

Ratio Proportion and Unitary Method

Ex 10A

Ratio and Proportion

- A **ratio** is a comparison of two values expressed as a quotient
 - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
 - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A **proportion** is an equation stating that two ratios are equal.
 - Example: $\frac{12}{18} = \frac{2}{3}$

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the first term or antecedent and b , the second term or consequent.

Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b .

Mean Proportional:

Mean proportional between a and b is \sqrt{ab} .

4. Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$

Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. Variations:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y , if $xy = k$ for some constant k and

we write, $x \propto \frac{1}{y}$

Properties of proportions:**Convertendo:** If $a : b :: c : d$, then $a : (a - b) :: c : (c - d)$.**Invertendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$.**Alternendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$.**Componendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$.**Dividendo:** $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ **Componendo and Dividendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.

Q1

Answer :

(i) $24:56 = \frac{24}{56} = \frac{24 \div 8}{56 \div 8} = \frac{3}{7}$

As the H.C.F. of 3 and 7 is 1, the simplest form of 24:56 is 3:7.

(ii) 84 paise to Rs 3 = Rs 0.84 to R. 3 = $\frac{0.84}{3} = \frac{0.84 \div 3}{3 \div 3} = \frac{0.28}{1} = \frac{28}{100} = \frac{28 \div 4}{100 \div 4} = \frac{7}{25}$

As the H.C.F. of 7 and 25 is 1, the simplest form of 0.84:3 is 7:25.

(iii) 4 kg:750 g = 4000 g:750 g = $\frac{4000 \div 250}{750 \div 250} = \frac{16}{3}$

As the H.C.F. of 16 and 3 is 1, the simplest form of 4000:750 is 16:3.

(iv) 1.8 kg:6 kg = $\frac{1.8}{6} = \frac{18}{60} = \frac{18 \div 6}{60 \div 6} = \frac{3}{10}$

As the H.C.F. of 3 and 10 is 1, the simplest form of 1.8:6 is 3:10.

(v) 48 minutes to 1 hour = 48 minutes to 60 minutes = $\frac{48}{60} = \frac{48 \div 12}{60 \div 12} = \frac{4}{5}$

As the H.C.F. of 4 and 5 is 1, the simplest form of 48:60 is 4:5.

(vi) 2.4 km to 900 m = 2400m:900m = $\frac{2400}{900} = \frac{24}{9} = \frac{24 \div 3}{9 \div 3} = \frac{8}{3}$

As the H.C.F. of 8 and 3 is 1, the simplest form of 2400:900 is 8:3.

Q2

Answer :

(i) 36:90 = $\frac{36}{90} = \frac{36 \div 18}{90 \div 18} = \frac{2}{5}$ (As the H.C.F. of 36 and 90 is 18.)

Since the H.C.F. of 2 and 5 is 1, the simplest form of 36:90 is 2:5.

(ii) 324:144 = $\frac{324}{144} = \frac{324 \div 36}{144 \div 36} = \frac{9}{4}$ (As the H.C.F. of 324 and 144 is 36.)

Since the H.C.F. of 9 and 4 is 1, the simplest form of 324:144 is 9:4.

(iii) 85:561 = $\frac{85}{561} = \frac{85 \div 17}{561 \div 17} = \frac{5}{33}$ (As the H.C.F. of 85 and 561 is 17.)

Since the H.C.F. of 5 and 33 is 1, the simplest form of 85:561 is 5:33.

(iv) 480:384 = $\frac{480}{384} = \frac{480 \div 96}{384 \div 96} = \frac{5}{4}$ (As the H.C.F. of 480 and 384 is 96.)

Since the H.C.F. of 5 and 4 is 1, the simplest form of 480:384 is 5:4.

(v) 186:403 = $\frac{186}{403} = \frac{186 \div 31}{403 \div 31} = \frac{6}{13}$ (As the H.C.F. of 186 and 403 is 31.)

Since the H.C.F. of 6 and 13 is 1, the simplest form of 186:403 is 6:13.

(vi) 777:1147 = $\frac{777}{1147} = \frac{777 \div 37}{1147 \div 37} = \frac{21}{31}$ (As the H.C.F. of 777 and 1147 is 37.)

Since the H.C.F. of 21 and 31 is 1, the simplest form of 777:1147 is 21:31.

Q3

Answer :

(i) Rs 6.30:Rs 16.80

$$\frac{6.30}{16.80} = \frac{63}{168} = \frac{63 \div 21}{168 \div 21} = \frac{3}{8} \quad (\text{H.C.F. of 63 and 168 is 21.})$$

Ratio = 3 : 8

(ii) 3 weeks:30 days = 21days:30 days (1 week = 7 days)

$$\frac{21}{30} = \frac{21 \div 3}{30 \div 3} = \frac{7}{10} \quad (\text{H.C.F. of 21 and 30 is 3.})$$

Ratio = 7 : 10

(iii) 3 m 5 cm:35 cm = 305 cm:35 cm (1 m = 100 cm)

$$\frac{305}{35} = \frac{305 \div 5}{35 \div 5} = \frac{61}{7} \quad (\text{H.C.F. of 305 and 35 is 5.})$$

Ratio = 61:7

(iv) 48 min:2 hours 40 min = 48 min:160 min (1 hour = 60 mins)

$$\frac{48}{160} = \frac{48 \div 16}{160 \div 16} = \frac{3}{10} \quad (\text{H.C.F. of 48 and 160 is 16.})$$

Ratio = 3:10

(v) 1 L 35 mL:270 mL = 1035 mL:270 mL (1 L = 1000 mL)

$$\frac{1035}{270} = \frac{1035 \div 45}{270 \div 45} = \frac{23}{6} \quad (\text{H.C.F. of 1035 and 270 is 45.})$$

Ratio = 23:6

(vi) 4 kg:2 kg 500 g = 4000 g:2500 g (1 kg= 1000 g)

$$\frac{4000}{2500} = \frac{40}{25} = \frac{40 \div 5}{25 \div 5} = \frac{8}{5} \quad (\text{H.C.F. of 40 and 25 is 5.})$$

Ratio = 8:5

Q4

Answer :

Mr Sahai's earning = Rs 16800

Mrs Sahai's earning = Rs 10500

$$(i) \text{ Ratio} = 16800:10500 = 168:105 = \frac{168 \div 21}{105 \div 21} = \frac{8}{5} \quad (\text{H.C.F. of 168 and 105 is 21.})$$

Mr Sahai's income:Mrs Sahai's income = 8:5

$$(ii) \text{ Ratio} = 10500:16800 = 105:168 = \frac{105 \div 21}{168 \div 21} = \frac{5}{8} \quad (\text{H.C.F. of 168 and 105 is 21.})$$

Mrs Sahai's income:Mr Sahai's income = 5:8

(iii) Total income = 16800 + 10500 = Rs 27300

$$\text{Ratio} = 16800:27300 = 168:273 = \frac{168}{273} = \frac{168 \div 21}{273 \div 21} = \frac{8}{13} \quad (\text{H.C.F. of 168 and 273 is 21.})$$

Mrs Sahai's income:Total income = 8:13

Q5

Answer :

Rohit's income = Rs 15300

Rohit's savings = Rs 1224

$$(i) \text{ Income:Savings} = 15300:1224 = \frac{15300 \div 612}{1224 \div 612} = \frac{25}{2} \quad (\text{H.C.F. of 15300 and 1224 is 612.})$$

Income:Savings = 25:2

(ii) Monthly expenditure = Rs (15300 - 1224) = Rs 14076

$$\text{Income:Expenditure} = 15300:14076 = \frac{15300 \div 612}{14076 \div 612} = \frac{25}{23} \quad (\text{H.C.F. of 15300 and 14076 is 612.})$$

Income:Expenditure = 25:23

$$(iii) \text{ Expenditure : Savings} = 14076:1224 = \frac{14076 \div 612}{1224 \div 612} = \frac{23}{2} \quad (\text{H.C.F. of 14076 and 1224 is 612.})$$

Expenditure:Savings = 23:2

Q6

Answer :

Number of male: Number of female = 5:3

Let the number be x.

Number of male = 5x

Number of female = 3x

Number of male workers = 115

$$\text{Now, } 5x = 115$$

$$\Rightarrow x = \frac{115}{5} = 23$$

Number of female workers in the mill = 3x = 3 × 23 = 69

Q7

Answer :

Boys:Girls = 9:5

Let the number of boys = 9x

Let the number of girls = 5x

Total strength of the school = 448

According to given condition, we have:

$$\begin{aligned} 9x + 5x &= 448 \\ \Rightarrow 14x &= 448 \\ \Rightarrow x &= \frac{448}{14} = 32 \end{aligned}$$

Number of boys = 9x = 9 × 32 = 288

Number of girls = 5x = 5 × 32 = 160

Q8

Answer :

$$\text{Kamal:Madhu} = 7:2$$

$$\text{Sum of the ratio terms} = 7 + 2 = 9$$

$$\text{Kamal's share} = \frac{7}{9} \times 1575 = \frac{11025}{9} = \text{Rs } 1225$$

$$\text{Madhu's share} = \frac{2}{9} \times 1575 = \frac{3150}{9} = \text{Rs } 350$$

Q9

Answer :

$$\text{A:B:C} = 3:5:7$$

$$\text{Sum of the ratio terms} = 3 + 5 + 7 = 15$$

$$\text{A's share} = \frac{3}{15} \times 3450 = \frac{10350}{15} = \text{Rs } 690$$

$$\text{B's share} = \frac{5}{15} \times 3450 = \frac{17250}{15} = \text{Rs } 1150$$

$$\text{C's share} = \frac{7}{15} \times 3450 = \frac{24150}{15} = \text{Rs } 1610$$

Q10

Answer :

Two number are in the ratio 11:12.

Let the numbers be $11x$ and $12x$.

$$\text{Given: } 11x + 12x = 460$$

$$\Rightarrow 23x = 460$$

$$\Rightarrow x = \frac{460}{23} = 20$$

$$\text{First number} = 11x = 11 \times 20 = 220$$

$$\text{Second number} = 12x = 12 \times 20 = 240$$

Hence, the numbers are 220 and 240.

Q11

Answer :

Ratio of the two parts of line segment = 4:3

$$\text{Sum of the ratio terms} = 4 + 3 = 7$$

$$\text{First part} = \frac{4}{7} \times 35 \text{ cm} = 4 \times 5 \text{ cm} = 20 \text{ cm}$$

$$\text{Second part} = \frac{3}{7} \times 35 \text{ cm} = 3 \times 5 \text{ cm} = 15 \text{ cm}$$

Q12

Answer :

Number of bulbs produced each day = 630

Out of 10 bulbs, 1 is defective.

$$\text{Number of defective bulbs} = \frac{630}{10} = 63$$

\therefore Number of defective bulbs produced each day = 63

Q13

Answer :

Price of pencil = Rs 96 per score

Price of ball pen = Rs 50.40 per dozen

Price per unit of pencil = $\frac{96}{20} = 4.8$

Price per unit of ball pen = $\frac{50.40}{12} = 4.2$

Ratio = $\frac{4.8}{4.2} = \frac{48}{42} = \frac{48 \div 6}{42 \div 6} = \frac{8}{7}$

Price of a pencil:Price of a ball pen = 8:7

Q14

Answer :

Length:Width = 5:3

Let the length and the width of the field be 5x m and 3x m, respectively.

Width = 42 m

$3x = 42$

$x = \frac{42}{3} = 14$

\therefore Length = $5x = 5 \times 14 = 70$ metres

Q15

Answer :

Income:Savings = 11:2

Let the income and the saving be Rs 11x and Rs 2x, respectively.

Saving = Rs 1520

$2x = 1520$

$x = \frac{1520}{2} = 760$

\therefore Income = Rs 11x = Rs (11 \times 760) = Rs 8360

Expenditure = Income – Saving
= Rs (8360 – 1520)
= Rs 6840

Q16

Answer :

Income:Expenditure = 7:6

Let the income and the expenditure be Rs 7x and Rs 6x, respectively.

Income = Rs 14000

$7x = 14000$

$x = \frac{14000}{7} = 2000$

Expenditure = Rs 6x = Rs 6 \times 2000 = Rs 12000

\therefore Saving = Income – Expenditure
= Rs (14000 – 12000)
= Rs 2000

Q17

Answer :

Let the weight of zinc be x kg.

Ratio of zinc and copper = 7:9

Weight of copper in the alloy = 11.7 kg

$\frac{7}{9} = \frac{x}{11.7}$

$\Rightarrow x = \frac{11.7 \times 7}{9} = \frac{81.9}{9} = 9.1$

Weight of zinc = 9.1 kg

Q18

Answer :

A bus covers 128 km in 2 hours.

$$\text{Speed of the bus} = \frac{\text{Distance}}{\text{Time}} = \frac{128 \text{ km}}{2 \text{ hr}} = 64 \text{ km/hr}$$

A train covers 240 km in 3 hours.

$$\text{Speed of the train} = \frac{\text{Distance}}{\text{Time}} = \frac{240}{3} = 80 \text{ km/hr}$$

$$\text{Ratio of their speeds} = 64:80 = \frac{64}{80} = \frac{64 \div 16}{80 \div 16} = \frac{4}{5}$$

∴ Ratio of the speeds of the bus and the train = 4:5

Q19

Answer :

(i) (3:4) or (9:16)

Making the denominator equal:

$$\frac{3 \times 4}{4 \times 4} = \frac{12}{16} \text{ and } \frac{12}{16} > \frac{9}{16}$$

∴ (3:4) > (9:16)

(ii) (5:12) or (17:30)

Making the denominator equal:

$$\frac{5 \times 5}{12 \times 5} = \frac{25}{60} \text{ and } \frac{17 \times 2}{30 \times 2} = \frac{34}{60}$$
$$\Rightarrow \frac{25}{60} < \frac{34}{60}$$

∴ (5:12) < (17:30)

(iii) (3:7) or (4:9)

Making the denominator equal:

$$\frac{3 \times 9}{7 \times 9} = \frac{27}{63} \text{ and } \frac{4 \times 7}{9 \times 7} = \frac{28}{63}$$
$$\Rightarrow \frac{27}{63} < \frac{28}{63}$$

(3:7) < (4:9)

(iv) (1:2) or (13:27)

Making the denominator equal:

$$\frac{1 \times 27}{2 \times 27} = \frac{27}{54} \text{ and } \frac{13 \times 2}{27 \times 2} = \frac{26}{54}$$
$$\Rightarrow \frac{27}{54} > \frac{26}{54}$$

(1:2) > (13:27)

Q20 **Answer :**

$$(i) \frac{24}{40} = \frac{24 \div 8}{40 \div 8} = \frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

$$(ii) \frac{36}{63} = \frac{36 \div 9}{63 \div 9} = \frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

$$(iii) \frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28} = \frac{5 \times 7}{7 \times 7} = \frac{35}{49}$$

Ratio Proportion and Unitary Method

Ex 10B

Q1

Answer :

(i) 4, 6, 8, 12

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}; \quad \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

Hence, 4:9::8:12 are in proportion.

(ii) 7, 42, 13, 78

$$\frac{7}{42} = \frac{7 \div 7}{42 \div 7} = \frac{1}{6}; \quad \frac{13}{78} = \frac{13 \div 13}{78 \div 13} = \frac{1}{6}$$

Hence, 7:42::13:78 are in proportion.

(iii) 33, 121, 9, 96

$$\frac{33}{121} = \frac{33 \div 11}{121 \div 11} = \frac{3}{11}; \quad \frac{9}{96} = \frac{9 \div 3}{96 \div 3} = \frac{3}{32}$$

Hence, 33:121::9:96 are not in proportion.

(iv) 22, 33, 42, 63

$$\frac{22}{33} = \frac{22 \div 11}{33 \div 11} = \frac{2}{3} \quad \text{and} \quad \frac{42}{63} = \frac{42 \div 21}{63 \div 21} = \frac{2}{3}$$

Hence, 22:33 :: 42 : 63 are not in proportion.

(v) 32, 48, 70, 210

$$\frac{32}{48} = \frac{32 \div 16}{48 \div 16} = \frac{2}{3}; \quad \frac{70}{210} = \frac{70 \div 70}{210 \div 70} = \frac{1}{3}$$

Hence, 32:48::70:210 are not in proportion.

(vi) 150, 200, 250, 300

$$\frac{150}{200} = \frac{150 \div 50}{200 \div 50} = \frac{3}{4}; \quad \frac{250}{300} = \frac{250 \div 50}{300 \div 50} = \frac{5}{6}$$

Hence, 150:200::250:300 are not in proportion.

Q2

Answer :

(i) 60:105::84:147

$$\frac{60}{105} = \frac{60 \div 15}{105 \div 15} = \frac{4}{7} \quad (\text{H.C.F. of 60 and 105 is 15.})$$

$$\frac{84}{147} = \frac{84 \div 21}{147 \div 21} = \frac{4}{7} \quad (\text{H.C.F. of 84 and 147 is 21.})$$

Hence, 60:105::84:147 are in proportion.

(ii) 91:104::119:136

$$\frac{91}{104} = \frac{91 \div 13}{104 \div 13} = \frac{7}{8} \quad (\text{H.C.F. of 91 and 104 is 13.})$$

$$\frac{119}{136} = \frac{119 \div 17}{136 \div 17} = \frac{7}{8} \quad (\text{H.C.F. of 119 and 136 is 17.})$$

Hence, 91:104::119:136 are in proportion.

(iii) 108:72::129:86

$$\frac{108}{72} = \frac{108 \div 36}{72 \div 36} = \frac{3}{2} \quad (\text{H.C.F. of 108 and 72 is 36.})$$

$$\frac{129}{86} = \frac{129 \div 43}{86 \div 43} = \frac{3}{2} \quad (\text{H.C.F. of 129 and 86 is 43.})$$

Hence, 108:72::129:86 are in proportion.

(iv) 39:65::141:235

$$\frac{39}{65} = \frac{39 \div 13}{65 \div 13} = \frac{3}{5} \quad (\text{H.C.F. of 39 and 65 is 13.})$$

$$\frac{141}{235} = \frac{141 \div 47}{235 \div 47} = \frac{3}{5} \quad (\text{H.C.F. of 141 and 235 is 47.})$$

Hence, 39:65::141:235 are in proportion.

Q3

Answer :

(i) 55:11::x:6

Product of extremes = Product of means

$$55 \times 6 = 11 \times x$$

$$\Rightarrow 11x = 330$$

$$\Rightarrow x = \frac{330}{11} = 30$$

(ii) 27:x::63:84

Product of extremes = Product of means

$$27 \times 84 = x \times 63$$

$$\Rightarrow 63x = 2268$$

$$\Rightarrow x = \frac{2268}{63} = 36$$

(iii) 51:85::57:x

Product of extremes = Product of means

$$51 \times x = 85 \times 57$$

$$\Rightarrow 51x = 4845$$

$$\Rightarrow x = \frac{4845}{51} = 95$$

(iv) x:92::87:116

Product of extremes = Product of means

$$x \times 116 = 92 \times 87$$

$$\Rightarrow 116x = 8004$$

$$\Rightarrow x = \frac{8004}{116} = 69$$

Q4

Answer :

(i) 51:68::85:102

Product of means = $68 \times 85 = 5780$

Product of extremes = $51 \times 102 = 5202$

Product of means \neq Product of extremes

Hence, (F).

(ii) 36:45::80:100

Product of means = $45 \times 80 = 3600$

Product of extremes = $36 \times 100 = 3600$

Product of means = Product of extremes

Hence, (T).

(iii) 30 bags:18 bags::Rs 450:Rs 270

or 30:18::450:270

Product of means = $18 \times 450 = 8100$

Product of extremes = $30 \times 270 = 8100$

Product of means = Product of extremes

Hence, (T).

(iv) 81 kg:45 kg::18 men:10 men

or 81:45::18:10

Product of means = $45 \times 18 = 810$

Product of extremes = $81 \times 10 = 810$

Product of means = Product of extremes

Hence, (T).

(v) 45 km:60 km::12 h:15 h

or,45:60::12:15

Product of means = $60 \times 12 = 720$

Product of extremes = $45 \times 15 = 675$

Product of means \neq Product of extremes

Hence, (F).

(vi) 32 kg:Rs 36::8 kg:Rs 9

Product of means = $36 \times 8 = 288$

Product of extremes = $32 \times 9 = 288$

Product of means = Product of extremes

Hence, (T).

Q5

Answer :

(i) 25 cm:1 m and Rs 40:Rs 160 (or) 25 cm:100 cm and Rs 40:Rs 160

$$\frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4} \text{ and } \frac{40}{160} = \frac{40 \div 40}{160 \div 40} = \frac{1}{4}$$

Hence, they are in proportion.

(ii) 39 litres:65 litres and 6 bottles:10 bottles

$$\frac{39}{65} = \frac{39 \div 13}{65 \div 13} = \frac{3}{5} \text{ and } \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Hence they are in proportion.

(iii) 200 mL:2.5 L and Rs 4:Rs 50 (or) 200 mL:2500 mL and Rs 4:Rs 50

$$\frac{200}{2500} = \frac{2}{25} \text{ and } \frac{4}{50} = \frac{4 \div 2}{50 \div 2} = \frac{2}{25}$$

Hence, they are in proportion.

(iv) 2 kg:80 kg and 25 g:625 kg (or) 2 kg:80 kg and 25 g:625000 g

$$\frac{2}{80} = \frac{2 \div 2}{80 \div 2} = \frac{1}{40} \text{ and } \frac{25}{625000} = \frac{25 \div 25}{625000 \div 25} = \frac{1}{25000}$$

Hence, they are not in proportion.

Q6

Answer :

Let the 3rd term be x.

Thus, 51:68::x:108

We know:

Product of extremes = Product of means

$$51 \times 108 = 68 \times x$$

$$\Rightarrow 5508 = 68x$$

$$\Rightarrow x = \frac{5508}{68} = 81$$

Hence, the third term is 81.

Q7

Answer :

Let the second term be x.

Then, 12:x::8:14

We know:

Product of extremes = Product of means

$$12 \times 14 = 8x$$

$$\Rightarrow 168 = 8x$$

$$\Rightarrow x = \frac{168}{8} = 21$$

Hence, the second term is 21.

Q8

Answer :

(i) 48:60, 60:75

$$\text{Product of means} = 60 \times 60 = 3600$$

$$\text{Product of extremes} = 48 \times 75 = 3600$$

Product of means = Product of extremes

Hence, 48:60::60:75 are in continued proportion.

(ii) 36:90, 90:225

$$\text{Product of means} = 90 \times 90 = 8100$$

$$\text{Product of extremes} = 36 \times 225 = 8100$$

Product of means = Product of extremes

Hence, 36:90::90:225 are in continued proportion.

(iii) 16:84, 84:441

$$\text{Product of means} = 84 \times 84 = 7056$$

$$\text{Product of extremes} = 16 \times 441 = 7056$$

Product of means = Product of extremes

Hence, 16:84::84:441 are in continued proportion.

Q9

Answer :

Given: 9:x::x:49

We know:

Product of means = Product of extremes

$$x \times x = 9 \times 49$$

$$\Rightarrow x^2 = 441$$

$$\Rightarrow x^2 = (21)^2$$

$$\Rightarrow x = 21$$

Q10

Answer :

Let the height of the pole = x m

Then, we have:

$$x:20::6:8$$

Now, we know:

Product of extremes = Product of means

$$8x = 20 \times 6$$

$$x = \frac{120}{8} = 15$$

Hence, the height of the pole is 15 m.

Q11

Answer :

5:3::x:6

We know:

Product of means = Product of extremes

$$3x = 5 \times 6$$

$$\Rightarrow x = \frac{30}{3} = 10$$

$\therefore x = 10$

Ratio Proportion and Unitary Method

Ex 10C

Q1

Answer :

Cost of 14 m of cloth = Rs 1890

Cost of 1 m of cloth = $\frac{1890}{14}$ = Rs 135

Cost of 6 m of cloth = 6×135 = Rs 810

Q2

Answer :

Cost of dozen soaps = Rs 285.60

Cost of 1 soap = $\frac{285.60}{12}$

Cost of 15 soaps = $15 \times \frac{285.60}{12} = \frac{4284}{12}$ = Rs 357

Q3

Answer :

Cost of 9 kg of rice = Rs 327.60

Cost of 1 kg of rice = $\frac{327.60}{9}$

Cost of 50 kg of rice = $50 \times \frac{327.60}{9} = \frac{16380}{9}$ = Rs 1820

Hence, the cost of 50 kg of rice is Rs 1820.

Q4

Answer :

Weight of 22.5 m of uniform iron rod = 85.5 kg

Weight of 1 m of uniform iron rod = $\frac{85.5}{22.5}$ kg

Weight of 5 m of uniform iron rod = $5 \times \frac{85.5}{22.5} = \frac{427.5}{22.5} = 19$ kg

Thus, the weight of 5 m of iron rod is 19 kg.

Q5

Answer :

Oil contained by 15 tins = 234 kg

Oil contained by 1 tin = $\frac{234}{15}$ kg

Oil contained by 10 tins = $10 \times \frac{234}{15} = \frac{2340}{15} = 156$ kg

Q6

Answer :

Distance covered by a car in 12 L diesel = 222 km

Distance covered by it in 1 L diesel = $\frac{222}{12}$ km

Distance covered by it in 22 L diesel = $22 \times \frac{222}{12} = \frac{4884}{12} = 407$ km

Q7

Answer :

Cost of transporting 25 tonnes of weight = Rs 540

Cost of transporting 1 tone of weight = $\frac{540}{25}$

Cost of transporting 35 tonnes of weight = $35 \times \frac{540}{25} = \frac{18900}{25} =$ Rs 756

Q8

Answer :

Let the weight of copper be x g.

Then, $4.5:3.5::18.9:x$

Product of extremes = Product of means

$$4.5 \times x = 3.5 \times 18.9$$

$$\Rightarrow x = \frac{66.15}{4.5} = 14.7$$

So, the weight of copper is 14.7 g.

Q9

Answer :

Number of inland letters whose total cost is Rs 87.50 = 35

Number of inland letters of whose cost is Re 1 = $\frac{35}{87.50}$

Number of inland letters whose cost is Rs 315 = $315 \times \frac{35}{87.50} = \frac{11025}{87.50} = 126$

Hence, we can buy 126 inland letters for Rs 315.

Q10

Answer :

Number of bananas that can be purchased for Rs 104 = 48 (4 dozen)

Number of bananas that can be purchased for Re 1 = $\frac{48}{104}$

Number of bananas that can be purchased for Rs 6.50 = $6.50 \times \frac{48}{104} = \frac{312}{104} = 3$

Hence, 3 bananas can be purchased for Rs 6.50.

Q11

Answer :

Number of chairs that can be bought for Rs 22770 = 18

Number of chairs that can be bought for Re 1 = $\frac{18}{22770}$

Number of chairs that can be bought for Rs 10120 = $10120 \times \frac{18}{22770} = \frac{182160}{22770} = 8$

Q12

Answer :

(i) Time taken by the car to travel 195 km = 3 hours

Time taken by it to travel 1 km = $\frac{3}{195}$ hours

Time taken by it to travel 520 km = $520 \times \frac{3}{195} = \frac{1560}{195} = 8$ hours

(ii) Distance covered by the car in 3 hours = 195 km

Distance covered by it in 1 hour = $\frac{195}{3} = 65$ km

Distance covered by it in 7 hours = $7 \times 65 = 455$ km

Q13

Answer :

(i) Earning of a labourer in 12 days = Rs 1980

Earning of the labourer in 1 day = $\frac{1980}{12} =$ Rs 165

Earning of the labourer in 7 days = $7 \times 165 =$ Rs 1155

(ii) Number of days taken by the labourer to earn Rs 1980 = 12 days

Number of days taken by him to earn Re 1 = $\frac{12}{1980}$ days

Number of days taken by him to earn Rs 2640 = $2640 \times \frac{12}{1980} = \frac{31680}{1980} = 16$ days

Q14

Answer :

Weight of 65 books = 13 kg

(i) Weight of 1 book = $\frac{13}{65}$ kg

Weight of 80 books = $80 \times \frac{13}{65} = \frac{1040}{65} = 16$ kg

(ii) Number of books weighing 13 kg = 65

Number of books weighing 1 kg = $\frac{65}{13} = 5$

Number of books weighing 6.4 kg = $6.4 \times 5 = 32$

Q15

Answer :

Number of boxes containing 6000 pens = 48

Number of boxes containing 1 pen = $\frac{48}{6000}$

Number of boxes containing 1875 pens = $1875 \times \frac{48}{6000} = \frac{90000}{6000} = 15$

15 boxes are needed for 1875 pens.

Q16

Answer :

Number of days taken by 24 workers to build a wall = 15 days

Number of days taken by 1 worker to build the wall = $15 \times 24 = 360$ days (less worker means more days)

Number of days taken by 9 workers to build the wall = $\frac{360}{9} = 40$ days

Q17

Answer :

Number of men required to complete the work in 26 days = 40

Number of men required to complete the work in 1 day = $40 \times 26 = 1040$ men (less men more days)

Number of men required to complete the work in 16 days = $\frac{1040}{16} = 65$

Q18

Answer :

Number of days the provisions will last for 550 men = 28 days

Number of days the provisions will last for 1 man = $28 \times 550 = 15400$ days (less men means more days)

Number of days the provisions will last for 700 men = $\frac{15400}{700} = 22$ days

The provision will last for 22 days.

Q19

Answer :

Number of days for which the given quantity of rice is sufficient for 60 persons = 3 days

Number of days for which it is sufficient for 1 person = $3 \times 60 = 180$ days (less men means more days)

Number of days for which it is sufficient for 18 persons = $\frac{180}{18} = 10$ days

Ratio Proportion and Unitary Method

Ex 10D

Q1

Answer :

(d) 4 : 5

$$92:115 = \frac{92 \div 23}{115 \div 23} = \frac{4}{5} \text{ (As H.C.F. of 92 and 115 is 23.)}$$

Q2

Answer :

(a) 95

57:x::51:85

$$\frac{57}{x} = \frac{51}{85}$$

$$\Rightarrow x = \frac{57 \times 85}{51}$$

$$\Rightarrow x = \frac{4845}{51} = 95$$

Q3

Answer :

(a) 63

25:35::45:x

$$\frac{25}{35} = \frac{45}{x}$$

$$\Rightarrow x = \frac{35 \times 45}{25} = \frac{1575}{25} = 63$$

Q4

Answer :

(c) 28

$$4:5::x:35$$

$$\Rightarrow \frac{4}{5} = \frac{x}{35}$$

$$\Rightarrow x = \frac{4 \times 35}{5} = 4 \times 7 = 28$$

Q5

Answer :

$$(b) ad = bc$$

Given:

a, b, c, d are in proportion.

$$a:b::c:d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow ad = bc$$

Q6

Answer :

$$(b) b^2 = ac$$

Given:

a, b, c are in proportion.

$$a:b::b:c$$

Product of means = Product of extremes

$$\Rightarrow b^2 = ac$$

Q7

Answer :

$$(b) (5 : 8) < (3 : 4)$$

We can write

$$(5 : 8) = \frac{5}{8} \text{ and } (3 : 4) = \frac{3}{4}$$

Making the denominator equal:

$$\frac{5}{8} \text{ and } \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$\text{As } 6 > 5, \frac{5}{8} < \frac{3}{4}$$

Q8

Answer :

(a) Rs 440

$$A:B = 8:11$$

$$\text{Sum of ratio terms} = 8 + 11 = 19$$

$$\text{B's share} = \frac{11}{19} \times 760 = \frac{8360}{19} = \text{Rs } 440$$

Q9

Answer :

(d) 147

$$\text{Ratio} = 5:7$$

Let x be any number such that we have:

$$5x + 7x = 252$$

$$\Rightarrow 12x = 252$$

$$\Rightarrow x = \frac{252}{12} = 21$$

$$\text{Now, } 5x = 5 \times 21 = 105$$

$$7x = 7 \times 21 = 147$$

The largest number is 147.

Q10

Answer :

(b) 50 cm

The sides of the triangle are in the ratio 1:3:5.

Let x be any number such that the sides are $1x$ cm, $3x$ cm and $5x$ cm.

$$1x + 3x + 5x = 90$$

$$\Rightarrow 9x = 90$$

$$\Rightarrow x = \frac{90}{9} = 10$$

First side = $1x = 1 \times 10 = 10$ cm

Second side = $3x = 3 \times 10 = 30$ cm

Third side = $5x = 5 \times 10 = 50$ cm

The length of the largest side is 50 cm.

Q11

Answer :

(c) 2856

Ratio of boys and girls = 12:5

Let x be any number such that the number of boys and girls are $12x$ and $5x$, respectively.

Number of girls = 840

$$5x = 840$$

$$\Rightarrow x = \frac{840}{5} = 168$$

Number of boys = $12x = 12 \times 168 = 2016$

Number of girls = 840

Total strength of the school = $2016 + 840 = 2856$

Q12

Answer :

(b) Rs 161

Cost of 12 pens = Rs 138

$$\text{Cost of 1 pen} = \text{Rs } \frac{138}{12}$$

$$\text{Cost of 14 pens} = \text{Rs } \frac{138}{12} \times 14 = \text{Rs } \frac{1932}{12} = \text{Rs } 161$$

Q13

Answer :

(b) 45 days

Time taken by 24 workers to build a wall = 15 days

Time taken by 1 worker to build a wall = $24 \times 15 = 360$ days (clearly less workers will take more time to build a wall)

$$\text{Time taken by 8 workers to build a wall} = \frac{360}{8} = 45 \text{ days}$$

Q14

Answer :

(a) 52

Number of men required to finish the work in 26 days = 40

Number of men required to finish it in 1 day = $40 \times 26 = 1040$ men (More men means less days)

$$\text{Number of men required to finish it in 20 days} = \frac{1040}{20} = 52$$

Q15

Answer :

(b) 185 km

Distance covered in 6 L of petrol = 111 km

$$\text{Distance covered in 1 L of petrol} = \frac{111}{6} \text{ km}$$

$$\text{Distance covered in 10 L of petrol} = \frac{111}{6} \times 10 = \frac{1110}{6} = 185 \text{ km}$$

Q16

Answer :

(a) 22 days

Number of days for which 550 men had provisions = 28 days

Number of days for which 1 man had provisions = $28 \times 550 = 15400$ days (more men means less days)

Number of days for which 700 men had provisions = $\frac{15400}{700} = 22$ days

Q17

Answer :

(c) 90°

Ratio of the angles of a triangle is 3:1: 2

Let x be any number such that the three angles are $(3x)^\circ$, $(1x)^\circ$ and $(2x)^\circ$.

We know, the sum of the angles of a triangle is 180° .

$$3x + 1x + 2x = 180$$

$$\Rightarrow 6x = 180$$

$$\Rightarrow x = \frac{180}{6} = 30$$

$$\therefore (3x)^\circ = (3 \times 30)^\circ = 90^\circ$$

$$(1x)^\circ = (1 \times 30)^\circ = 30^\circ$$

$$(2x)^\circ = (2 \times 30)^\circ = 60^\circ$$

The measure of the largest angle is 90° .

Q18

Answer :

(b) 45 m

Length:Breadth = 5:4

Let x be any number such that the length and the breadth are $5x$ and $4x$, respectively.

Now , $4x = 36$

$$x = \frac{36}{4} = 9$$

Length = $5x = 5 \times 9 = 45$ m

Q19

Answer :

(a) 13 : 15

Speed = $\frac{\text{Distance}}{\text{Time}}$

Speed of the bus = $\frac{195 \text{ km}}{3 \text{ hr}} = 65 \text{ km/hr}$

Speed of the train = $\frac{300 \text{ km}}{4 \text{ hr}} = 75 \text{ km/hr}$

Ratio = $\frac{65}{75} = \frac{65 \div 5}{75 \div 5} = \frac{13}{15} = 13:15$

Q20

Answer :

(c) Rs 198

Cost of 5 bars of soap = Rs 82.50

Cost of 1 bar of soap = $\frac{82.50}{5} = \text{Rs } 16.5$

Cost of 12 (1 dozen) bars of soap = $16.5 \times 12 = \text{Rs } 198$

Q21

Answer :

(b) Rs 750

Cost of 30 packets of 8 pencils each = Rs 600

Cost of 1 packet of 8 pencils = $\frac{600}{30}$ = Rs 20

Cost of 1 pencil = Rs $\frac{20}{8}$

Cost of 1 packet of 12 pencils = $12 \times \frac{20}{8} = \frac{240}{8}$ = Rs 30

Cost of 25 packets of 12 pencils each = 25×30 = Rs 750

Q22

Answer :

(a) Rs 344

Cost of rail journey of 75 km = Rs 215

Cost of rail journey of 1 km = Rs $\frac{215}{75}$

Cost of rail journey of 120 km = $120 \times \frac{215}{75} = \frac{25800}{75}$ = Rs 344

Q23

Answer :

(d) 8

Let the third term be x.

Then, we have:

12:21::x:14

We know:

Product of means = Product of extremes

$$21x = 12 \times 14$$

$$\Rightarrow 21x = 168$$

$$\Rightarrow x = \frac{168}{21} = 8$$

The third term is 8

Q24

Answer :

(b) 15 h

Time taken by 10 boys to dig a pitch = 12 hours

Time taken by 1 boy to dig a pitch = $12 \times 10 = 120$ hours (less boys means more time)

Time taken by 8 boys to dig a pitch = $\frac{120}{8} = 15$ hours