4}{5}x-\frac{3}{4}x=4$$

$$\Rightarrow \frac{16x-15x}{20}=4$$

⇒ x = 80

14. Question

The ages of A and B are in the ratio 5 : 7. Four years from now the ratio of their ages will be 3 : 4. The present age of B is

A. 20 years

B. 28 years

C. 15 years

D. 21 years

Answer

Let the ages of A and B be 5x and 7x

$$\Rightarrow \frac{5x + 4}{7x + 4} = \frac{3}{4}$$

By cross multiplication

⇒ 4(5x + 4) = 3(7x + 4)

⇒ 21x - 20x = 16 - 12

⇒ x = 4

Age of B = 7x = 28 years

15. Question

The base of an isosceles triangle is 6 cm and its perimeter is 16 cm. Length of each of the equal sides is

A. 4 cm

B. 5 cm

C. 3 cm

D. 6 cm

Answer

Let the length of equal sides be x cm.

We know that, Perimeter = 16 cm

⇒ x + x + 6 = 16

⇒ 2x = 10

⇒ x = 5cm

16. Question

Sum of three consecutive integers is 51. The middle one is

A. 14

B. 15

C. 16

D. 17

Answer

Let the consecutive integers be x, x + 1 and x + 2

x + x + 1 + x + 2 = 51

⇒ 3x = 51 - 3

$$\Rightarrow x = \frac{48}{3} = 16$$

Middle one = x + 1 = 16 + 1 = 17

17. Question

The sum of two numbers is 95. If one exceeds the other by 15, then the smaller of the two is

A. 40

B. 35

C. 45

D. 55

Answer

Let the numbers be x and 95 - x

⇒ 95 - x - x = 15

By cross multiplication

⇒ - 2x = - 80

⇒ x = 40

So, the numbers are 40 and 95 – 40 = 55

18. Question

Number of boys and girls in a class are in the ratio 7 : 5. The number of boys is 8 more than the number of girls. The total class strength is

A. 56

B. 52

C. 48

D. 36

Answer

Let the number of girls and boys be 5x and 7x respectively

According to the question

$7x = 8 + 5x$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

Boys = $7x = 28$

Girls = $5x = 20$

Total strength = $20 + 28 = 48$

CCE Test Paper-8

1. Question

Subtract $4a^2 + 5b^2 - 6c^2 + 8$ from $2a^2 - 3b^2 - 4c^2 - 5$.

Answer

$(2a^2 - 3b^2 - 4c^2 – 5) – (4a^2 + 5b^2 - 6c^2 + 8)$

$= 2a^2 - 3b^2 - 4c^2 – 5 - 4a^2 - 5b^2 + 6c^2 – 8$

$= - 2a^2 - 8b^2 + 2c^2 - 13$

2. Question

Find each of the following products:

(i) $(4a + 5b) \times (5a - 6b)$ (ii) $(6x^2 - x + 8) \times (x^2 - 3)$

Answer

$(4a + 5b) \times (5a - 6b)$

$= 4a (5a - 6b) + 5b (5a - 6b)$

$= 20a^2 - 24ab + 25ab - 30b^2$

$= 20a^2 + ab - 30b^2$

(ii) $(6x^2 - x + 8) \times (x^2 - 3)$

$(6x^2 - x + 8) \times (x^2 - 3)$

$= x^2 (6x^2 - x + 8) - 3 (6x^2 - x + 8)$

$= 6x^4 - x^3 + 8x^2 - 18 x^2 + 3x – 24$

$6x^4 - x^3 – 10x^2 + 3x - 24$

3. Question

Divide $(5a^3 - 4a^2 + 3a + 18)$ by $(a^2 - 2a + 3)$.

Answer

$(5a^3 - 4a^2 + 3a + 18) = (5a + 6)(a^2 - 2a + 3)$

On dividing

$$\frac{(5a+6)(a^2-2a+3)}{(a^2-2a+3)} = (5a + 6)$$

4. Question

If $\left(x-\frac{1}{x}\right) = 4$, find the value of

(i) $\left(x^2+\frac{1}{x^2}\right)$, (ii) $\left(x^4+\frac{1}{x^4}\right)$.

Answer

(i) $x-\frac{1}{x}=4$

Squaring both the sides,

$$\left(x-\frac{1}{x}\right)^2 = 4^2$$

Using the identity, $(a - b)^2 = a^2 - 2ab + b^2$

$$x^2-2+\frac{1}{x^2}=4^2$$

$$\Rightarrow x^2+\frac{1}{x^2}=16+2=18 \text{ ----------(1)}$$

(ii) Squaring equation (1) using the identities, $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow x^4+2+\frac{1}{x^4}=324$$

$$\Rightarrow x^4+\frac{1}{x^4}=324-2=322$$

5. Question

Evaluate $\{(83)^2 - (17)^2\}$.

Answer

Using the identity: $a^2 - b^2 = (a + b)(a-b)$

$\{(83)^2 - (17)^2\} = (83 - 17)(83 + 17)$

$= 66 \times 100 = 6600$

6. Question

Factorize:

(i) $x^3 - 3x^2 + x - 3$

(ii)$63x^2y^2 - 7$

(iii) $1 - 6x + 9x^2$

(iv) $7x^2 - 19x - 6$

Answer

(i) $x^3 - 3x^2 + x - 3$

By hit and trial method we find that x = 3 is a factor of it

So, on dividing $x^3 - 3x^2 + x - 3$ by $(x^2 + 1)$we get $(x^2 + 1)$

$x^3 - 3x^2 + x - 3 = (x^2 + 1)(x - 3)$

(ii) $63x^2y^2 - 7$

$= 7(9x^2y^2 - 1) = 7(3xy - 1)(3xy + 1)$ { Using the identity : $a^2 - b^2 = (a + b)(a - b)$}

(iii) $1 - 6x + 9x^2$

Using the identity : $a^2 + b^2 - 2ab = (a - b)^2$

$1 - 6x + 9x^2 = (3x - 1)^2$

(iv) $7x^2 - 19x - 6$

Using middle term splitting, we get

$7x^2 - (21 - 2)x - 6 = 7x^2 - 21x + 2x - 6 = 7x(x - 3) + 2(x - 3) = (7x + 2)(x - 3)$

7. Question

Solve:

$$\frac{2x+7}{3x+5} = \frac{15}{17}$$

Answer

$$\frac{2x+7}{3x+5} = \frac{15}{17}$$

By cross multiplication, 17(2x + 7) = 15(3x + 5)

⇒ (34x + 119) = 45x + 75

⇒ 11x = 44

⇒ x = 4

8. Question

5 years ago a man was 7 times as old as his son. After 5 years he will be thrice as old as his son. Find their present ages.

Answer

Let the age of son be $\frac{1}{7}$x years, 5 years ago and that of father be x years

According to the question

$$x + 10 = 3\left(\frac{1}{7}x + 10\right)$$

$$\text{Tking LCM of 1 and 7} = 7$$

$$\Rightarrow \frac{7x - 3x}{7} = 30 - 10$$

⇒ 4x = 140

⇒ x = 35

So the present age of father = 35 + 5 = 40 years and that of son is $\frac{1}{7}$x + 5 = 5 + 5 = 10 years

9. Question

ab - a - b + 1 = ?

A. (1 - a)(1 - b)

B. (1 - a)(b - 1)

C. (a - 1)(b - 1)

D. (a - 1)(1 - b.)

Answer

ab - a - b + 1Taking 'a' as common from first two terms of the above polynomial.

= a(b - 1) - (b - 1)

Taking (b - 1) as common, in the above equation

= (b - 1)(a - 1)= (a - 1)(b - 1)

10. Question

$3 + 23x - 8x^2$ = ?

A. (1 - 8x)(3 + x)

B. (1 + 8x)(3 - x)

C. (1 - 8x)(3 - x)

D. none of these

Answer

$3 + 23x - 8x^2$

By using Splitting the middle term

$= 3 + 23x - 8\ x^2$

$= 3 + (24 - 1)x - 8\ x^2$

$= 3(1 + 8x) - x(1 + 8x)$

$= (1 + 8x)(3 - x)$

11. Question

$7x^2 - 19x - 6 = ?$

A. (x - 3)(7x + 2)

B. (x + 3)(7x - 2)

C. (x - 3) (7x - 2)

D. (7x - 3)(x + 2)

Answer

$7x^2 - 19x - 6$

By using splitting the middle term

$= 7\ x^2 - 19x - 6$

$= 7x^2 + (- 21 + 2)x - 6$

$= 7x(x - 3) + 2(x - 3)$

$= (x - 3)(7x + 2)$

12. Question

$12x^2 + 60x + 75 = ?$

A. (2x + 5)(6x + 5)

B. $(3x + 5)^2$

C. $3(2x + 5)^2$

D. none of these

Answer

$12x^2 + 60x + 75$

By using Splitting the middle term

$12\ x^2 + 60x + 75$

$= 3(4\ x^2 + (10 + 10)x + 25)$

$= 3(2x(2x + 5) + 5(2x + 5))$

$= 3(2x + 5)(2x + 5)$

13. Question

$10p^2 + 11p + 3 = ?$

A. $(2p + 3)(5p + 1)$

B. $(5p + 3)(2p + 1)$

C. $(5p - 3)(2p - 1)$

D. none of these

Answer

$10p^2 + 11p + 3$

By using Splitting the middle term

$10\ p^2 + 11p + 3$

$= 10\ p^2 + (5 + 6)p + 3$

$= 5p(2p + 1) + 3(2p + 1)$

$= (2p + 1)(5p + 3)$

14. Question

$8x^3 - 2x = ?$

A. $(4x - 1)(2x - 1)x$

B. $(2x^2 + 1)(2x - 1)$

C. $2x(2x - 1)(2x + 1)$

D. none of these

Answer

$8x^3 - 2x$

Using the identity: $a^2 - b^2 = (a + b)(a - b)$

$8\ x^3 - 2x$

= 2x(4 x^2 - 1)

= 2x(2x - 1)(2x - 1)

15. Question

$\frac{x+5}{2}+\frac{x-5}{3}=\frac{25}{6}$ gives

A. x = 3

B. x = 4

C. x = 5

D. x = 2

Answer

$\frac{x+5}{2}+\frac{x-5}{3}=\frac{25}{6}$

Taking LCM of 2 and 3 = 6

$\frac{3(x+5)+2(x-5)}{6}=\frac{25}{6}$

⇒ 5x + 5 = 25

⇒ x = 4

16. Question

Fill in the blanks.

(i) x^2 - 18 x + 81 = (…)

(ii) 4 - 36x^2 = (…)(...)

(iii)x^2 - 14x + 13 = (…)(…)

(iv) 9z^2 - x^2 - 4y^2 + 4xy = (…)(…)

(v) abc - ab - c + 1 = (…)(…)

Answer

Using the identity : $a^2 - b^2$ = (a + b)(a-b)

(i) x^2 - 18x + 81 = x^2 - (9x) + 81 = (x - 9)(x - 9) = $(x - 9)^2$

(ii) (4 - 36 x^2) = 4(1 - 9x^2) = 4(1 - 3x)(1 + 3x)

(iii) x^2 - 14x + 13 = x^(2)–(13 + 1)x + 13 = x(x - 13) - 1(x - 13) = (x - 13)(x - 1)

(iv) $9z^2 - x^2 - 4y^2 + 4xy = 9z^2 - (x - 2y)^2 = (3z - x + 2y)(3z + x - 2y)$

(v)abc - ab - c + 1 = ab(c - 1) - (c - 1) = (ab - 1)(c - 1)

17. Question

Write 'T' for true and 'F' for false for each of the following:

(i) $(5 - 3x^2)$ is a binomial.

(ii) - 8 is a monomial.

(iii) (5a - 9b) - (- 6a + 2b) = (- a - 7b).

(iv) When x = 2 and y = 1, the value of $\frac{-8}{7}x^3y^4$ is $\frac{-64}{7}$

(v) $\frac{x}{4}+\frac{x}{6}-\frac{x}{2}=\frac{3}{4}\Rightarrow x=-9$

(vi) $2x - 5 = 0 \Rightarrow x = \frac{2}{5}$

Answer

(i) True

It has two terms so binomial.

(ii) True

It has single term so monomial.

(iii) False

(5a - 9b) - (- 6a + 2b) = 5a + 6a - 9b - 2b = 11a - 11b

(iv) True

$\frac{-8}{7}x^3y^4 = \frac{-8}{7} \times 8 \times 1 = \frac{-64}{7}$

(v) True

Taking the LCM of 4,6 and 2 = 12

$$\frac{3x + 2x - 6x}{12} = \frac{3}{4}$$

$$\Rightarrow -4x = 36$$

$$\Rightarrow x = -9$$

(vi) False

2x - 5 = 0

$\Rightarrow x = \frac{5}{2}$